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Impact of illegal mining and pollution on the ecosystem : A mathematical model

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Abstract

A deterministic mathematical model is put out to investigate how pollution and mining operations affect valuable plant and animal resources. The level of pollutants in the environment increases steadily and is further exacerbated by rampant mining activities

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occurring in the area. The formulation of the model employs differential equations, which are subsequently analyzed through the principles of stability theory and numerical simulation techniques. The findings indicate that both plant and animal life are negatively impacted, either directly or indirectly, as a result of excessive mining operations and environmental contamination. Our analysis indicates that maintaining ecological balance is crucial for the preservation of flora and fauna. This balance involves the interplay between plant resources, animal populations that depend on them, mining activities, and environmental pollution. It is essential for government involvement in regulatory measures. Controlling the parameters that govern excessive mining activities significantly mitigates other forms of pollution, leading to substantial improvements in the health of flora and fauna.

Subject Classification: *34D20, 49K15.*

Keywords: *Mathematical modeling, Mining activities, Fauna, Flora, Pollutants, Stability.*

1. Introduction

Mathematical modelling improves mining efficiency, minimises environmental effect, and facilitates decision-making through deterministic, stochastic, optimisation, and simulation models. The mining sector plays a crucial role in the global economy by supplying essential metals and minerals, creating employment opportunities for local communities and an international workforce. Additionally, it offers significant advantages for both national and local economies, contributing substantially to the GDP of various countries such as China, Australia, the United States, and Canada. etc. A significant number of lower and middle-income countries rich in minerals have witnessed robust growth for a decade or more, driven by the swift increase in their mineral exports and their values in the global market [21]. In Nigeria mining industry contributes only 0.3% of its GDP as a result of its vast oil resources which was discovered in 1956 in addition many expatiate mining experts left the country because of civil war in early 1960 when the sector has not developed fully. The efficiency and safety of mining operations at all phases, including extraction, processing, and refining, are greatly improved by mathematical modelling. The dynamic analysis of mining vehicles is made possible by sophisticated simulation approaches, such as those that use Lagrange equations and software like Wolfram Mathematica. This optimizes the design and operational strategies of mining vehicles to efficiently handle dynamic loads [20]. Additionally, resource estimation and geomechanical stability are improved through the visualisation and evaluation of coal deposit qualities made possible by geological block modelling, which is made possible by software such as Geovia Surpac and Micromine Origin [25]. Furthermore, prospective mining engineers' professional competency is increased when mathematical methods are incorporated into their education since it helps them apply these models in real-world situations [11]. Notwithstanding developments, the mining sector has not properly utilised mathematical optimisation methods, which could markedly enhance project outcomes and profitability [16]. There is substantial evidence indicating that nations implementing contemporary mining regulations and providing a supportive environment can successfully draw private sector investment in mining exploration and production [9]. Regardless of these enormous benefits of mining activities, it could lead to deforestation and other environmental degradation. The environmental damages include alterations to the ecological state, pollution of air, water, and soil, destruction of soil flora and fauna, loss of vegetation, degradation of landscapes, and radiation emissions, among others [13], [15], [1]. In contemporary society, excessive mining activities have become a significant contributor to air pollution, releasing particulate matter, carbon dioxide, persistent free radicals, volatile organic compounds, and heavy metal particles. These pollutants pose serious health risks, particularly for individuals with heart and lung conditions, including children, leading to disease and mortality [14, 7]. In general air pollution is responsible for over 6.6 million deaths worldwide in 2020 [23]. Numerous evidence have shown that northern part of Edo state, Ogoni land land and Shagamu all in Nigeria have suffered immensely from damage of vegetation and plantation, oil spillage, soil erosion and flooding. Radiation exposure, according to Aliyu and Ramil [2] and [5], causes immunological and physiological changes, point mutations, and an increase in the incidence of disease. In 2010, approximately 200 children under the age of 5 lost their lives to lead poisoning in Zamfara state, Nigeria. Additionally, residents in Liakpe, Kogi state, experience symptoms such as catharsis, dehydration, dry skin, and discoloration of teeth as a result of decades of iron ore exploration in the area [4]. The examination of the dynamics of flora and fauna in relation to factors such as population growth, industrialization, and pollution has been a well-explored area by numerous studies.

Many researchers have used mathematical model to study various life problem, ranging from epidemiology [5], psychiatric disorder [3, 12], finance [10, 17] etc. Modeling was employed to investigate the reduction of forestry biomass in relation to population dynamics and pollution levels [18, 19]. The conservation of forestry biomass was examined, revealing that resources can be sustained at an optimal level through effective forestry conservation and growth management strategies. A ratio dependent model was developed by [13] to explore the interactions between forestry biomass, wildlife populations, and industrialization. The findings indicate that there is an inverse relationship between biomass and wildlife densities and the level of industrialization. Other researchers [19, 13, 22, 24] have worked extensively on conservation of forestry biomass with wonderful results. By reason of inordinate mining carried out by locals and companies mostly in the forest(including but not limited to cash crops, economics trees,shrubs etc) dense area several flora is cleared alongside. As a result the fauna within the ecosystem lose their food, habitats, ecological balance is badly affected. to curtail these excess government's regulatory bodies should embrace the task of inspecting, enforce the mine environmental compliance laws,and other cadastral related issues. In this context, we present a nonlinear mathematical model that examines the effects of governmental initiatives, excessive mining operations, and the subsequent pollutants on the populations of flora and fauna within the mining area.

