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A unified flexible Weibull-H family for modeling real-life data with properties, Bayesian and non-Bayesian inference

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Abstract

The Weibull distribution is a cornerstone in reliability analysis. In this study, we propose a new family of distributions derived from the Weibull model, called the new Weibull-H (NW-H) family. This family features sub-models that can represent diverse failure rate patterns, including bathtub, increasing, unimodal, decreasing, J-shaped, and inverted J-shaped shapes. Additionally, these sub-models produce various density shapes, such as left-skewed, unimodal, symmetric, bimodal, right-skewed, and J-shaped. We explore the mathematical properties of the NW-H family and focus on the parameters of a specific sub-model, the NW-exponential (NWEx), employing ten different estimation techniques, both classical and Bayesian. Bayesian estimators are calculated using three distinct loss functions. Through numerical simulations, we compare and rank these estimation methods based on partial and overall performance. Our results demonstrate that the Bayesian approach consistently yields the most effective parameter estimates for the NWEx across various loss functions. Moreover, we apply the NWEx distribution to three real-world datasets from engineering and industry, showcasing its superior fit and flexibility compared to several competing exponential models.

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1. Introduction

Over the past three decades, numerous approaches have been proposed to construct new families or to extend classical distributions. These innovative families and generalized distributions offer greater flexibility in various applied fields, including public health, biology, medicine, computer science, engineering, industry, insurance, and life-testing. One common method for creating these extended distributions involves adding one or more parameters to a baseline model, thereby enhancing its adaptability and application across different domains.

Consequently, the statistical literature features numerous families designed to generate new distributions. Among these, some well-known families include the "exponentiated-G" by Mudholkar and Srivastava [1], "Kumaraswamy-G" by Cordeiro and Castro [2], "exponentiated generalized-G" by Yousof et al. [3], "Kumaraswamy transmuted-G" by Afify et al. [4], "transmuted generalized transmuted-G" by Nofal et al. [5], "Marshall–Olkin alpha-power" by Nassar et al. [6], "flexible Weibull-G" by Alizadeh et al. [7], "log–logistic tan-G" by Zaidi et al. [8], "Marshall–Olkin Burr-III-G" by Afify et al. [9], "exponential alpha power-G" by Eghwerido, Ibidoja, and Karokatose [10], "shifted exponential-G" by Eghwerido, Agu, and Ibidoja [11], "new transmuted-G" by Bakouch et al. [12], "weighted Lindley-G" by Alnssyan et al. [13], "generalized Kavya–Manoharan-G" by Mahran et al. [14], "Lambert-G" by Al Abbasi et al. [15], "exponentiated half-logistic type II Topp–Leone-G" by Sengweni, Oluyede, and Rannona [16], and "unit modified Burr III Weibull-G" by Abraham, Nasiru, and Wiredu [17] families, among many others.

This paper introduces a new class of distributions based on the Weibull distribution and the T-X family by Alzaatreh, Lee, and Famoye, [18], termed the new Weibull-H (NW-H) family. A key feature of this NW-H family is its inclusion of two additional shape parameters, which enable its special sub-models to exhibit bimodal density shapes. The NW-H family has several advantageous properties:

(i) The "probability density function" (pdf) of the NW-H family is expressed in a straightforward closed form, making its special sub-models suitable for modeling and analyzing real-life data across various fields; (ii) The addition of two extra parameters allows the special sub-models within the NW-H family to capture a wide range of monotonic and non-monotonic hazard rate (hr) shapes; (iii) The densities of the NW-H sub-models can accommodate various shapes such as right-skewed, reversed J-shaped, symmetric, decreasing-increasing-decreasing, and left-skewed patterns; (iv) The NWEx distribution, a specific sub-model of the NW-H family, provides a strong fit for real-life datasets.

