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Predicting a random determinant with i.i.d. Beta variates of first kind

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Abstract

Chebyshev's inequality is a powerful probabilistic inequality based on which we can set up confidence interval of any random variable irrespective of its distribution. In the present paper, we make use of this inequality to set up confidence (fiducial) limits of a second and third order random determinant whose entries are independently and identically distributed Beta variates of first kind. The results would be useful in computations involving

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determinants where the entries are random variables and in areas where we are dealing with random square matrices.

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I. Introduction

Chebyshev's inequality is a powerful probabilistic inequality based on which we can set up confidence interval of any random variable irrespective of its distribution. In the present paper, we make use of this inequality to set up confidence (fiducial) limits of a random determinant of order second and third order whose entries are independently and identically distributed (i.i.d.) Beta variates of first kind. For any random variable X, $\mathrm{P}\{\mathrm{E}(\mathrm{X}) - \mathrm{cSD}(\mathrm{X}) < \mathrm{X} < \mathrm{E}(\mathrm{X}) + \mathrm{cSD}(\mathrm{X})\} \geq 1 - \frac{1}{\mathrm{c}^2}$, where c is a constant which is usually taken as some positive integer. This is a well known inequality credited to Chebyshev. Here E(X) and SD(X) denoted the mean (expectation) and the standard deviation of the probability distribution of X in the population.

II. Previous Work

Analysis of a random determinant of second order whose elements are continuous i.i.d. U(0,1) variates can be found in the paper by Wise and Hall [1]. This study was extended by Saha and Chakraborty [3], [4] for continuous i.i.d U(0, θ) variates and also for discrete i.i.d. U(1, 2, 3,….t) and some more variates using Chebyshev's inequality.

We shall not prove the powerful inequality of Chebyshev here which holds for any probability distribution, discrete or continuous. The interested reader may see Gupta and Kapoor [5]. Muir [2] may be consulted for further literature on determinants.

III. Beta Distribution of first kind

A continuous random variable X follows Beta Distribution of first kind with parameters 'a' and 'b' (a > 0, b > 0) if its probability density function is given by:

$$\mathrm{f(x)} = \begin{cases} \frac{1}{\mathrm{B(a,b)}} x^{a-1}(1-x)^{b-1} & ; \mathrm{(a,b)} > 0,\ 0 < \mathrm{x} < 1 \\ 0\,;\ \text{otherwise} \end{cases}$$

where B(a,b) is the Beta function

IV. Our Contribution

Let us start with a second order random determinant $\mathrm{D} = \begin{vmatrix} \mathrm{x}_{11} & \mathrm{x}_{12} \\ \mathrm{x}_{21} & \mathrm{x}_{22} \end{vmatrix} = (\mathrm{x}_{11}\mathrm{x}_{22} - \mathrm{x}_{12}\mathrm{x}_{21})$

Let the x_{ij}'s be i.i.d Beta variates of first kind.

Remark: For a Beta variate X of first kind, with parameters a and b,

$$\text{Mean} = E(X) = \frac{a}{a+b} \tag{i}$$

and

$$\text{Variance} = Var(X) = \frac{ab}{(a+b)^2(a+b+1)} \tag{ii}$$

We shall not prove these results and refer the interested reader to Gupta and Kapoor (2014).

Then, $E(D) = E(x_{11}x_{22} - x_{12}x_{21})$

$= E(x_{11}x_{22}) - E(x_{12}x_{21})$

$= E(x_{11})E(x_{22}) - E(x_{12})E(x_{21})$ (Since, x_{ij}'s are independent)

$= \left(\frac{a}{a+b}\right)\left(\frac{a}{a+b}\right) - \left(\frac{a}{a+b}\right)\left(\frac{a}{a+b}\right)$

