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Predicting a random determinant of order n filled with independently and identically distributed (i.i.d) exponential variates

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Abstract

The paper derives the fiducial limits for a determinant D of order 4, filled with independently and identically distributed (i.i.d) Exponential variates, using Chebyshev's inequality and then, comparing the derived result with the fiducial limits for D of order 2 and 3 obtained in previous research, generalizes the result for order n. If D is of order n with

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i.i.d Exponential variates (θ), $P\left(-k\sqrt{\frac{(n+1)!}{\theta^{2n}}}<D<k\sqrt{\frac{(n+1)!}{\theta^{2n}}}\right)\geq\left(1-\frac{1}{k^2}\right)$ where k is a positive integer with $E(D) = 0$ and $\mathrm{Var}(D) = \frac{(n+1)!}{\theta^{2n}}$

Several applications are pointed out. The study may be extended to other probability distributions.

Subject Classification: *62P99.*

Keywords: *Random determinant, Exponential distribution, Chebyshev's inequality.*

I. Introduction

The paper addresses the problem of predicting a random determinant of order n filled with independent and identically distributed (i.i.d) Exponential variates with the help of . Chebyshev's inequality which states that for a random variable X, $P\{E(X) - kSD(X) < X < E(X) + kSD(X)\} \geq 1 - \frac{1}{k^2}$, for some constant k which is usually some positive integer [1]. Readers interested in further literature on determinants may consult [2].

II. Previous Work

Unfortunately there is not much work in this problem except for the paper [3] in which the distribution of a determinant of order 2 for i.i.d U(0,1) variates has been given. The study has been extended for i.i.d U(0, θ) variates and i.i.d. discrete uniform U(1, 2, 3,….t) variates and, using Chebyshev's inequality, the fiducial limits of D for order 2 and 3 have been obtained for several other probability distributions in [4] and [5].

III. Our Contribution

We would first derive the fiducial limits for D of order 4, filled with independent and identically distributed (i.i.d) Exponential variates, using Chebyshev's inequality and then, comparing the derived result with the fiducial limits for D of order 2 and 3, we shall generalize the result for order n.

Consider a determinant D of order 4 as $D = \begin{vmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{vmatrix}$

$$\therefore D = a\begin{vmatrix} f & g & h \\ j & k & l \\ n & o & p \end{vmatrix} - b\begin{vmatrix} e & g & h \\ i & k & l \\ m & o & p \end{vmatrix} + c\begin{vmatrix} e & f & h \\ i & j & l \\ m & n & p \end{vmatrix} - d\begin{vmatrix} e & f & g \\ i & j & k \\ m & n & o \end{vmatrix}$$

$$= a\{f(kp - lo) - g(jp - ln) + h(jo - kn)\} - b\{e(kp - lo) - g(ip - lm) + h(io - km)\} + c\{e(jp - ln) - f(ip - lm) + h(in - jm)\} - d\{e(jo - kn) - f(io - km) + g(in - jm)\}$$

= (afkp – aflo – agjp + agln + ahjo – ahkn – bekp + belo + bgip – bglm – bhio + bhkm + cejp – celn – cfip + cflm + chin – chjm – dejo + dekn + dfio – dfkm – dgin + dgjm)

where a,b,c,d,e………m,n,o,p are i.i.d. Exponential variates.

Remark:

1. A random variable X is said to have an exponential distribution with parameter $\theta > 0$, if its p.d.f is given by:

$$f(x, \theta) = \begin{cases} \theta e^{-\theta x} ; \ x \geq 0 \\ 0 ; \ \text{otherwise} \end{cases}$$

2. Mean

$$E(x) = \frac{1}{\theta} \tag{i}$$

and Variance

$$V(x) = \frac{1}{\theta^2} \tag{ii}$$

3. We know that

$$V(X) = E(X^2) - \{E(X)\}^2$$

$$\therefore \frac{1}{\theta^2} = E(X^2) - \frac{1}{\theta^2}$$

$$\Rightarrow E(X^2) = \frac{2}{\theta^2} \tag{iii}$$

Then

E(D) = E(afkp – aflo – agjp + agln + ahjo – ahkn – bekp + belo + bgip – bglm – bhio + bhkm + cejp – celn – cfip + cflm + chin – chjm – dejo + dekn + dfio – dfkm – dgin + dgjm)