2. Model Formulation

We consider a mining site situated at a location where the human and other living population within, depend greatly on the flora that grows within its environment. Here, flora stands for all food trees, economic trees and aesthetic trees and fauna is consider as all the animals that live in a particular area. Flora and fauna within the mining site grow logistically [8] with nonlinear interactions, intrinsic growth rate r_1 , r_2 and carrying capacity of the ecosystem K, L respectively. r_0 and s_0 are the modification parameters of flora and fauna respectively. Let $f_1(t)$ be the density of flora in an ecosystem which refers to the number of plant species or individual plants per unit area within a habitat. This is a crucial ecological metric that assesses the health, productivity, and diversity of an ecosystem. Let $f_2(t)$ be the density of fauna in an ecosystem which refers to the quantity of individual animals per unit area, indicating the abundance and dispersion of species within that environment. Also, let $f_3(t)$ be the measure of mining activities and $f_4(t)$ be the measure of the concentration of pollutants in the ecosystem.The flora density is depleted by its interaction, β_1 with fauna either as food, herb or other necessary usages, its rate of contact, γ_1 with the pollutants realized within the site as a result of mining activities either directly or indirectly; and rate of deforestation, α_1 as a result of mining site expansion or urbanization . The dependency of the fauna on the flora within the ecosystem is limited which is at the rate π, hence there is no total dependency; its density is depleted by the rate, β_2 which is the effect of the pollutants release during mining activities either directly or indirectly by inhalation, contamination of food materials, polluting of water bodies, soil's biodiversity, killing of various form of actinomycetes. The mining activities which includes all attributes of processes involved with mining projects is assumed to have constant growth rate, M_0 because of regular mining activities required for power, energy, infrastructure etc., and it also grows by increasing population at rate, α in and within the mining site; in addition it is controlled by the government authorities at rate θ to avoid excesses and to checkmate mining activities. Pollutants emitted/introduced into the ecosystem by different sources excluding mining activities like other industries, transport, agriculture, domestic and sources is supposed to be a constant, Q while the introduction as a result of mining activities is at the rate, λ. The the density of fauna is depleted naturally under observable condition at the rate,μ, likewise the density of pollution at the rate, μ_2 and c_1, c_2 are controls imposed on flora and fauna respectively to reduce pollution in the ecosystem.

With these assumptions, we propose a mathematical model that is portrayed in the form of following nonlinear ordinary differential equations:

$$\begin{aligned}
\frac{df_1}{dt} &= f_1\left(r - \frac{r_0 f_1}{K}\right) - \beta_1 f_1 f_2 - \alpha_1 f_1 f_4 - \gamma_1 f_1 f_3,\\
\frac{df_2}{dt} &= f_2\left(s - \frac{s_0 f_2}{L}\right) + \pi\beta_1 f_1 f_2 - \beta_2 f_2 f_4 - \mu_1 f_2,\\
\frac{df_3}{dt} &= M_0 - (1-\theta) f_3 + \alpha f_2,\\
\frac{df_4}{dt} &= Q + \lambda f_3 - c_1\alpha_1 f_1 f_4 + c_2\beta_2 f_2 f_4 - \mu_2 f_4,
\end{aligned} \tag{1}$$

where $f_1 \geq 0, f_2 \geq 0, f_3 \geq 0, f_4 \geq 0, 0 \leq \pi \leq 1, 0 \leq c_1 \leq 1, 0 \leq \theta \leq 1$ and $0 \leq c_2 \leq 1$.

3. Qualitative Analysis

Lemma 3.1: *The closed set* $\Omega = \{(f_1, f_2, f_3, f_4) \in \mathbb{R}_+^4 : 0 \leq f_1 \leq \frac{Kr_1}{r_0}, 0 \leq f_2 \leq w_m, 0 \leq f_3 \leq M_\theta, 0 \leq f_4 \leq P_\theta\}$, *where* $w_m = \frac{K}{s_0}(s + \pi\gamma_2 \frac{r}{r_0} L), \theta \neq 1, M_\theta = \frac{1}{1-\theta}(M_0 + \alpha L), P_\theta = \frac{Q + \frac{\lambda}{1-\theta}(M_0 + \alpha L)}{c_1\alpha_1 K + \mu_2 - c_2\beta_2 L}$ *and* $c_1\alpha_1 K + \mu_2 > c_2\beta_2 L$ *is positively invariant with respect to model (1) and attracts all positive solutions of the model.*

Proof: Assume that all the state variables are continuous and differentiable. Then applying weak comparison theorem of system of differential equations on (1), we obtain,

$$\frac{df_1}{dt} \leq (r - \frac{r_0}{K} f_1) f_1,$$

and $0 \leq f_1 \leq \frac{Kr}{r_0}$. Also

$$\frac{df_2}{dt} \leq f_2(s - \frac{s_0}{L} f_2) + \pi\beta_1 f_1 f_2,$$

which gives $0 \leq f_2 \leq w_m$, where $w_m = \frac{K}{s_0}(s + \pi\gamma_2 \frac{r}{r_0} L)$. The third equation of (1) implies

$$\frac{df_3}{dt} \leq M_0 - (1 - \theta) f_3 + \alpha w_m$$

which gives

$$0 \leq f_3 \leq \frac{1}{1-\theta}(M_0 + \alpha w_m).$$

In addition, we have that

$$\frac{df_4}{dt} \leq Q + \frac{\lambda}{1-\theta}(M_0 + \alpha w_m) - c_1\alpha_1 K \frac{r}{r_0} f_4 + c_2\beta_2 w_m f_4 - \mu_2 f_4;$$

which gives

$$0 \leq f_4 \leq \frac{Q + \frac{\lambda}{1-\theta}(M_0 + \alpha w_m) - c_1\alpha_1 K \frac{r}{r_0} f_4 + c_2\beta_2 w_m}{\frac{c_1\alpha_1 Kr}{r_0} + \mu_2 - c_2\beta_2 w_m}$$

provided that $\mu_2 + \frac{c_1\alpha_1 Kr}{r_0} > c_2\beta_2 w_m$ which ensures that the are on which pollutants are being depleted out-shoots its introduction to the atmosphere by mining activities.

3.1 *Equilibrium Analysis*

The model (1) has four non - trivial, non - negative equilibrium points namely, $E_0(0,0,\hat{f}_3,\hat{f}_4)$, $E_1(\tilde{f}_1,0,\tilde{f}_3,\tilde{f}_4)$, $E_2(0,\bar{f}_2,\bar{f}_3,\bar{f}_4)$ and $E_3(f_1^*,f_2^*,f_3^*,f_4^*)$ which depicts mining in an arid area where the flora and fauna growth is negligible, growth of fauna is not considerate due to the ecological dispose, flora density is minute with regards to the fauna density and coexistence of the four variables respectively.

Existence of $E_0(0,0,\hat{f}_3,\hat{f}_4)$

We obtain $E_0(0,0,\hat{f}_3,\hat{f}_4)$ by solving the following algebraic equations with cognizance that $f_1(t) = f_2(t) = 0$ in (1);

$$M_0 - (1-\theta)f_3 = 0, \tag{2}$$

$$Q + \lambda f_3 - \mu_2 f_4 = 0. \tag{3}$$

Therefore from (2) and (3), we obtain that $f_3 = \frac{M_0}{1-\theta}$ and $f_4 = \frac{1}{\mu_2}(Q + \frac{\lambda M_0}{1-\theta})$. Hence $E_0(0,0,f_3,f_4) = (0,0,\frac{M_0}{1-\theta},\frac{1}{\mu_2}(Q+\frac{\lambda M_0}{1-\theta}))$ exists provided that $0 \le \theta < 1$.

