

Risk management and return trade-offs in gold and silver pair trading : Quantitative approach

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Abstract

Volatility is a critical determinant of market dynamics, shaping trading decisions and outcomes. Extreme volatility events, often precipitated by geopolitical developments

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or policy shifts, significantly influence asset prices. Recent empirical evidence underscores the heightened risks faced by individual traders in derivative markets, with a substantial proportion incurring losses. This study comprehensively examines risk management and return trade-offs within the context of gold and silver pair trading. Pair trading, a market-neutral strategy, leverages the historical correlation between two assets—Gold and Silver—to generate returns. This paper explores methodologies for identifying cointegration and establishing trading signals, employing statistical tools such as Johansen's Cointegration Test for short-term returns and the Granger Causality Test for long-term returns. The study aims to develop a practical framework for implementing and managing Gold and silver pair trading strategies, emphasizing the balance between profit potential and inherent risks. Utilizing historical price data, the study back tests and evaluates the performance of proposed strategies, providing insights into the efficacy of various risk management techniques and their impact on overall returns.

Subject Classification: 91G20.

Keywords: *Pair trading strategy, Gold and silver, Strategy effectiveness, Risk management.*

Introduction

Gold and Silver, known as safe-haven assets and inflation hedges, share a complex price relationship influenced by macroeconomic factors. While they typically move in correlation, price divergences create opportunities for pair trading—a market-neutral strategy that exploits temporary misalignments with the expectation of eventual convergence. Traders capitalize on deviations from the historical gold-silver price ratio by establishing long and short positions. However, gold-silver pair trading carries risks such as correlation breakdowns, high volatility, and liquidity constraints. To manage these risks effectively, a quantitative framework is essential. This study adopts a data-driven approach, using statistical models like Johansen's Cointegration Test for short-term relationships and the Granger Causality Test for long-term interactions. By analysing historical trends, identifying optimal entry and exit points, and balancing risk and return, the research aims to provide a structured methodology for strategic decision-making in gold-silver trading. Ultimately, this study contributes to quantitative trading literature by developing a robust framework for implementing and managing pair trading strategies [1]. It optimizes risk-adjusted returns [1]., helping traders navigate volatile market conditions effectively.

Literature Review

- Trading Gold-Silver Ratio with Machine Learning Algorithms [2] explores how machine learning enhances trading strategies for the gold-silver ratio. The study finds that machine learning-based approaches surpass conventional buy-and-hold and technical analysis-driven pair trading strategies, particularly when rebalancing occurs less frequently.
- Gold-Silver Chart Ratio Strategy: Rules and Back test [3] assesses the gold-silver ratio as a trading indicator for optimizing entry & exit decisions. The research incorporates extensive back testing to evaluate the effectiveness of various trading strategies across different market conditions.
- Gold & Silver Trading Strategies: RSI Explained [4] examines the use of the Relative Strength Index (RSI) in gold and silver trading. The study emphasizes RSI's flexibility in adapting to diverse market conditions and its relevance in technical analysis.
- Ingram [5] discussed a pair trading strategy in their article that combines neural networks and multi-objective programming. The aim of finding statistical arbitrage opportunities in correlated assets is to improve trading performance.
- The study of Platania et al. [6] inspects the long-term correlations among the prices of Silver and Gold, the corresponding stock indexes (GDX & SLV), and CBOE market sentiment metrics. The results support Gold and Silver's role as stand-in assets in financial markets by confirming a substantial link.
- Spread Trading and Correlation in Portfolio Selection [7] evaluates the role of correlation in portfolio diversification across oil-importing and oil-exporting nations. The study concludes that correlation plays a pivotal role in optimizing portfolio allocation [8].
- Pair Trading and Equal-Weighted Strategies [8] investigates pair trading strategies where assets are allocated equal weight. The study presents empirical evidence supporting the effectiveness of equal-weighted pair trading.
- Cointegration Approach in Pair Trading [9] compares statistical measures like percentage deviation, residual standard deviation, Bollinger Bands in pair trading. The results indicate that Bollinger Bands & offer the favorable risk-adjusted returns.

- Historical Development of Pair Trading [10] traces the origins of pair trading back to the 1980s, attributing its development to Nunzio Tartaglia and his team. The review discusses the “distance method,” which relies on normalized price differences and historical volatility to identify mean reversion-based trading opportunities.