= E(afkp) – E(aflo) – E(agjp) + E(agln) + E(ahjo) – E(ahkn) − E(bekp) + E(belo) + E(bgip) – E(bglm) – E(bhio) + E(bhkm) + E(cejp) – E(celn) − E(cfip) + E(cflm) + E(chin) – E(chjm) – E(dejo) + E(dekn) + E(dfio) – E(dfkm) − E(dgin) + E(dgjm)

$$\Rightarrow E(D) = \frac{1}{\theta^4} - \frac{1}{\theta^4} - \frac{1}{\theta^4} + \frac{1}{\theta^4} + \frac{1}{\theta^4} - \frac{1}{\theta^4} - \frac{1}{\theta^4} + \frac{1}{\theta^4} + \frac{1}{\theta^4} - \frac{1}{\theta^4} - \frac{1}{\theta^4} + \frac{1}{\theta^4}$$
$$+ \frac{1}{\theta^4} - \frac{1}{\theta^4} - \frac{1}{\theta^4} + \frac{1}{\theta^4} + \frac{1}{\theta^4} - \frac{1}{\theta^4} - \frac{1}{\theta^4} + \frac{1}{\theta^4} + \frac{1}{\theta^4} - \frac{1}{\theta^4} - \frac{1}{\theta^4} + \frac{1}{\theta^4}$$

$$\Rightarrow E(D) = 0 \tag{iv}$$

$$\therefore V(D) = E(D^2) - \{E(D)\}^2$$

= E[(afkp – aflo – agjp + agln + ahjo – ahkn – bekp + belo + bgip – bglm – bhio + bhkm + cejp – celn – cfip + cflm + chin – chjm – dejo + dekn + dfio – dfkm – dgin + dgjm)^2] − 0

= E[(afkp – aflo – agjp + agln + ahjo – ahkn – bekp + belo + bgip – bglm – bhio + bhkm + cejp – celn – cfip + cflm + chin – chjm – dejo + dekn + dfio – dfkm – dgin + dgjm)^2]

= E[(afkp – aflo – agjp + agln + ahjo – ahkn – bekp + belo + bgip – bglm – bhio + bhkm + cejp – celn – cfip + cflm + chin – chjm – dejo + dekn + dfio – dfkm – dgin + dgjm) (afkp – aflo – agjp + agln + ahjo – ahkn – bekp + belo + bgip – bglm – bhio + bhkm + cejp – celn – cfip + cflm + chin – chjm – dejo + dekn + dfio – dfkm – dgin + dgjm)]

Taking,
$$A = \left(E(X)\right)^8 = \frac{1}{\theta^8}$$
$$B = E(X^2)\left(E(X)\right)^6 = \frac{2}{\theta^2}\frac{1}{\theta^6}$$
$$C = [E(X^2)]^2\left(E(X)\right)^4 = \frac{4}{\theta^4}\frac{1}{\theta^4}$$
$$D = [E(X^2)]^4 = \left(\frac{2}{\theta^2}\right)^4$$

$$\begin{aligned}Var(D) = {} & 12(D – C – C + B + B – C – C + A + B – A – A + B + B – A – C + B + A – A - A + B + B – C – A + A) + 12(- C + D + B – C – C + B + A – C – A + B + B -A – A + B + B – C – A + A + B – A – C + B + A – A) \\ & = 12(D – 6C + 8B – 3A) + 12(D – 6C + 8B – 3A) \\ & = - 72A + 192B – 144C + 24D \\ & = - 72\frac{1}{\theta^8} + 192\frac{2}{\theta^2}\frac{1}{\theta^6} – 144\frac{4}{\theta^4}\frac{1}{\theta^4} + 24\left(\frac{2}{\theta^2}\right)^4 \\ & = -\frac{72}{\theta^8} + \frac{384}{\theta^8} - \frac{576}{\theta^8} + \frac{384}{\theta^8} \\ & = \frac{120}{\theta^8}\end{aligned}$$
$$Var(D) = \frac{5!}{\theta^8}$$

∴ Standard Deviation(D)
$$\sigma = \sqrt{\frac{5!}{\theta^8}}$$

∴ using Chebyshev’s inequality yields
$$P(E(D) – k\ SD(D) < D < E(D) + k\ SD(D)) \geq \left(1 - \frac{1}{k^2}\right)$$
$$\Rightarrow P\left(-k\sqrt{\frac{5!}{\theta^8}} < D < k\sqrt{\frac{5!}{\theta^8}}\right) \geq \left(1 - \frac{1}{k^2}\right)$$