Existence of $E_1(\tilde{f}_1,0,\tilde{f}_3,\tilde{f}_4)$

To obtain E_1 we equate $f_2 = 0$ in (1) to obtain the following algebraic equations

$$r - \frac{r_0}{K}f_1 - \alpha_1 f_4 - \gamma_1 f_3 = 0, \tag{4}$$

$$M_0 - (1-\theta)f_3 = 0, \tag{5}$$

$$Q + \lambda f_3 - c_1\alpha_1 f_1 f_4 - \mu_2 f_4 = 0. \tag{6}$$

From (5) and (6) we obtain that

$$f_4 = \frac{Q + \frac{\lambda M_0}{1-\theta}}{\mu_2 + c_1\alpha_1 f_1}. \tag{7}$$

Substituting (7) and (5) into (4) and simplifying, we obtain

$$\begin{aligned}\frac{(1-\theta)r_0c_2\alpha_1}{K}f_1^2 &+ ((1-\theta)(rc_2\alpha_1 - \frac{r_0M_0c_2}{K}) - \gamma_1M_0c_2\alpha_1)f_1 \\ &+(1-\theta)(\alpha Q - r\mu_2) + M_0(\alpha\lambda + \gamma_1\mu_2) = 0.\end{aligned} \tag{8}$$

To obtain at least one positive root $f_1 = \tilde{f}_1$, we will have the following parametric restriction

$$\frac{K}{(1-\theta)r_0c_2\alpha_1}(1-\theta)(\alpha Q - r\mu_2) + M_0(\alpha\lambda + \gamma_1\mu_2) < 0. \tag{9}$$

Existence of $E_2(0,\bar{f}_2,\bar{f}_3,\bar{f}_4)$

Since the fauna within the ecosystem can source its food relatively from environment outside the mining site, then E_2 may exist when we equate $f_1 = 0$ in (1), hence we obtain the following algebraic equations at equilibrium

$$s - \frac{s_0}{L}f_2 - \beta_2 f_4 - \mu_1 = 0, \tag{10}$$

$$M_0 - (1-\theta)f_3 + \alpha f_2 = 0, \tag{11}$$

$$Q + \lambda f_3 + c_2\beta_2 f_3 f_4 - \mu_2 f_4 = 0. \tag{12}$$

From (12) and (11) we obtain that

$$f_4 = \frac{Q+\lambda(\frac{M_0+\alpha f_2}{1-\theta})}{\mu_2 - c_2\beta_2 f_2}. \tag{13}$$

For (13) to exist and for positive value $0 \leq \theta < 1$ and $\mu_2 > c_2\beta_2 f_2$. Substituting (13) into (10) and simplifying, we obtain

$$(1-\theta)c_2 s_0 \beta_2 f_2^2 + ((1-\theta)(c_2\beta_2(\mu_1 - s) - s_0\mu_2) - \beta_2\lambda\alpha)f_2$$
$$-((1-\theta)(-\mu_2(s-\mu_1) + Q\beta_2) + \beta_2\lambda M_0) = 0.$$

For existence of real and positive value of $f_3 = \bar{f}_3$, we have the parametric restriction that

$$Q\beta_2 - \mu_2(s-\mu_1) + \beta_2\lambda\frac{M_0}{1-\theta} > 0. \tag{14}$$

Existence of $E_3(f_1^*, f_2^*, f_3^*, f_4^*)$

To obtain this nontrivial equilibrium point we solve these algebraic equations gotten from (1) at critical point

$$r - \frac{r_0}{K}f_1 - \beta_1 f_2 - \alpha_1 f_4 - \gamma_1 f_3 = 0, \tag{15}$$

$$s - \frac{s_0}{L}f_2 + \pi\beta_1 f_1 - \beta_2 f_4 - \mu_1 = 0, \tag{16}$$

$$Q + \lambda f_3 - c_1\alpha_1 f_1 f_4 + c_2\beta_2 f_2 f_4 - \mu_2 f_4 = 0. \tag{17}$$

From simplification of (17) gives

$$f_4 = \frac{Q+\lambda f_3}{c_1\alpha_1 f_1 + \mu_2 - c_2\beta_2 f_2}. \tag{18}$$

Evaluating (18) with reference to (11), we obtain that

$$f_4 = \frac{Q+\lambda(\frac{M_0+\alpha f_2}{1-\theta})}{c_1\alpha_1 f_1 + \mu_2 - c_2\beta_2 f_2} \equiv g(f_1, f_2). \tag{19}$$

For f_4 to positive $c_1\alpha_1 f_1 + \mu_2 > c_2\beta_2 f_2$ and it exists if $0 \leq \theta < 1$. Substituting (11) and (19) into (16) and (17), we have

$$R_1(f_1, f_2) \equiv r - \frac{r_0}{K}f_1 - \beta_1 f_2 - \alpha_1 g(f_1, f_2) - \gamma_1(\frac{M_0+\alpha f_2}{1-\theta}) = 0, \tag{20}$$

$$R_2(f_1, f_2) \equiv s - \frac{s_0}{L}f_2 + \pi\beta_1 f_1 - \beta_2 g(f_1, f_2) - \mu_1 = 0. \tag{21}$$

We analyse the isocline of (20):

(I) i. When $(f_2 = 0)$, we have

$$\begin{aligned} R_1(f_1, 0) &= r - \frac{r_0}{K}f_1 - \alpha_1 g(f_1, 0) - \gamma_1\frac{M_0}{1-\theta} \\ &= r - \frac{r_0}{K}f_1 - \alpha_1(\frac{Q+\frac{\lambda M_0}{1-\theta}}{\alpha_1 c_1 f_1 + \mu_2}) - \gamma_1\frac{M_0}{1-\theta}. \end{aligned} \tag{22}$$