Research Methodology

Research Problem

A study on the pair trading strategy of Gold and Silver addresses the critical gap in existing financial theory and practice. There is no established theory, method, or strategy for trading in commodity instruments and pair trading strategies. This lack of a structured approach forms the core research problem that this study aims to address.

Objectives

1. To study the effectiveness of the pair trading strategy between Gold and Silver
2. To develop a trading strategy based on the price difference of Gold and Silver
3. To study the causal relationship between the two variables.

Data description

Silver and gold futures prices were the two main data sets used in the study. The Multi Commodity Exchange of India’s (MCX) official website,

Duration of the Study

The study’s data, which covered the years April 2003 through December 2024, came from (<https://in.investing.com>) & (<https://www.mcxindia.com>).

Research Methods

This study employed a variety of research methods for analysis, drawing primarily from established econometric principles. The foundational texts, “Econometrics by Examples” [11] and “Basic Econometrics” [12], served as the primary resources for understanding and applying econometric techniques. The specific methodologies and

econometric tools utilized for data analysis, detailed individually within this chapter, are commonly employed in stock market research, as evidenced by studies such as [13].

Data Analysis & Interpretation

Unit Root Test

The Stationarity is an essential necessity for any analysis related to time-series data. A Unit Root Test was applied to the log-transformed daily prices of Gold and Silver to assess this property. The test was conducted at a 95% confidence level, using a 5% critical value for t-statistic. The findings of Unit Root Test on the daily price data are summarized below:

Unit Root Test Results (Daily Prices) Without first differencing;

(H₀): The series exhibits a unit root, indicating that it is not stationary

(H₁): The series lacks a unit root, suggesting that it is

Comparing Test Statistic to Critical Values and p-value:

Unit Root Test results suggest that computed test statistic (-1.404352) lies within the acceptance region of the null hypothesis, as it is less negative than all the critical values. Additionally, p-value of 0.5818 exceeds the commonly accepted significance thresholds of 0.05 and 0.10. This suggests that there is not enough evidence to reject the null hypothesis, indicating that the time series is likely non-stationary.

With the first difference

Comparing the Test Statistic to Critical Values and p-value:

The (ADF) test conducted on the first-differenced logarithm of gold prices (D(GOLD_L_PRICE)) provides robust evidence of stationarity. The test statistic, with an absolute value of 75.44107, exceeds all critical values by a significant margin, resulting in a definitive rejection of the null hypothesis that a unit root is present. Furthermore, the p-value of 0.0001 is substantially lower than conventional significance thresholds (0.05, 0.01, and 0.001), reinforcing the conclusion that the differenced series is indeed stationary.

Comparing Test Statistic to Critical Values and p-value

ADF test results for first-differenced log price of silver show that the test statistic (-3.087801) is more negative than critical values by worth points (0.05 & 0.10) (-2.861876 and -2.566991, respectively), but not more negative than the 1% critical value (-3.431372). The corresponding p-value of 0.0275 lies between 0.01 and 0.05.

This indicates that the null hypothesis was rejected at the 5% significance level, providing adequate evidence that differenced series is stationary. However, at the more stringent 1% level, we remained unsuccessful to discard null hypothesis, indicating weaker evidence for stationarity. Overall, since both gold and silver series become stationary after first differencing, further analysis can be carried out using the first-differenced data.

Cointegration Test

Null Hypothesis (H_0): There is no statistically significant evidence of cointegration among Silver & Gold prices.

Alternative Hypothesis (H_1): There exists a statistically significant cointegration relationship among Silver & Gold prices.

Long-term association can be known by performing a cointegration test. Before performing a cointegration test, it is essential to identify the optimum lag length that can be used to run the test of cointegration.

Johansen Cointegration Assessment

The test was conducted to assess the long-term relationship between the two points Using *Akaike Information Criterion* (AIC), ideal lag distance was found to be 2. The Maximum Eigenvalue and Trace Statistic are the two main statistics produced by the Johansen test. The following table offers a thorough synopsis of the test result.