The remainder of the paper is outlined as follows: The NW-H family is defined in Section 2. Section 3 presents four special cases within the NW-H family. Section 4 discusses the mathematical properties of the family. Section 5 explores both classical and Bayesian methods for estimating the parameters of the NWEx distribution. Simulations results are obtained in Section 6. Section 7 presents three real data applications. Section 8 offers final remarks and conclusions.

2. The NW-H Family

The NW-H family can be derived based on the Weibull density and by replacing x with the quantity $\phi(x;\boldsymbol{\varphi}) = H(x;\boldsymbol{\varphi})\big(2 - H(x;\boldsymbol{\varphi})\big)/\bar{H}(x;\boldsymbol{\varphi})$.

The Weibull density takes the form $f(x) = \alpha\,\beta\,x^{\beta-1}\exp\big(-\alpha\,x^{\beta}\big),\ x > 0,$ $\alpha\,\beta > 0$.

Hence, the "cumulative distribution function" (cdf) of the NW-H family follows (for $x > \Re$) as

$$G(x;\alpha,\beta,\boldsymbol{\varphi}) = \alpha\,\beta \int_0^{\phi(x;\boldsymbol{\varphi})} x^{\beta-1}\exp\big(-\alpha\,x^{\beta}\big)dx$$

$$= 1 - \exp\left\{-\alpha\left[\frac{H(x;\boldsymbol{\varphi})\big(2-H(x;\boldsymbol{\varphi})\big)}{1-H(x;\boldsymbol{\varphi})}\right]^{\beta}\right\},\ \alpha,\beta > 0, \tag{2.1}$$

where $H(x;\boldsymbol{\varphi})$ represents the baseline cdf, which is parameterized by the vector $\boldsymbol{\varphi}$ and α and β are shape parameters. It is important to note that the new exponential-H (NEx-H) family, introduced by Jamal et al. [19], emerges as a special case of the NW-H family when $\beta = 1$. Furthermore, a new family, known as the new Rayleigh-H (NR-H) family, arises when $\beta = 2$.

The pdf of the NW-H family reduces to

$$g(x;\alpha,\beta,\boldsymbol{\varphi}) = \frac{\alpha\,\beta\,h(x;\boldsymbol{\varphi})}{[1-H(x;\boldsymbol{\varphi})]^2}\{1 + [1 - H(x;\boldsymbol{\varphi})]^2\}$$

$$[\phi(x;\boldsymbol{\varphi})]^{\beta-1}exp\{-\alpha[\phi(x;\boldsymbol{\varphi})]^{\beta}\}, \tag{2.2}$$

where $h(x;\boldsymbol{\varphi}) = dH(x;\boldsymbol{\varphi})/dx$ denotes the baseline pdf associated with $H(x;\boldsymbol{\varphi})$.

The hr function (hrf) of the NW-H family is obtained as follows:

$$r(x;\alpha,\beta,\boldsymbol{\varphi}) = \frac{\alpha\,\beta\,h(x;\boldsymbol{\varphi})\{1+[1-H(x;\boldsymbol{\varphi})]^2\}}{[1-H(x;\boldsymbol{\varphi})]^2}[\phi(x;\boldsymbol{\varphi})]^{\beta-1}.$$

The "survival function" (sf) and cumulative hrf of the NW-H family are

$$S(x;\alpha,\beta,\boldsymbol{\varphi}) = \exp\left\{-\alpha\left[\frac{H(x;\boldsymbol{\varphi})\big(2-H(x;\boldsymbol{\varphi})\big)}{1-H(x;\boldsymbol{\varphi})}\right]^{\beta}\right\}$$

and

$$R(x;\alpha,\beta,\boldsymbol{\varphi}) = \alpha\left[\frac{H(x;\boldsymbol{\varphi})\big(2-H(x;\boldsymbol{\varphi})\big)}{1-H(x;\boldsymbol{\varphi})}\right]^{\beta}.$$

3. Special Sub-Models

In this section, we present four specific sub-models of the NW-H family, derived from the baseline "exponential" (Ex), "Weibull" (W), "Lomax" (Lo), and "Burr III" (BIII) models. These new models are named the NW-exponential (NWEx), NW-Weibull (NWW), NW-Lomax (NWLo), and NW-Burr III (NWBIII) distributions. Each of these sub-models is designed to accommodate a variety of hrf shapes, including J-shaped, increasing, modified bathtub, decreasing, reversed-J-shaped, bathtub, and upside-down bathtub forms. Additionally, the densities of these models can exhibit various characteristics such as reversed J, bimodal, unimodal, right-skewed, symmetric, increasing, left-skewed, and decreasing shapes.