On analyzing (21) further, we obtain the following

a. $R_1(0) = r - \alpha_1(\frac{Q+\frac{\lambda M_0}{1-\theta}}{\mu_2}) - \gamma_1\frac{M_0}{1-\theta}$ which is positive provided

$$r - \frac{\alpha_1 Q(1-\theta) - M_0(\alpha_1\lambda + \mu_2\gamma_1}{\mu_2(1-\theta)} > 0. \tag{23}$$

b.

$$R_1\left(\frac{Kr}{r_0}\right) = -\alpha_1 r_0\left(\frac{Q+\frac{\lambda M_0}{1-\theta}}{\alpha_1 c_1 Kr + r_0\mu_2}\right) - \frac{\gamma_1 M_0}{1-\theta} < 0. \tag{24}$$

c.

$$R'_1 f_1 = -\left(\frac{r_0}{K} - \frac{\alpha_1^2 c_1\left(Q+\frac{\lambda M_0}{1-\theta}\right)}{(\alpha_1 c_1 f_1 + \mu_2)^2}\right) < 0,$$

provided that

$$r_0(\alpha_1 c_1 f_1 + \mu_2)^2 - K\alpha_1^2 c_1\left(Q + \frac{\lambda M_0}{1-\theta}\right) > 0. \tag{25}$$

Therefore $R_1(f_1) = 0$ has a unique positive solution in $0 < f_1 < \frac{Kr}{r_0}$.

ii. $f_1 < 0$ as $f_2 \to \infty$.

iii. $(\frac{df_2}{df_1})_1 = -\frac{\frac{\partial R_1}{\partial f_1}}{\frac{\partial R_1}{\partial f_2}} = -\left(\frac{\frac{r_0}{K} - \alpha_1\frac{\partial g}{\partial f_1}}{-\beta_1 - \alpha_1\frac{\partial g}{\partial f_2}}\right) < 0.$

(II) We then analyse the isocline of (21), which leads to the following

i. When $f_2 = 0, f_1$ is obtained by the following equations

$$R_2(f_1) = s + \pi\beta_1 f_1 - \beta_2\left(\frac{Q+\lambda\frac{M_0}{1-\theta}}{c_1\alpha_1 f_1 + \mu_2}\right) - \mu_1, \tag{26}$$

which we analysed as

a.

$$R_2(0) = s - \beta_2\left(\frac{Q+\lambda\frac{M_0}{1-\theta}}{\mu_2}\right) - \mu_1,$$

which is positive provided that

$$s - \beta_2(\frac{Q+\lambda\frac{M_0}{1-\theta} - \mu_1\mu_2}{\mu_2}) > 0. \tag{27}$$

b.

$$R_2\left(\frac{K}{s_0}\left(s + \pi\gamma_2\frac{r}{r_0}l\right)\right) = s + \pi\beta_1\frac{K}{s_0}(s + \pi\gamma_2\frac{r}{r_0}l) - \beta_2\left(\frac{Q+\lambda\frac{M_0}{1-\theta}}{c_1\alpha_1 f_1 + \mu_2}\right) - \mu_1 > 0$$

c.

$$R'_2(f_1) = \pi\beta_1 + \beta_2 c_1\alpha_1\frac{Q+\lambda\frac{M_0}{1-\theta}}{(c_1\alpha_1 f_1 + \mu_2)^2} > 0$$

Hence a unique positive root of (21) is guaranteed within the limits under consideration.

ii. $f_2 > 0$ as $f_1 \to \infty$.

iii. $(\frac{df_2}{df_1})_2 = -(\frac{\pi\beta_1 - \beta_2\frac{\partial g}{\partial f_1}}{-\frac{s_0}{L} - \beta_2\frac{\partial g}{\partial f_2}}) > 0.$

Hence there exist a unique positive root $f_2 = f_2^*$ of (21) in the region $(0, w_m)$.

Theorem 3.2: *If the density of flora and fauna is decreased by increase in mining activities then* $\frac{df_1}{df_3} < 0$ *and* $\frac{df_2}{df_3} < 0$.

Proof: Define

$$D_1(f_1, f_2, f_3) \equiv r - \frac{r_0}{K} f_1 - \beta_1 f_3 - \alpha_1 g(f_1, f_2, f_3) - \frac{\gamma_1(M_0 + \alpha f_2)}{1-\theta} = 0, \qquad (28)$$

$$D_2(f_1, f_2, f_3) \equiv s \frac{s_0}{L} f_2 + \pi\beta_1 f_1 - \beta_1 g(f_1, f_2, f_3) - \mu_1 = 0. \qquad (29)$$

We need to show that $\frac{df_1}{df_3} < 0$ and $\frac{df_2}{df_3} < 0$. Differentiating (28) with respect to f_3 by chain rule, we have that

$$\left(-\frac{r_0}{K} - \alpha_1 \frac{\partial g}{\partial f_1}\right) df_1 = \left(\beta_1 + \alpha_1 \frac{\partial g}{\partial f_2}\right) df_2 + \alpha_1 \frac{\partial g}{\partial f_3} df_3 + \left(\frac{\gamma_1 \alpha}{1-\theta} + \alpha_1 \frac{\partial g}{\partial f_2} df_2\right).$$

$$\Rightarrow A_1 \frac{df_1}{df_3} = A_2 \frac{df_2}{df_3} + A_3,$$

where $A_1 = -\frac{r_0}{K} - \alpha_1 \frac{\partial g}{\partial f_1}, A_2 = \beta_1 + 2\alpha_1 \frac{\partial g}{\partial f_1} + \frac{\gamma_1 \alpha}{1-\theta}, A_3 = \alpha_1 \frac{\partial g}{\partial f_3}$. Differentiating (29) with respect to f_3 we have

$$\left(\pi\beta_1 - \beta_2 \frac{\partial g}{\partial f_1}\right) df_1 = \left(\frac{s_0}{L} + \beta_2 \frac{\partial g}{\partial f_3}\right) df_3 + \beta_2 \frac{\partial g}{\partial f_3} df_3;$$

$$\Rightarrow B_1 \frac{df_1}{df_3} = B_2 \frac{df_2}{df_3} + B_3$$

where $B_1 = \pi\beta_1 - \beta_2 \frac{\partial g}{\partial f_1}, B_2 = \frac{s_0}{L} + \beta_2 \frac{\partial g}{\partial f_3}, B_3 = \beta_2 \frac{\partial g}{\partial f_3}$. From (18) we observe that $\frac{\partial g}{\partial f_1} < 0, \frac{\partial g}{\partial f_2} > 0$ and $\frac{\partial g}{\partial f_3}$ hence $A_1 < 0, A_2 > 0, A_3 \geq 0, B_1 > 0, B_2 > 0,$ and $B_3 \geq 0$. Rewriting our results in matrix form

$$\begin{pmatrix} A_1 & -A_2 \\ B_1 & -B_2 \end{pmatrix} \begin{pmatrix} \frac{df_1}{df_3} \\ \frac{df_2}{df_3} \end{pmatrix} = \begin{pmatrix} A_3 \\ B_3 \end{pmatrix}$$