Table 1

CE(s)	Eigenvalue	Trace Statistic	C Value (0.05)	Probability
None	.0034	20.28603	15.49471	.0088
At most one	.000324	1.764128	3.841465	.1841

Source: E View Output

Table 2

Hypothesized No. of CE(s)	Eigenvalue	Max-Eigen Statistic	Critical Value (0.05)	Prob.**
None *	0.0034	18.5219	14.2646	0.01
At most 1	0.000324	1.764128	3.841465	0.1841

Source: E-view Output

The Null hypothesis for the Cointegration test can be mentioned as below:

H_0 : There is no cointegration between the daily prices of Gold and Silver.

H_1 : Cointegration exists between the daily prices of Gold and Silver.

Comparing the Test Statistic to Critical Values and p-value:

The test statistic (-3.087801) is more negative than the critical values at the 5% and 10% significance levels (-2.861876 and -2.566991, respectively), but does not exceed the 1% critical value (-3.431372) (Table 1 and 2). The associated p-value of 0.0275 is below 5% significance level but exceeds 1% threshold. This implies that at the 5% level, the null hypothesis of a unit root is rejected, providing strong evidence that the first-differenced logarithmic price of silver is stationary. However, at more stringent 1% significance level, null hypothesis cannot be rejected, suggesting that evidence for stationarity is not strong enough.

Table 3

CE(s)	Eigen value	Trace Statistic	Benchmark value at 0.05	Probability
None	.003225	19.36203	15.49471	.0124
At most one	.000331	1.797749	3.841465	.18

Source: E-view Output

Table 4

CE(s)	Eigen value	Max-Eigen Statistic	Benchmark value at 0.05	Probability
None	.003225	17.56428	14.2646	.0145
At most one	.000331	1.797749	3.841465	.18

Source: E-view Output

Null hypothesis for cointegration test is mentioned below:

(H₀): There is no cointegration among daily of Silver & Gold.

(H₁): A integrating relationship exists between the daily of Silver and Gold

For “Null” hypothesis, the p-values (.0124 and .0145) are below .05 verge, justifying null assumption is invalidated of no cointegration (Table 3 and 4).

For “Alternative” hypothesis, the p-value of .1800 exceeds .05, indicating that Assumed hypothesis of at most one cointegrating equation cannot be rejected (Table 3 and 4).

Cointegrating Coefficients:

Normalized cointegrating coefficients: SILVER_PRICE = 1.000000, GOLD_PRICE = -0.937757.

This implies that in the long run, a 1-unit upsurge in the price of Gold is related with a 0.937757-unit reduction in price of Silver.

Hypothesis Testing: p-values for the “None” hypothesis is .0124 and .0145, both of which are under the .05 threshold. This leads to refusal of Baseline hypothesis of no cointegration. Conversely, the p-value for the “At most 1” hypothesis is .1800, which exceeds .05, so the null hypothesis of at most one cointegrating equation cannot be rejected.

Cointegration Equation: The normalized cointegration coefficients are as follows:

SILVER_PRICE = 1.000000, GOLD_PRICE = -0.937757. This suggests that, in the long run, a one-unit increase in gold prices corresponds to an approximate 0.937757-unit decline in silver prices.

VAR Model–Vector Autoregressive Model–Both variables attain stationarity upon first differencing. Consequently, (VAR) model is estimated using first-differenced series of Gold & Silver. The variables in the model are organized in a system framework—meaning the equations are specified simultaneously within a system of equations, as illustrated below.

$$D(LN_Gold_DAILY) = C(1)*D(LN_Gold_DAILY(-1)) + C(2)*D(LN_gold_DAILY(-2)) + C(3)*D(LN_Silver_DAILY(-1)) + C(4)*D(LN_Silver_DAI- \\ LY(-2)) + C(5) \quad [] \quad \text{Eq.1}$$

$$D(\text{LN_Silver_DAILY}) = C(6)*D(\text{LN_Gold_DAILY}(-1)) + C(7)*D(\text{LN_Gold_DAILY}(-2)) + C(8)*D(\text{LN_Silver_DAILY}(-1)) + C(9)*D(\text{LN_Silver_DAILY}(-2)) + C(10) \quad \text{Eq.2}$$

Cointegrating Equation:

- SILVER_PRICE (-1) = 1
- GOLD_PRICE (-1) = -0.951380 (t-statistic = -9.90316)
- C = -15382.58 (constant)
- This equation suggests that for every 1 unit increase in the lagged Gold price, the Silver price is expected to decrease by approximately 0.95 units in the long run.