These shapes are shown in Figures 1-4, which present the plots of the density and failure rate functions for the NWEx, NWW, NWLo, and NWBIII distributions.

3.1 *NWEx Distribution*

The pdf of the Ex distribution is given by $h(x) = \theta\ e^{-\theta x},\ x > 0,\ \theta > 0.$ The cdf and pdf of the NWEx model are expressed (for $\alpha, \beta, \theta > 0$) as

$$G(x) = 1 - \exp\left\{-\alpha\left[e^{\theta x} - e^{-\theta x}\right]^{\beta}\right\}, \quad x > 0$$

and

$$g(x) = \alpha\ \beta\ \theta\ e^{\theta x}\left[e^{\theta x} - e^{-\theta x}\right]^{\beta-1}\left\{1 + e^{-2\theta x}\right\}\exp\left\{-\alpha\left[e^{\theta x} - e^{-\theta x}\right]^{\beta}\right\}.$$

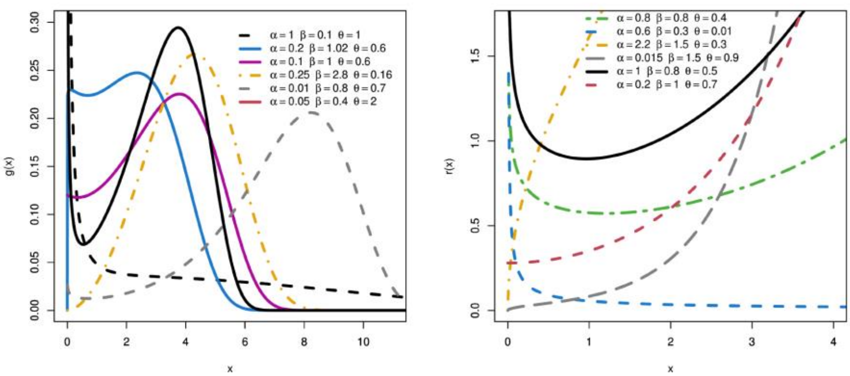

Figure 1
The pdf and hrf plots for the NWEx distribution for various parameter values.

3.2 *NWW Distribution*

Consider the W distribution with pdf given by $h(x) = \theta\ a x^{a-1}\ e^{-\theta x^{a}}, x > 0, \theta, a > 0$. Then, the cdf and pdf of the NWW distribution take the forms (for $x > 0$)

$$G(x) = 1 - \exp\left\{-\alpha\left[e^{\theta x^{a}} - e^{-\theta x^{a}}\right]^{\beta}\right\},\ \alpha, \theta, \beta, a > 0$$

and

$$g(x) = \alpha\, \theta\, \beta\, a\ x^{a-1}\ e^{\theta x^{a}}\left[e^{\theta x^{a}} - e^{-\theta x^{a}}\right]^{\beta-1}\left\{1 + e^{-2\theta x^{a}}\right\}$$
$$\exp\left\{-\alpha\left[e^{\theta x^{a}} - e^{-\theta x^{a}}\right]^{\beta}\right\}.$$

It is noted that the NWEx and NW-Rayleigh models emerge as special cases of the NWW distribution when It is important to note that the NWEx and NW-Rayleigh models are special cases of the NWW distribution, occurring when $a = 1$ and $a = 2$, respectively. Additionally, two new models-namely the

NExW (Jamal et al. [19]) and NRW distributions-emerge when $\beta = 1$ and $\beta = 2$, respectively. Moreover, the NExW model simplifies to the NExEx model when $a = \beta = 1$, and to the NExR model when $a = 2$ and $\beta = 1$. Finally, the NRW distribution reduces to the NREx model when $a = 1$ and $\beta = 2$, and to the NRR model when $a = \beta = 2$.