$\therefore \frac{df_1}{df_3} = \frac{A_2 B_3 - A_3 B_2}{A_2 B_1 - A_1 B_2} < 0$ provided $A_2 B_3 - A_3 B_2 < 0$ since $A_2 B_1 - A_1 B_2 \geq 0$. This depicts that the density of flora within the mining site decreases as mining activities increases. In addition $\frac{df_2}{df_3} = \frac{A_1 B_3 - A_3 B_1}{A_2 B_1 - A_1 B_2} < 0$ since $A_1 B_3 - A_3 B_1$ is always negative; which implies that the density of fauna decreases as a result of lower life expectancy cause by hazardous effect of particles released within mining site.

Theorem 3.3: *For the existence and uniqueness of the equilibria* E_0, E_1, E_2 *and* E_3*, the following conditions must hold*

a. E_0 exists provided $\theta \neq 1$, it implies that government should regulate the mining activities of companies/miners but should not take total control of the mining activities.

b. E_1 exists provided $(1-\theta)(r\mu_2-\alpha_1 Q)-M_0(\alpha_1\lambda-\gamma_1\mu_2)>0$.

c. E_2 exists provided

i. $\mu_2>c_2\beta_2 f_2$,

ii. $Q\beta_2-\mu_2(s-\mu_1)+\beta_2\lambda\frac{M_0}{1-\theta}>0$.

d. E_3 exists provided

i. $c_1\alpha_1 f_1+\mu_3>c_2\beta_2 f_2$,

ii. $r-\frac{\alpha_1 Q}{\mu_2}-\frac{M_0}{\mu_2(1-\theta)}(\alpha_1\lambda+\mu_2\gamma_1)>0$,

iii. $r_0(\alpha_1 c_1+\mu_2)^2-K\alpha_1^2 c_1(Q+\lambda\frac{M_0}{1-\theta})>0$,

iv. $s-\frac{\beta_2}{\mu_1}(Q+\lambda\frac{M_0}{1-\theta})-\mu_2>0$.

4. Stability Analysis

To obtain the stability of the equilibrium points we obtain the Jacobian matrix of (1);

$$J(f_1,f_2,f_3,f_4)=\begin{pmatrix} W_1 & -\beta_1 f_1 & -\gamma f_1 & -\alpha_1 f_1 \\ \pi\beta_1 f_2 & W_2 & 0 & -\beta_2 f_2 \\ 0 & \alpha & -(1-\theta) & 0 \\ -c_1\alpha_1 f_4 & c_2\beta_2 f_2 & \lambda & c_2\beta_2 f_2-c_1\alpha_1 f_1-\mu_2 \end{pmatrix}, \tag{30}$$

where $W_1=r-\frac{2r_0}{K}f_1-(\beta_1 f_2+\alpha_1 f_4+\gamma_1 f_3)$ and $W_2=s-\frac{2s_0}{L}f_2+\pi\beta_1 f_1-\beta_2 f_4-\mu_1$. To study the stability of the model at the equilibria, we evaluate (30) at the various points. Evaluating (30) at E_0 we have

$$J(E_0)=J_0=\begin{pmatrix} r-\alpha_1\hat{f}_4-\gamma_1\hat{f}_3 & 0 & 0 & -\alpha_1\hat{f}_4 \\ 0 & s-\beta_2\hat{f}_4-\mu_1 & 0 & 0 \\ 0 & \alpha & -(1-\theta) & 0 \\ -c_1\alpha_1\hat{f}_4 & 0 & \lambda & -\mu_2 \end{pmatrix}.$$

The characteristic equation corresponding for J_0 is given as

$$\lambda^4+A_1\lambda^3+A_2\lambda^2+A_3\lambda+A_4=0,$$

where

$$\begin{aligned}
A_1 &= (\alpha_1+\beta_2)\hat{f}_4+\gamma_1\hat{f}_3+\mu_1+\mu_2+(1-\theta)-r-s,\\
A_2 &= (r-\alpha_1\hat{f}_4-\gamma_1\hat{f}_3)(s-\beta_2\hat{f}_4-\mu_1-(1-\theta_1))+(s-\beta_2\hat{f}_4-\mu_1)(\theta-1)\\
&\quad +\alpha_1^2 c_1\hat{f}_4^{\,2}-\mu_2(s-\beta_2\hat{f}_4-\mu_1-(1-\theta)+r-\alpha_1\hat{f}_4-\gamma_1\hat{f}_3),\\
A_3 &= (r-\alpha_1\hat{f}_4-\gamma_1\hat{f}_3)(s-\beta_2\hat{f}_4-\mu_1)(\theta-\mu_2-1)+(s-\beta_2\hat{f}_4\\
&\quad -\mu_1)(\mu_2(1-\theta)+\alpha_1^2 c_1\hat{f}_4^{\,2})+(1-\theta)(r-\alpha_1\hat{f}_4-\gamma_1\hat{f}_3)\mu_2+\alpha_1^2 c_1\hat{f}_4^{\,2}),\\
A_4 &= (s-\beta_2\hat{f}_4-\mu_1)(1-\theta)(r-\alpha_1\hat{f}_4-\gamma_1\hat{f}_3)\mu_2+\mu_2\alpha_1\hat{f}_4).
\end{aligned}$$

With the knowledge that $f_3>0, f_4>0$, then by Routh - Hurwitz stability criterion the characteristic equation has two positive eigenvalues, therefore at E_0

the model solution trajectory is a hyperbolic equilibrium with saddle node that is unstable in f_1 direction. Evaluating (30) at E_1 we have

$$J(E_1) = J_1 =$$
$$\begin{pmatrix} r - \frac{2r_0}{K}\tilde{f}_1 - \alpha_1\tilde{f}_4 - \gamma_1\tilde{f}_3 & -\beta_1\tilde{f}_1 & -\gamma_1\tilde{f}_1 & -\alpha_1\tilde{f}_1 \\ 0 & s + \pi\beta_1\tilde{f}_1 - \beta_2\tilde{f}_4 - \mu_1 & 0 & 0 \\ 0 & \alpha & -(1-\theta) & 0 \\ -c_1\alpha_1\tilde{f}_4 & 0 & \lambda & -c_1\alpha_1\tilde{f}_1 - \mu_2 \end{pmatrix}.$$