$= 0$

Since, by definition, $Var(X) = E[X - E(X)]^2 = E(X^2) - \{E(X)\}^2$ after simplification,

$$E(X^2) = V(X) + \{E(X)\}^2$$

$$= \left(\frac{ab}{(a+b)^2(a+b+1)}\right) + \left(\frac{a}{a+b}\right)^2 \qquad \text{using (i) and (ii)}$$

$$= \frac{ab + a^2(a+b+1)}{(a+b)^2(a+b+1)}$$

$$= \frac{ab + a^3 + a^2b + a^2}{(a+b)^2(a+b+1)}$$

$$= \frac{a(b + a^2 + ab + a)}{(a+b)^2(a+b+1)}$$

$$= \frac{a\{a+b+a(a+b)\}}{(a+b)^2(a+b+1)}$$

$$= \frac{a\{(a+b)(a+1)\}}{(a+b)^2(a+b+1)}$$

$$= \frac{a(a+1)}{(a+b)(a+b+1)}$$

$$\therefore E(X^2) = \frac{a(a+1)}{(a+b)(a+b+1)} \tag{iii}$$

Since E(D) = 0, using the technique as in Saha and Chakraborty (2021),

$Var(D) = 2[E(X^2)]^2 - 2[E(X)]^4$

$$= 2\left(\frac{a(a+1)}{(a+b)(a+b+1)}\right)^2 - 2\left(\frac{a}{a+b}\right)^4 \qquad \text{using(i) and (iii)}$$

$$= 2\frac{a^2(a+1)^2}{(a+b)^2(a+b+1)^2} - 2\frac{a^4}{(a+b)^4}$$

$$= 2\frac{a^2}{(a+b)^2}\left\{\frac{(a+1)^2}{(a+b+1)^2} - \frac{a^2}{(a+b)^2}\right\}$$

$$= 2\frac{a^2\{(a+1)^2(a+b)^2 - a^2(a+b+1)^2\}}{(a+b)^4(a+b+1)^2}$$

$$= 2\frac{a^2\{2a^2b + 2ab^2 + 2ab + b^2\}}{(a+b)^4(a+b+1)^2}$$

∴ Standard Deviation (D)

$$\sigma = \frac{\sqrt{2}\,a\{2a^2b+2ab^2+2ab+b^2\}^{\frac{1}{2}}}{(a+b+1)(a+b)^2}$$

∴ using Chebyshev's inequality yields, since E(D) = 0,

$$P(-c\,\sigma < D < c\,\sigma) \geq \left(1 - \frac{1}{c^2}\right)$$

where
$$\sigma = \frac{\sqrt{2}\,a\{2a^2b+2ab^2+2ab+b^2\}^{\frac{1}{2}}}{(a+b+1)(a+b)^2}$$

Now, for the third order case, let $D = \begin{vmatrix} p & q & r \\ s & t & u \\ v & w & x \end{vmatrix}$

= ptx – puw – qsx + quv + rsw – rvt, after simplification
where p, q, r, s, t, u, v, w, x are i.i.d. Beta variates of first kind.

Repeating the analysis when D was of second now for D being of third order, we get

$$E(D) = E(ptx - puw - qsx + quv + rsw - rvt)$$
$$= E(ptx) - E(puw) - E(qsx) + E(quv) + E(rsw) - E(rvt)$$

Let $E(a_{ij}) = \left(\frac{a}{a+b}\right) = \lambda$

Then, considering the fact that a_{ij}'s are iid Beta variates with mean λ, it is easy to see that

$$E(D) = 3\lambda^3 - 3\lambda^3 = 0 \qquad \text{(iv)}$$

So

$$\text{Var}(D) = E(D^2) - \{E(D)\}^2 = E(D^2) \text{ using (iv)}$$
$$= E\{(ptx - puw - qsx + quv + rsw - rvt)^2\}$$