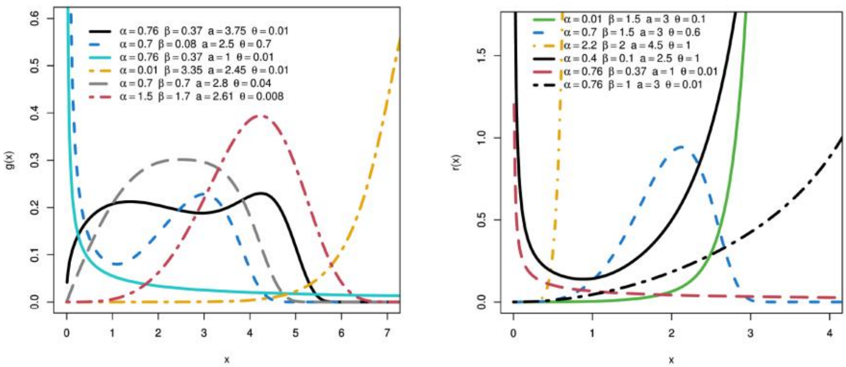

Figure 2
The pdf and hrf plots for the NWW distribution for various parameter values.

3.3 *NWLo Distribution*

Consider the Lo distribution with pdf $h(x) = \theta(x+1)^{-\theta-1}, x > 0, \theta > 0$. The cdf and pdf of the NWLo reduce to

$$G(x) = 1 - \exp\left\{-\alpha\left[\delta(x)^{\theta} - \delta(x)^{-\theta}\right]^{\beta}\right\},\ \ \alpha, \theta, \beta > 0$$

and

$$g(x) = \alpha\, \theta\, \beta\, \delta(x)^{\theta-1}\left[1 + \delta(x)^{-2\theta}\right]\left[\delta(x)^{\theta} - \delta(x)^{-\theta}\right]^{\beta-1}$$
$$\exp\left\{-\alpha\left[\delta(x)^{\theta} + \delta(x)^{-\theta}\right]^{\beta}\right\},$$

where $\delta(x) = (x+1)$.

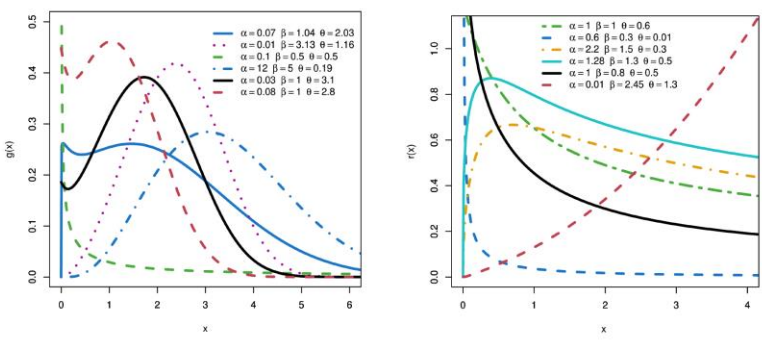

Figure 3
The pdf and hrf plots for the NWLo distribution for various parameter values

3.4 *NWBIII Distribution*

Consider the BIII distribution (Burr [20]) with pdf $h(x) = a\, b\, (x^{-a} + 1)^{-b-1} x^{-a-1},\ x > 0,\ a, b > 0$. Hence, the cdf of NWBIII distribution follows (for $\alpha, \beta, a, b > 0$) as

$$G(x) = 1 - \exp\left\{-\alpha\left[\frac{2-\zeta(x)^{-b}}{\zeta(x)^{b}-1}\right]^{\beta}\right\},\ x > 0.$$

Its pdf has the form

$$g(x) = \alpha\,\beta\, a\, b\, \zeta(x)^{-b-1}\, x^{-a-1}\, \{[1-\zeta(x)^{-b}]^{-2}+1\} \left[\frac{2-\zeta(x)^{-b}}{\zeta(x)^{-b}-1}\right]^{\beta-1} \exp\left\{-\alpha\left[\frac{2-\zeta(x)^{-b}}{\zeta(x)^{-b}-1}\right]^{\beta}\right\},$$

where $\zeta(x) = x^{-a} + 1$.