Error Correction Term Coefficients:

- For D(SILVER_PRICE): -0.009721 (t-statistic = -4.40505)
- For D(GOLD_PRICE): 0.000701 (t-statistic = 1.30868)
- The significant negative coefficient for D(SILVER_PRICE) suggests that the Silver price adjusts more strongly to deviations from the long-run equilibrium.

Short-Term Dynamics:

- The coefficients for the lagged first differences (D (SILVER_PRICE (-1)) and D (GOLD_PRICE (-1))) capture the short-term relationships between price changes.

R-squared:

- For D(SILVER_PRICE): 0.105439
- For D(GOLD_PRICE): 0.000674
- The low R-squared values suggest that the model explains only a small proportion of the variance in price changes,

Wald Test.

The Wald Test was applied to check the joint significance of the coefficients.

Null: $H_0: C(2) = C(3) = C(4) = C(7) = C(9) = C(10) = 0$ i.e. They have no joint significance for the model.

Alt:H1:C (2), C (3), C (4), C (7), C (9) and C (10)c have joint significance for the model

The detailed result of the Wald Test is mentioned below.

R-squared Value (0.823670): This Value implies that around 82.37% of the fluctuations in silver prices are accounted for by changes in gold prices, indicating a solid linear association among two variables.

Adjusted R-squared (0.823638): The adjusted R-squared is nearly identical to the R-squared Value, suggesting that the model maintains a strong explanatory power even after adjusting for the number of predictors—here, only one. This reinforces the model's good fit.

Coefficients:

GOLD_PRICE (0.990423): Slope of the Regression Line: The estimated slope indicates that for each \$1 rise in rate of Gold, rate of Silver is expected to increase by roughly \$0.99, highlighting a nearly one-to-one positive relationship between the two. The positive coefficient indicates a positive relationship – as gold prices increase; silver prices also tend to go up. C (14207.32): This is constant term. It represents predicted silver price when the gold price is zero. However, in this context, it's important to note that a gold price of zero is not realistic, so the intercept should be interpreted cautiously. It mainly serves to position the regression line.

Statistical Significance:

t-Statistic and p-Value for GOLD_PRICE (159.4091, 0.0000): The exceptionally high t-value and the p-value of 0.0000 for GOLD_PRICE suggest that it is a highly significant explanatory variable for SILVER_PRICE. This implies that observed relationship among silver and gold prices is statistically robust and no more likely due to random variation.

F-statistic and Prob(F-statistic) (25411.25, 0.0000): The large F-statistic, along with p-value of 0.0000, confirms overall significance of regression model. This further supports the conclusion that gold prices strongly and statistically meaningfully impact silver prices.

Deriving VAR Equations with significant coefficients. Removing non-significant coefficients from equations 4.1 and 4.2, both equations can be written as follows.

$$D(\text{LN_Gold_DAILY}) = C(1) * D(\text{LN_Gold_DAILY}(-1)) + C(5)[\quad] \quad \text{Eq. 3}$$

$$D(\text{LN_Silver_DAILY}) = C(6) \cdot D(\text{LN_Silver_DAILY}(-1)) + C(8) \cdot D(\text{LN_Silver_DAILY}(-1)) \quad \text{Eq.4}$$

Interpretation of VAR Equations

For Eq. 3, it can be interpreted that Gold is caused by its own lag but not impacted by Silver

ForEq.4: It can be interpreted that prices of Silver are caused by its own lag & one lag of Gold.

t-statistic: 159.4091 (with 5440 degrees of freedom)

F-statistic: 25411.25 (with 1 and 5440 degrees of freedom)

Chi-square statistic: 25411.25 (with 1 degree of freedom)

Probability: 0.0000 (for all three test statistics)

$$(H_0): C(1) = 0$$

Summary of Ho:

$$\text{Normalized Restriction } (= 0): C(1) = 0.990423$$

$$\text{Std. Err.: } 0.006213$$

Interpretation:

Test Statistics and Probability: t-statistic, F-statistic, & Chi-square statistic all produce a p-value of 0.0000. This extremely low probability offers strong evidence against the null hypothesis.