If t is the eigenvalue then the characteristic equation can be written as

$$(s + \pi\beta_1\tilde{f}_1 - \beta_2\tilde{f}_4 - \mu_1 - t)(\theta - 1 - t)$$
$$\begin{vmatrix} r - \frac{2r_0}{K}\tilde{f}_1 - \alpha_1\tilde{f}_4 - \gamma_1\tilde{f}_3 - t & -\alpha_1\tilde{f}_4 \\ -c_1\alpha_1\tilde{f}_4 & -c_1\alpha_1\tilde{f}_4 - mu_2 - t \end{vmatrix} = 0. \tag{31}$$

Observe that if the determinant part of (31) is represented as J_{1*}, then we can observe that $Tr(J_{1*}) < 0$ and $Det(J_{1*}) > 0$ since $\tilde{f}_1 > 0, \tilde{f}_3 > 0$ and $\tilde{f}_4 > 0$; hence we get eigenvalues with a negative real part. Therefore, (1) is stable at E_1 since we obtain eigenvalues that have negative real parts. Evaluating (30) at E_2 we obtain

$$J(E_2) = J_2 =$$
$$\begin{pmatrix} r - \beta_1\bar{f}_2 - \alpha_1\bar{f}_4 - \gamma_1\bar{f}_3 & 0 & 0 & 0 \\ \pi\beta_1\bar{f}_2 & s - \dfrac{2s_0}{L}\bar{f}_2 - \beta_2\bar{f}_4 - \mu_1 & 0 & \beta_2\bar{f}_2 \\ 0 & \alpha & -(1-\theta) & 0 \\ -c_1\alpha_1\bar{f}_4 & 0 & \lambda & c_2\beta_2\bar{f}_2 - \mu_2 \end{pmatrix}.$$

The characteristic polynomial is given as

$$(M_0 - t^*)(t^{*3} + M_1 t^{*2} + M_2 t^* + M_3) = 0, \tag{32}$$

where

$$\begin{aligned} M_0 &= r - \beta_1\bar{f}_2 - \alpha_1\bar{f}_4 - \gamma_1\bar{f}_3, \\ M_1 &= \tfrac{2s_0}{L}\bar{f}_2 + \beta_2\bar{f}_4 + \mu_1 + \mu_2 + 1 - s - \theta - c_2\beta_2\bar{f}_2, \\ M_2 &= (s - \tfrac{2s_0}{L}\bar{f}_2 - \beta_2\bar{f}_4 - \mu_1)(c_2\beta_2\bar{f}_2 - \mu_2 - (1-\theta) - \mu_2) \\ &\quad -(1-\theta)(c_2\beta_2\bar{f}_2 - \mu_2), \\ M_3 &= (1-\theta)\left(s - \tfrac{2s_0}{L}\bar{f}_2 - \beta\bar{f}_4 - \mu\right)(c_2\beta_2 f_2 - \mu_2) - \beta_2 f_2\alpha\lambda. \end{aligned}$$

From (32) we observe that the first eigenvalue is negative but since $M_3 < 0$, using Routh - Hurwitz criterion we expect the remaining eigenvalues to have atleast one with a positive real part, hence (1) has hyperbolic equilibrium that is a saddle point and unstable at E_2.

We use Lyapunov's method in the stability behaviour of E_3. Consider the following Goh - Volterra type of Lyapunov function

$$V = f_1 - f_1^* - f_1^* \ln\frac{f_1}{f_1^*} + d_1\left(f_2 - f_2^* - f_2^* \ln\frac{f_2}{f_2^*}\right) + \frac{d_2}{2}(f_3 - f_3^*)^2 + \frac{d_3}{2}(f_4 - f_4^*)^2; \tag{33}$$

where d_1, d_2 and d_3 are positive constants, f_1^*, f_2^*, f_3^* and f_4^* are small perturbation around E_3.

$$\therefore \dot{V} = \dot{f_1} - \frac{f_1^*}{f_1}\dot{f_1} + d_1(\dot{f_2} - \frac{f_2^*}{f_2}\dot{f_2}) + d_2(f_3 - f_3^*)\frac{df_3}{dt} + d_3(f_4 - f_4^*)\frac{df_4}{dt}. \tag{34}$$

Substituting (1) into (34) at the steady state E_3 we have

$$\begin{aligned}\dot{V} \ &= f_1(\tfrac{r_0}{K}f_1^* + \beta_1 f_2^* + \alpha_1 f_4^* + \gamma_1 f_3^* - \tfrac{r_0}{K}f_1^*) - \beta_1 f_1 f_2 - \alpha_1 f_1 f_4 - \gamma_1 f_1 f_3 \\ &\quad -f_1^*(\tfrac{r_0}{K}f_1^* + \beta_1 f_2^* + \alpha_1 f_4^* + \gamma_1 f_3^* - \tfrac{r_0}{K}f_1^* - \beta_1 f_2 - \alpha_1 f_4 - \gamma_1 f_3) \\ &\quad +d_1(f_2(\tfrac{s_0}{L}f_2^* - \pi\beta_1 f_1^* + \beta_2 f_4^* + \mu_1 f_2^* - \tfrac{s_0}{L}f_2) + \pi\beta_1 f_1 f_2 - \beta_2 f_2 f_4 \\ &\quad -\mu f_2 - f_2^*(\tfrac{s_0}{L}f_2^* - \pi\beta_1 f_1^* + \beta_2 f_4^* + \mu_1 f_2^* - \tfrac{s_0}{L}f_2 + \pi\beta_1 f_1 - \beta_2 f_4 + \mu_1)) \\ &\quad +d_2(f_3 - f_3^*)((1-\theta)f_3^* - \alpha f_3^* - (1-\theta)f_3 - \alpha f_3) \\ &\quad +d_3(f_4 - f_4^*)(c_1\alpha_1 f_1^* f_4^* + \mu_2 f_4^* - c_2\beta_2 f_3^* f_4^* - \lambda f_3^* - c_1\alpha_1 f_1 f_4 \\ &\quad -\mu_2 f_4 + c_2\beta_2 f_3 f_4 + \lambda f_3).\end{aligned} \tag{35}$$