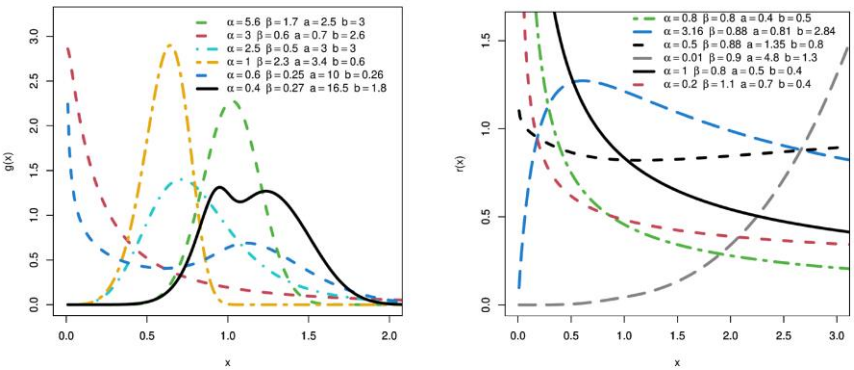

Figure 4

The pdf and hrf plots for the NWBIII distribution for various parameter values.

4. Mathematical Properties

This section is dedicated to presenting the key characteristics of the NW-H family.

4.1 *Quantile Function*

The quantile function (qf) of the NW-H family follows as

$$Q(u) = H^{-1}\left\{\frac{\left[\frac{-\log(1-u)}{\alpha}\right]^{\frac{1}{\beta}}+2\pm\sqrt{\left[\frac{\log(1-u)}{\alpha}\right]^{\frac{2}{\beta}}+4}}{2}\right\}, \qquad (4.1)$$

where $H^{-1}(.)$ is the baseline qf and u is a uniform random number on the interval $(0,1)$.

4.2 *Linear Representation*

The mixture form for the cdf (2.1) of the NW-H family is derived using the following series expansions:

$$\exp(-a\,x) = \sum_{i=0}^{\infty} \frac{(-a)^i}{i!}\, x^i$$

$$(1-z)^q = \sum_{i=0}^{\infty} \binom{q}{i} (-1)^i z^i$$

and

$$(1+z)^q = \sum_{i=0}^{\infty} \binom{q}{i} z^i.$$

Hence, the cdf (2.1) can be express as

$$G(x) = 1 - \exp\left\{-\alpha \left[\frac{H(x)(2-H(x))}{1-H(x)}\right]^{\beta}\right\}$$

$$= 1 - \sum_{i=0}^{\infty} \frac{(-\alpha)^i\; H(x)^{i\beta}}{i!\;[1-H(x)]^{i\beta}} [1 + (1-H(x))]^{i\beta}.$$

The quantity $\{1 + [1 - H(x)]\}^{i\beta} = \sum_{k=0}^{\infty} \binom{i\beta}{k} [1 - H(x)]^k$. Hence, the above equation follows as

$$G(x) = 1 - \sum_{i,k=0}^{\infty} \frac{(-\alpha)^i\, H(x)^{i\,\beta}}{i!} \binom{i\beta}{k} [1 - H(x)]^{k-i\beta}$$

$$= 1 - \sum_{i,k,j=0}^{\infty} \frac{(-\alpha)^i (-1)^j}{i!} \binom{i\beta}{k} \binom{k-i\beta}{j} H(x)^{j+i\beta}.$$

Hence, the cdf of NW-H family is expressed as

$$G(x) = 1 + \sum_{i,j=0}^{\infty} \omega_{i,j}\;\; H(x)^{j+i\beta}, \tag{4.2}$$

where $\omega_{i,j} = \sum_{k=0}^{\infty} \frac{(-\alpha)^i\;(-1)^{j+1}}{i!} \binom{i\beta}{k} \binom{k-i\beta}{j}$.