Null Hypothesis Rejection: Given that p-value is effectively zero, Ho that C (1) equals zero is rejected with high confidence.

Coefficient C (1): The estimated coefficient C (1) is 0.990423, with a standard error of 0.006213. This result indicates that C (1) is significantly different from zero, confirming its statistical relevance in the model.

4. Connecting to the Regression Analysis

From the previous output, we know this is a simple linear regression with:

- Dependent Variable: SILVER_PRICE
- Independent Variable: GOLD_PRICE
- Coefficient for GOLD_PRICE (C (1) in this Wald test) = 0.990423

Wald Test Interpretation: The Wald test assesses whether the coefficient for GOLD_PRICE significantly differs from zero. As null hypothesis has been rejected, it confirms that GOLD_PRICE is a statistically significant variable in explaining variations in SILVER_PRICE.

Granger Causality Test

H₀₁: Lagged values of the gold price do not provide statistically significant information for predicting the silver price.

H₀₂: Lagged values of the silver price do not provide statistically significant information for predicting the gold price

Results:

Lagged values of GOLD_RATE do not significantly predict SILVER_RATE.

- o F-Statistic: 8.65482
- o Probability (p-value): 0.0002

Lagged values of SILVER_RATE do not significantly predict GOLD_RATE.

- o F-Statistic: 1.22701
- o Probability (p-value): 0.2932

Interpretation:

GOLD_PRICE → SILVER_PRICE:

The p-value of 0.0002 falls significantly below the 0.05 threshold, leading to the rejection of the null hypothesis. This indicates that changes in gold prices Granger-cause changes in silver prices. In other words, historical gold price data contain valuable predictive information for forecasting silver prices, even after considering the past behaviour of silver itself.

SILVER_PRICE → GOLD_PRICE:

The p-value of .2932 exceeds the .05 level of significance, means silver prices do not Granger-cause gold prices. That is, the historical values of silver prices do not offer any substantial improvement in predicting future gold prices beyond what is already explained by gold's own past values.

(H₀): GOLD_RATES do not have predictive power over SILVER_RATES in the Granger causality framework.

(H₁): GOLD_RATES Granger-causes SILVER_RATES, indicating that past values of gold prices help predict silver prices.

Table 5

Null Hypothesis	Obs	F-Statistic	(p-value)
Past movements in GOLD_RATE do not statistically influence SILVER_RATE.	5440	8.65482	.0002
Past values of SILVER_RATE do not influence future movements in GOLD_RATE.	5440	1.22701	.2932

Source: E-view Output

Interpretation of Granger Causality Results:

Test 1 showed a probability of .0002, which is well below .05 determinant point. This led to rejecting the null hypothesis, indicating strong statistical evidence that changes in GOLD_RATE are useful for predicting future movements in SILVER_RATE. In other words, GOLD_RATE Granger-causes SILVER_RATE (Table 5).

Conversely, Test 2 had a p-value of 0.2932, exceeding the 0.05 significance level. Therefore, the null hypothesis was not rejected, meaning there isn't enough evidence to claim that SILVER_RATE Granger-causes GOLD_RATE. This suggests that past SILVER_RATE values don't significantly improve the prediction of GOLD_RATE.

Findings and Conclusions: Key findings include:

Correlation & Stationarity: Initial graphical analysis showed correlated price movements but confirmed non-stationarity. Logarithmic transformation did not induce stationarity, but first-differencing did, making both series unit of order one (I (1)). **Persistent connection:** Johansen cointegration tests indicated a significant long-term equilibrium between gold and silver prices, suggesting they tend to move together over time despite short-term fluctuations. The cointegration results pointed to an inverse long-term relationship. **Short-Term Dynamics:**

A Vector Autoregressive (VAR) model showed that gold price movements depend only on their own past values, while silver prices are influenced by both past Gold and silver values. Causality: Granger causality tests revealed a unidirectional effect—gold prices predict silver prices, but Silver does not influence gold prices. This suggests that gold acts as a leading indicator for silver price movements. Investment Implications: These results provide insights for investors, indicating that gold price trends may help predict silver price fluctuations. However, low R-squared values in the VAR model highlight the role of other unmodelled factors. Future research could explore additional influencing variables and refine the model.

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