Simplifying algebraically and on proper rearrangement we have

$$\begin{aligned}\dot{V} \ &= -\tfrac{r_0}{K}(f_1 - f_1^*)^2 - d_1\tfrac{s_0 - L}{L}(f_2 - f_2^*)^2 - d_2(1-\theta)(f_3 - f_3^*)^2 \\ &\quad +(-\beta_1 + d_1\pi\beta_1)(f_1 - f_1^*)(f_2 - f_2^*) \\ &\quad +(-d_1\beta_2 + d_3 c_2\beta_2 f_4^*)(f_4 - f_4^*)(f_2 - f_2^*) \\ &\quad +(-\alpha_1 + d_3 c_1\alpha_1 f_4^*)(f_4 - f_4^*)(f_1 - f_1^*) - d_3\mu_2(f_4 - f_4^*)^2 \\ &\quad +\gamma_1(f_3 - f_3^*)(f_1 - f_1^*).\end{aligned} \tag{36}$$

We choose $\beta_1 + d_1\pi\beta_1 = 0, -d_1\beta_2 + d_3 c_2\beta_2 f_4^* = 0$ and $d_2 = 1$ then

$$d_1 = \frac{1}{\pi}, d_3 = \frac{1}{\pi c_2 f_4^*},$$

which are positive constants. Then

$$\begin{aligned}\dot{V} \ &= -\tfrac{r_0}{K}(f_1 - f_1^*)^2 - (\tfrac{s_0}{\pi L} + \tfrac{\mu_1}{\pi})(f_2 - f_2^*)^2 - (1-\theta)(f_3 - f_3^*)^2 \\ &\quad -\tfrac{\mu_2}{\pi c_2 f_4^*}(f_4 - f_4^*)^2 + \alpha_1(\tfrac{c_1}{\pi c_2 f_4^*} - 1)(f_4 - f_4^*)(f_1 - f_1^*) \\ &\quad +\gamma_1(f_3 - f_3^*)(f_1 - f_1^*) \\ &\leq 0.\end{aligned}$$

Since V is positive definite, $d_1, d_2, d_3 > 0$, and the derivative $\dot{V}$ is semi - negative definite in the invariant set, therefore by La Salle's invariance principle E_3 is globally asymptotically stable.

5. Numerical Simulation

We numerically simulate model (1) based on Runge Kutta iteration using the following parameters

$r = 10, r_0 = 15, \alpha = 0.3, \alpha_1 = 0.4, \beta_1 0.05, \beta_2 = 0.03, \gamma = 0.01, s = 8,$
$s_0 = 16, \pi = 0.8, \mu_1 = 0.005, \mu_2 = 0.0004, m_0 = 25, \theta = 0.1,$
$Q = 40, \lambda = 12, c_1 = 0.03, c_2 = 0.09, K = 300, L = 100;$

with the following initial values

$$f_1(0) = 120, f_2 = 20, f_3 = 25, f_4 = 50.$$

Mining generally either open pit, underground and In Situ leach (ISL) is still on a developmental level in Nigeria and are basically are artisanal which is also applicable in most of the developing nations. The above premises makes shows that pollution in the forms of water, dust, noise, vibration and radiation is inevitable in the ecosystem.

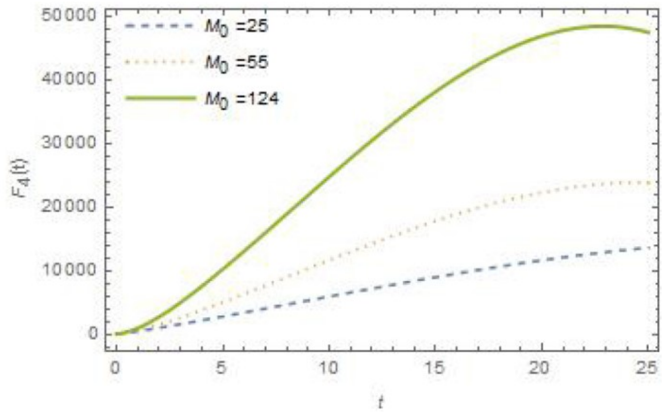

Figure 1
Variation in the density of Pollution f_4 with time for different values of m_0.

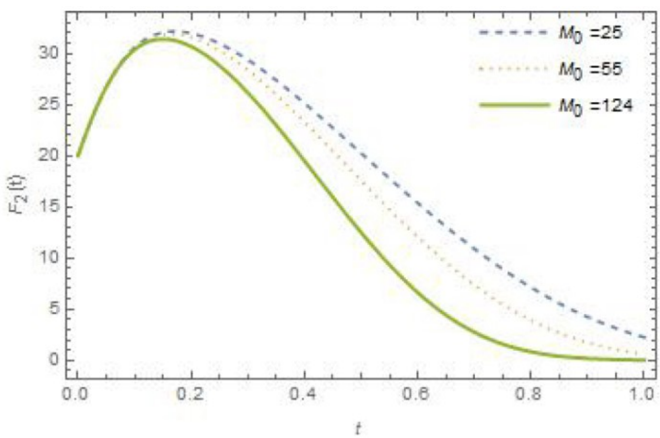

Figure 2
Variation in the population density of fauna f_2 with time for different values of m_0.

The increase in mining activities increases the amount of pollutants introduced into the environment which is clearly shown in Figure 1; these introduction of pollutants into the ecosystem dampen the growth densities of fauna and flora which can be drawn from Figure 2 with the knowledge that densities of fauna and flora are inversely related. It can be obtained from Figure 3 that vegetation loss is more rapid because the first casualty within a mining site is vegetation which are often economical [6]. This problem is linked with increased carbon IV Oxide content and global warming within the site and environment which are as a result of clearing of vegetation on whose majority are protective plants, cash crops and soil binders.