By differentiating (4.2), it follows the NW-H density

$$g(x) = \sum_{i,j=0}^{\infty} \omega_{i,j}\;\; \tau_{j+i\beta}(x), \tag{4.3}$$

where $\tau_{j+i\beta}(x) = (j + i\beta)\, H(x)^{j+i\beta-1}\, h(x)$ denotes the exp-H density with power parameter $j + i\beta$.

4.3 *Moments*

The expression from the Equation (4.3) can be used to obtain the The r^{th} moments of the NW-H family as follows

$$\mu_r' = E(X^r) = \int_{-\infty}^{\infty} x^r\; g(x)dx = \sum_{i,j=0}^{\infty} \omega_{i,j}\;\; \int_0^{\infty} x^r \tau_{j+i\beta}(x)dx.$$

Therefore

$$\mu_r' = E(X^r) = \sum_{i,j=0}^{\infty} \omega_{i,j}\;\; Q_{j+i\beta,r}(x),$$

where $Q_{j+i\beta,r}(x) = \int_0^{+\infty} x^r \tau_{j+i\beta}(x)dx$.

The m^{xh} "incomplete moment" of the NW-H family is expressed by

$$\mu^m(t) = \int_{-\infty}^{t} x^m g(x)dx = \sum_{i,j=0}^{\infty} \omega_{i,j} \; K_{j+i\beta,m}(x),$$

where $K_{j+i\beta,m}(x) = \int_0^t x^m \tau_{j+i\beta}(x)dx$.

The "negative moments" of the NW-H family is defined as

$$\mu'_{-r}(X^{-r}) = \int_{-\infty}^{+\infty} x^{-r} g(x)dx = \sum_{i,j=0}^{\infty} \omega_{i,j} \; L_{j+i\beta}(x),$$

where $L_{j+i\beta}(x) = \int_0^{\infty} x^{-r} \tau_{j+i\beta}(x)dx$.

The "moment generating function" of the NW-H family follows as

$$M_X(t) = E(e^{tX}) = \sum_{i,j,l=0}^{\infty} \frac{t^l}{l!} \omega_{i,j} \; T(x),$$

where $T(x) = \int_0^{\infty} x^l \tau_{j+i\beta}(x)dx$.

4.4 *Order Statistics*

Consider a "random sample $X_1, \ldots, X_n$" from the NW-H family with cdf (4.2) and pdf (4.3). Let $X_{i:n}$ is i^{th} order statistic, then the pdf of $X_{i:n}$ is

$$g_{i:n}(x) \; = \frac{n!}{(i-1)!(n-i)!} g(x) \; G(x)^{i-1} \; \bar{G}(x)^{n-i},$$

where

$$G(x)^{i-1} = \left[1 + \sum_{i,j=0}^{\infty} \omega_{i,j} \; H(x)^{j+i\beta}\right]^{i-1} = \sum_{l=0}^{\infty} \binom{i-1}{l}$$
$$\left[\sum_{i,j=0}^{\infty} \omega_{i,j} \; H(x)^{j+i\beta}\right]^{l}$$

and

$$\bar{G}(x)^{n-i} = \left[\sum_{i,j=0}^{\infty} \omega_{i,j} \; H(x)^{j+i\beta}\right]^{n-i}.$$

Hence, we have

$$g(x) \, G(x)^{i-1} \, \bar{G}(x)^{n-i} = \kappa_{i,j,l} \; \tau_{j+i\beta}(x) \left[H(x)^{j+i\,\beta}\right]^{n+l-i},$$

where $\kappa_{i,j,l} = \left[\sum_{i,j=0}^{\infty} \omega_{i,j}\right]^{n+l-i+1} \sum_{l=0}^{\infty} \binom{i-1}{l}$.