Whence, using the technique as shown in Saha and Chakraborty (2021), it can be shown that

$$\text{Var}(D) = 12P - 18Q + 6R \qquad \text{(v)}$$

where
$$P = [E(X)]^6$$
$$= \left(\frac{a}{a+b}\right)^6$$
$$Q = E(X^2)[E(X)]^4$$
$$= \frac{a(a+1)}{(a+b)(a+b+1)}\left(\frac{a}{a+b}\right)^4$$

and
$$R = [E(X^2)]^3$$
$$= \left(\frac{a(a+1)}{(a+b)(a+b+1)}\right)^3$$

From equation (v), substituting the expressions for P, Q and R, we get

$$\operatorname{Var}(D) = 12\left(\frac{a}{a+b}\right)^6 - 18\,\frac{a(a+1)}{(a+b)(a+b+1)}\left(\frac{a}{a+b}\right)^4 + 6\left(\frac{a(a+1)}{(a+b)(a+b+1)}\right)^3$$

∴ Standard Deviation(D)

$$\sigma = \left\{12\left(\frac{a}{a+b}\right)^6 - 18\,\frac{a(a+1)}{(a+b)(a+b+1)}\left(\frac{a}{a+b}\right)^4 + 6\left(\frac{a(a+1)}{(a+b)(a+b+1)}\right)^3\right\}^{\frac{1}{2}}$$

Accordingly, since $E(D) = 0$, Chebyshev's inequality yields

$$P(-c\,\sigma < D < c\sigma) \geq \left(1 - \frac{1}{c^2}\right)$$

where

$$\sigma = SD(D) = \left\{12\left(\frac{a}{a+b}\right)^6 - 18\,\frac{a(a+1)}{(a+b)(a+b+1)}\left(\frac{a}{a+b}\right)^4 + 6\left(\frac{a(a+1)}{(a+b)(a+b+1)}\right)^3\right\}^{\frac{1}{2}}$$

V. Discussion

It is quite interesting to compare the results on confidence limits of D obtained for i.i.d. Beta variates of first kind as inputs with those obtained for inputs from other probability distributions. For this the reader is referred to tables 1 and 2 of the paper [4] and the research monograph[3].

VI. Conclusion

1) For a second order random determinant D with i.i.d Beta variates of first kind as input,

$$P(-c\sigma < D < c\sigma) \geq \left(1 - \frac{1}{c^2}\right)$$

where $\sigma = \frac{\sqrt{2}\,a\{2a^2b+2ab^2+2ab+b^2\}^{\frac{1}{2}}}{(a+b)^2(a+b+1)}$

2) For a third order random determinant D with i.i.d Beta variates of first kind as input,

$$P(-c\sigma < D < c\sigma) \geq \left(1 - \frac{1}{c^2}\right)$$

But in this case

$$\sigma = \left\{12\left(\frac{a}{a+b}\right)^6 - 18\,\frac{a(a+1)}{(a+b)(a+b+1)}\left(\frac{a}{a+b}\right)^4 + 6\left(\frac{a(a+1)}{(a+b)(a+b+1)}\right)^3\right\}^{\frac{1}{2}}$$

The results would be useful in computations involving determinants where the entries are random variables and in areas where we are dealing with random square matrices. In particular, the situations where we encounter Beta variates of first kind are of interest, e.g., when we are dealing with probabilities or proportions whose range is from 0 to 1, certain environmental and genetic studies, in Bayesian analysis, in Program Evaluation and Review Techniques (PERT) etc.

Conflict of interest statement

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References

[1] G. L. Wise and E. B. Hall, "A note on the distribution of the determinant of a random matrix," *Statistics and Probability Letters*, vol. 11, pp. 147–148 (1991).

[2] T. Muir, A Treatise on the Theory of Determinants, Nabu Press, 2011.

[3] N. Saha and S. Chakraborty, Probabilistic Analysis of a Random Determinant, GRIN Verlag (2019).

[4] N. Saha and S. Chakraborty, "Using Chebyshev's inequality to predict a random determinant for i.i.d. Gamma and Weibull distributions," *Journal of Statistics and Management Systems*, vol. 24, no. 3, pp. 613–623 (2021).

[5] S. C. Gupta and V. K. Kapoor, Fundamentals of Mathematical Statistics, Sultan Chand and Sons (2014).

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