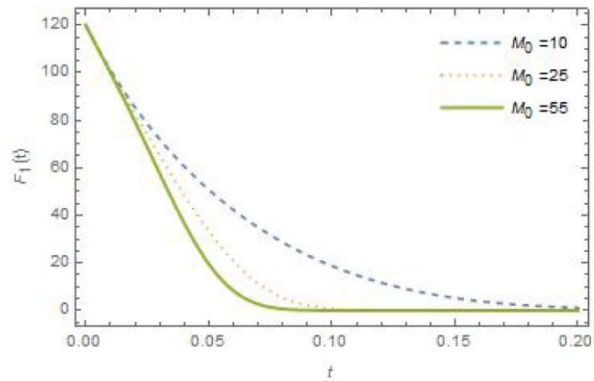

Figure 3
Variation in the population density of flora f_4 with time for different values of m_0.

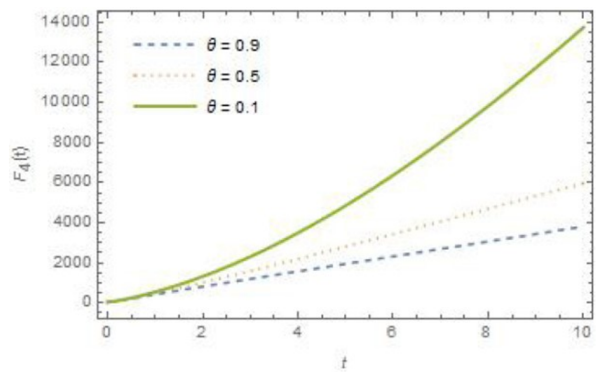

Figure 4
Variation in the density of Pollution f_4 with time for different values of θ.

Figure 4 shows the effect of government in regulating the activities and sanctioning companies based on the damages they cause to the ecosystem. In our analysis, when government takes total control of the mining sectors, the operational and viability of the sector is damped greatly. Proper legislation and regulation of mining activities will help in reducing the amount of pollutants introduced into the environment which will in turn affect the flora and fauna within the ecosystem directly or indirectly. Figure 5 shows that increase in the parameter λ will lead to depletion of the fauna density as a result of upsurged pollution as shown in Figure 6 which if not regulated will have a devastating impact on the fauna population, be it human, wild life and other living organisms which happens concomitantly with total extinction of the flora within the ecosystem.

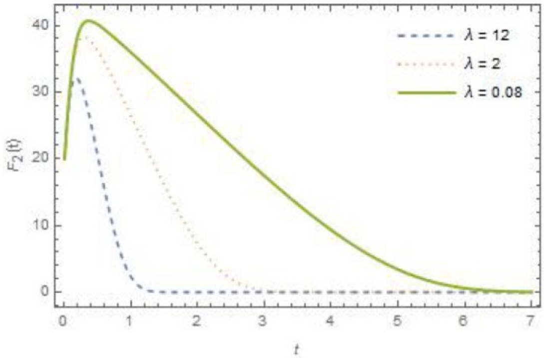

Figure 5
Variation in the density of Pollution f_4 with time for different values of λ.

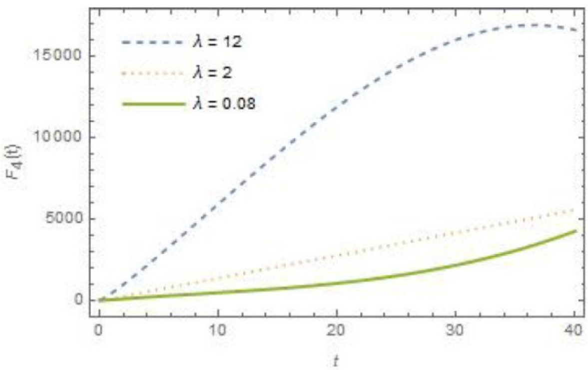

Figure 6
Variation in the population density of Pollution f_2 with time for different values of λ.

When the value of β_2 is increased, there is an upsurge of pollutants that is necessitated my increased mining activities that consequently affects the fauna population adversely which is shown by Figure 7. The increase in flow of liquids and gaseous pollutants from mining sites like sulphur dioxide, siliceous dust, nitrogen dioxide, carbon monoxide, black smoke, and mine wastes containing magnetite, zircon, monazite, thorite etc causes health issues that can lead to death or associated diseases.

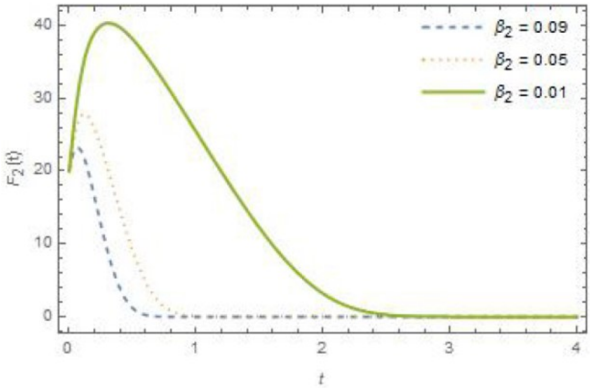

Figure 7
Variation in the density of Pollution f_4 with time for different values of β_2.

6. Conclusion

The economic plant resources (flora) and animal populations (fauna) are both negatively impacted by mining activities and environmental degradation and the deterministic mathematical model that is provided in this paper offers important insights into these negative consequences. Differential equations are used to formulate the problem and stability theory and numerical simulations are used to analyze it. The study illustrates the grave repercussions of unrestrained mining and pollution by highlighting these implications. Furthermore, the findings indicate that both flora and fauna are subjected to direct and indirect harm as a consequence of excessive mining and rising pollution concentrations in the environment.

In order to lessen the impact of these negative impacts, it is vital to preserve an ecological equilibrium between the fauna, plant-dependent animal populations, mining activities and pollution of the environment. For the purpose of reducing pollution levels and regulating indiscriminate mining, the intervention of the government through stringent regulatory measures is absolutely necessary. Additionally, the investigation suggests that by managing the parameter that is connected with excessive mining activities, it is possible to considerably reduce other types of pollution as well, which will result in considerable improvements in the health and sustainability of flora and fauna. The result of this model shows that when government takes total control of the mining sites, it is bound not to produce desirable result, rather it should participate at the level of regulatory, supervision and sanctioning.

In the end, this study highlights the significance of environmentally responsible mining methods and strong environmental legislation in order to protect biodiversity and guarantee the viability of the ecosystem over the long term. To prevent irreparable harm to plant and animal populations, it is vital to take preventative steps, such as increasing the stringency of mining restrictions and implementing pollution control measures.

Conflicts of Interest

The authors declare no conflict of interest.

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