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Maximum penalty method to solve transportation problem

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Abstract

In this present article, we have proposed a method to obtain an Initial Basic Feasible Solution (IBFS) for the Transportation Problem. It is based on Maximum penalty where row penalty and column penalty is obtained by the average of Maximum and Minimum element of newly formed Transportation Table. Proposed method is illustrated with numerical examples and compared results with existing well known methods.

Subject Classification (2020): *90B06.*

Keywords: *IBFS, Average, Penalty, Transportation cost.*

1. Introduction

Transportation Problem is one of the subclass of linear programming problem, it aims to transport various quantities of a single homogeneous commodity, that are initially stored at various source to different

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destination in such that the transportation cost, transportation distance or time is minimum [4].For this research work I have analyzed some research papers: In [2] a new method (Summation and Ratio Method) is proposed to obtain solution. In [3], the LCJ algorithm developed to determine the best way to solve the transportation problem. A rough multi-objective linear fractional transportation problem (RMOLFTP) covered in [4]. The Multi-objective linear plus linear fractional-transportation problem (MOLPLFTP) solved using a new method presented in [5].Firas Hamid Thanoon [6] proposed modified Vogel approximation method (Mean Absolute Deviation) for solving transportation problems. LCMM proposed to obtain an IBFS for the minimization of transportation cost [7]. In [8] max min penalty approach applied to obtain IBFS. An effective methodology for solving transportation problem is introduced in [9]. The Average of Cost Approach is taken into consideration in [10] when doing a comparative study of IBFS for solving transportation problems. The total opportunity cost and maximum minimum penalty approach [11] is used to solve transportation problem. In [1], the best optimal solution was calculated by using triangular fuzzy numerals to answer the fuzzy-transportation compliance using the VAM approach. [12] provides a comprehensive and detailed survey of various types of transportation problems.

This paper is organized as follows: Detailed algorithm for solving Transportation Problem is discussed in Section II. In Section III illustrated the numerical examples for the new method and comparing these results in the Section IV. Conclusions are given at the end.

II. Algorithm of Proposed Method:

Step 1. Check whether the transportation problem is balanced or not. If it is balanced then go to next step.

Step 2. Find the largest cost from each row and subtract each element from the largest cost.

Step 3. Find the average of minimum and maximum element of each row and column which is called as row penalty and column penalty. Write row penalty after the supply and column penalty below the demand.

Step 4. From penalties select maximum penalty. Choose the largest element along the selected row or column at which maximum penalty lies. At the largest element allocate the minimum of supply or demand. Then cross off the row or column corresponding to whose supply or demand is satisfied.

Step 5. In above step,

If tie occurs in largest penalty then observe the largest element in corresponding row or column and allocate there.

If tie occurs in largest element then choose such a element at which concern cost is minimum.

Step 6. Repeat the steps 3 to 5 until satisfaction of all the supply and demand is met.

Step 7. At the end, calculate the total transportation by taking sum of product cost and corresponding allocated value.

III. Illustrations:

Example 1:

Illustrate: The transportation problem given in table 1

Table 1

Transportation Problem

Sources	**Destinations**			**Supply**
	D_1	**D_2**	**D_3**	
S_1	3	3	5	9
S_2	6	5	4	8
S_3	6	10	7	10
Demand	7	12	8	

Solution: The given transportation problem is balanced.

Row differences obtained at this stage are presented in table 2

Table 2

Row differences

Sources	**Destinations**			**Supply**
	D_1	**D_2**	**D_3**	
S_1	2	2	0	9
S_2	0	1	2	8
S_3	4	0	3	10
Demand	7	12	8	

Penalties and allocations are shown in table 3

Table 3

Penalties and allocations

Sources	Destinations			Supply	Row Penalties		
	D_1	D_2	D_3				
S_1	2	$^{(9)}2$	0	9	1	1	1
S_2	0	$^{(3)}1$	$^{(5)}2$	8	1	1.5	1.5
S_3	$^{(7)}$ 4	0	$^{(3)}3$	10	2	1.5	-
Demand	7	12	8				
Column Penalties	2	1	1.5				
	-	1	1.5				
	-	1	1.5				

Allocations in the original transportation table are shown in table 4

Table 4

Allocations

Sources	Destinations			Supply
	D_1	D_2	D_3	
S_1	3	$^{(9)}3$	5	9
S_2	6	$^{(3)}5$	$^{(5)}4$	8
S_3	$^{(7)}6$	10	$^{(3)}7$	10
Demand	7	12	8	

The transportation cost is 9*3+3*5+5*4+7*6+3*7=125.

Example 2:

Illustrate: The transportation problem given in table 5

Table 5

Transportation Problem

Sources	Destinations				Supply
	D_1	D_2	D_3	D_4	
S_1	9	8	5	7	12
S_2	4	6	8	7	14
S_3	5	8	9	5	16
Demand	8	18	13	3	

Solution: The given transportation problem is balanced.