Proceeding as in the above case when D was of order 4, we get

If D is of order 'n' then E(D) = 0 and $Var(D) = \frac{(n+1)!}{\theta^{2n}}$

∴ Standard Deviation (D)
$$\sigma = \sqrt{\frac{(n+1)!}{\theta^{2n}}}$$

∴ using Chebyshev’s inequality yields
$$P(E(D) – k\ SD(D) < D < E(D) + k\ SD(D)) \geq \left(1 - \frac{1}{k^2}\right)$$

$$\Rightarrow P\left(-k\sqrt{\frac{(n+1)!}{\theta^{2n}}} < D < k\sqrt{\frac{(n+1)!}{\theta^{2n}}}\right) \geq \left(1 - \frac{1}{k^2}\right)$$

IV. Discussion

1. When D is of order 2 with i.i.d Exponential variates,

$$E(D) = 0\ ,\ Var(D) = \frac{6}{\theta^4} = \frac{3!}{\theta^4}$$

$$\text{and S.D} = \sqrt{\frac{6}{\theta^4}} = \sqrt{\frac{3!}{\theta^4}}$$

2. When D is of order 3 with i.i.d Exponential variates,

$$E(D) = 0\ ,\ Var(D) = \frac{24}{\theta^6} = \frac{4!}{\theta^6}$$

$$\text{and S.D} = \sqrt{\frac{24}{\theta^6}} = \sqrt{\frac{4!}{\theta^6}}$$

For proof**,** we refer the reader to [4].

It is of interest to compare the results on fiducial limits of D obtained for Exponential Distribution as inputs with those obtained for other probability distributions.

The Fiducial limits of D with Exponential distribution constituting its elements is given in tabular form (tables 1A and 1B) as follows:

Table 1A

Fiducial limits of D when D is of order 2
(Elements of D are iid Exponential variates)

Elements of D	Fiducial limits of D
Exponential variates	$P\left(-k\sqrt{\frac{3!}{\theta^4}} < D < k\sqrt{\frac{3!}{\theta^4}}\right) \geq \left(1 - \frac{1}{k^2}\right)$

Table 1B

Fiducial limits of D when D is of order 3
(Elements of D are iid Exponential variates)

Elements of D	Fiducial limits of D
Exponential variates	$P\left(-k\sqrt{\frac{4!}{\theta^6}} < D < k\sqrt{\frac{4!}{\theta^6}}\right) \geq \left(1 - \frac{1}{k^2}\right)$

If D is of order 4 with i.i.d Exponential variates (θ), we have obtained

$$P\left(-k\sqrt{\frac{5!}{\theta^8}} < D < k\sqrt{\frac{5!}{\theta^8}}\right) \geq \left(1 - \frac{1}{k^2}\right)$$

with $E(D) = 0$ and $Var(D) = \frac{120}{\theta^8} = \frac{5!}{\theta^8}$

Comparing this result of order 4 with the results of order 2 and order 3 as given in tables 1A and 1B and generalizing, we have, If D is of order n with i.i.d Exponential variates (θ),

$$P\left(-k\sqrt{\frac{(n+1)!}{\theta^{2n}}} < D < k\sqrt{\frac{(n+1)!}{\theta^{2n}}}\right) \geq \left(1 - \frac{1}{k^2}\right)$$

with E(D) = 0 and Var(D) = $\frac{(n+1)!}{\theta^{2n}}$

As immediate applications, predicting a random determinant can in turn help us in addressing the several interesting problems that we may encounter in real life in science, engineering and other disciplines including business and economics:-

a) In finding the area of a triangle whose vertices are random variables
b) In finding the eigen values and eigen vectors for square random matrices
c) For predicting the solution vector of simultaneous linear equations whose coefficients are random variables using Cramer's rule
d) In predicting the cross product of two random vectors
e) In Transportation and Assignment Models
f) In solving some optimization problems

V. Conclusion

The paper investigates an intriguing problem of predicting a random determinant in which not much previous literature is available. For such a determinant of order n filled with i.i.d Exponential variates, the fiducial limits of D are obtained using Chebyshev's inequality. It is found that if D is of order n with i.i.d Exponential variates (θ), then

$$P\left(-k\sqrt{\frac{(n+1)!}{\theta^{2n}}} < D < k\sqrt{\frac{(n+1)!}{\theta^{2n}}}\right) \geq \left(1 - \frac{1}{k^2}\right)$$

with E(D) = 0 and Var(D) = $\frac{(n+1)!}{\theta^{2n}}$

Predicting a random determinant could be helpful in tackling several real life problems in science, engineering and other disciplines.

Conflict of interest statement

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