Row differences obtained at this stage are presented in table 6

Table 6

Row differences

Sources	**Destinations**				**Supply**
	D_1	**D_2**	**D_3**	**D_4**	
S_1	0	1	4	2	12
S_2	4	2	0	1	14
S_3	4	1	0	4	16
Demand	8	18	13	3	

Penalties and allocations are shown in table 7

Table 7

Penalties and allocations

Sources	**Destinations**				**Supply**	**Row Penalties**			
	D_1	**D_2**	**D_3**	**D_4**					
S_1	0	1	$^{(12)}4$	2	12	2	2	-	-
S_2	$^{(8)}4$	$^{(6)}2$	0	1	14	2	2	2	1
S_3	4	$^{(12)}1$	$^{(1)}0$	$^{(3)}4$	16	2	2	2	0.5
Demand	8	18	13	3					
Column Penalties	2	1.5	2	2.5					
	2	1.5	2	-					
	2	1.5	0	-					
	-	1.5	0	-					

Allocations in the original transportation table are shown in table 8

Table 8

Allocations

Sources	**Destinations**				**Supply**
	D_1	**D_2**	**D_3**	**D_4**	
S_1	9	8	$^{(12)}5$	7	12
S_2	$^{(8)}4$	$^{(6)}6$	8	7	14
S_3	5	$^{(12)}8$	$^{(1)}9$	$^{(3)}5$	16
Demand	8	18	13	3	

The transportation cost is 12*5+8*4+6*6+12*8+1*9+3*5=248.

Example 3:
Illustrate: The transportation problem given in table 9

Table 9

Transportation Problem

Sources	Destinations			Supply
	D_1	**D_2**	**D_3**	
S_1	4	3	5	9
S_2	6	5	4	8
S_3	8	10	7	10
Demand	7	12	8	

Solution: The given transportation problem is balanced.

Row differences obtained at this stage are presented in table 10

Table 10

Row differences

Sources	Destinations			Supply
	D_1	**D_2**	**D_3**	
S_1	1	2	0	9
S_2	0	1	2	8
S_3	2	0	3	10
Demand	7	12	8	

Penalties and allocations are shown in table 11

Table 11

Penalties and allocations

<table>
<tr><th rowspan="2">Sources</th><th colspan="3">Destinations</th><th rowspan="2">Supply</th><th colspan="3" rowspan="2">Row Penalties</th></tr>
<tr><th>D_1</th><th>D_2</th><th>D_3</th></tr>
<tr><td>S_1</td><td>1</td><td>$^{(9)}2$</td><td>0</td><td>9</td><td>1</td><td>1.5</td><td>-</td></tr>
<tr><td>S_2</td><td>$^{(5)}0$</td><td>$^{(3)}1$</td><td>2</td><td>8</td><td>1</td><td>0.5</td><td>0.5</td></tr>
<tr><td>S_3</td><td>$^{(2)}2$</td><td>0</td><td>$^{(8)}3$</td><td>10</td><td>1.5</td><td>1</td><td>1</td></tr>
<tr><td>Demand</td><td>7</td><td>12</td><td>8</td><td></td><td></td><td></td><td></td></tr>
<tr><td rowspan="3">Column Penalties</td><td>1</td><td>1</td><td>1.5</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>1</td><td>-</td><td></td><td></td><td></td><td></td></tr>
<tr><td>1</td><td>0.5</td><td>-</td><td></td><td></td><td></td><td></td></tr>
</table>

Allocations in the original transportation table are shown in table 12

Table 12

Allocations

Sources	Destinations			Supply
	D_1	D_2	D_3	
S_1	4	$^{(9)}3$	5	9
S_2	$^{(5)}6$	$^{(3)}5$	4	8
S_3	$^{(2)}8$	10	$^{(8)}7$	10
Demand	7	12	8	

The transportation cost is 9*3+5*6+3*5+2*8+8*7=144.

IV. Comparison of results:

Table 13 presents a comparison of results obtained by the proposed method and existing methods

Table 13

Comparison

Methods	Solutions		
	Example 1	Example 2	Example 3
Proposed Method	125	248	144
NWCM	143	320	150
LCM	159	248	145
VAM	143	248	150
Modi Method	125	240	139

Graphical representation of Comparison is shown in following figure 1

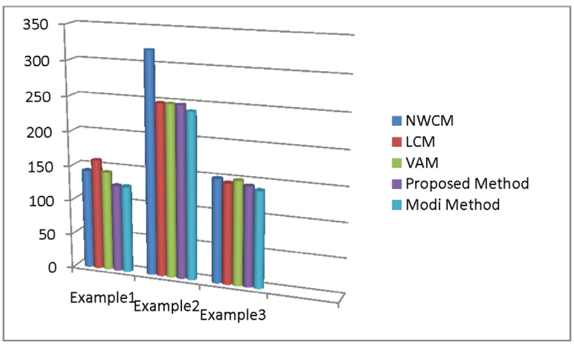

Figure 1
Graphical representation

Conclusion

In our study, in most of the cases proposed method is more superior to NWCM, LCM and VAM. Sometimes it gives solution same or nearly same to the optimal solution.

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