Then, the pdf of the i^{th} order statistic for the NW-H family takes the form

$$g_{i:n}(x) = \frac{n! \; \kappa_{i,j,l}}{(i-1)!(n-i)!} \tau_{j+i\beta}(x) \left[H(x)^{j+i\beta}\right]^{n+l-i},$$

where $H(x)$ and $\tau_{j+i\beta}(x)$ are the cdf and pdf of a baseline distribution.

4.5 *Rényi Entropy*

The "Rényi entropy" of a random variable (rv) X, denoted as $I_\theta(X)$, is a measure of the variation in uncertainty. The "Rényi entropy" is defined as:

$$I_\theta(X) = \frac{1}{1-\theta}\log\left[\int_{-\infty}^{\infty} g(x)^\theta dx\right],$$

where $g(x)$ is the pdf of the NW-H family.

Based on Equation (4.3), the "Rényi entropy" of the NW-H family follows as

$$I_\theta(X) = \frac{1}{1-\theta}\log\left[\left(\sum_{i,j=0}^{\infty} \omega_{i,j}\right)^\theta \int_{-\infty}^{\infty} \tau_{j+i\beta}(x)^\theta dx\right].$$

The Shannon entropy follows from $I_\theta(X)$ when $\theta \rightarrow 1.$

5. Estimation of the NWEx Parameters

In this Section, non-Bayesian and Bayesian estimators methods are considered. In first subsection, we will consider seven non-Bayesian estimators methods such as the classical estimators include the "maximum likelihood" estimators (MLE), "least squares" estimators (LSE), "weighted least squares" estimators (WLSE), "maximum product of spacings" (MPS), "Cramér von-Mises" estimators (CRVME), "Anderson–Darling" estimators (ADE), and right-tailed ADE (RTADE). In the second subsection, the Bayesian estimators (BEs) method under the three loss functions including the "squared error" (SE), "general entropy" (GE), and "linear exponential" (LN).

5.1 *Non-Bayesian Estimators*

The log-likelihood function for the NWEx distribution, given the parameter vector $\mathbf{\Omega} = (\alpha, \beta, \theta)^T$, is:

$$\ell(\mathbf{\Omega}) = n\log(\alpha) + n\log(\beta) + n\log(\theta) + \theta\sum_{i=1}^{n} x_i + (\beta-1)\sum_{i=1}^{n} \log\rho_i + \sum_{i=1}^{n} \log\left[1+e^{-2\theta x_i}\right] - \alpha\sum_{i=1}^{n} \rho_i{}^\beta,$$

where $\rho_i = e^{\theta x_i} - e^{-\theta x_i}$.

The MLE of α, β and θ are determined by solving the following equations:

$$\frac{\partial}{\partial\alpha}\ell(\mathbf{\Omega}) = \frac{n}{\alpha} - \sum_{i=1}^{n} \rho_i{}^\beta,$$

$$\frac{\partial}{\partial\beta}\ell(\mathbf{\Omega}) = \frac{n}{\beta} + \sum_{i=1}^{n} \log \rho_i - \alpha\sum_{i=1}^{n} \rho_i{}^\beta \log \rho_i$$

and

$$\frac{\partial}{\partial\theta}\ell(\mathbf{\Omega}) = \frac{n}{\theta} + \sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \frac{-2x_i e^{-2\theta x_i}}{1+e^{-2\theta x_i}} + (\beta-1)\sum_{i=1}^{n} \frac{x_i(e^{\theta x_i} + e^{-\theta x_i})}{\rho_i} - \alpha\sum_{i=1}^{n} \frac{\beta\rho_i{}^{\beta-1} x_i(e^{\theta x_i} + e^{-\theta x_i})}{\rho_i}.$$

The LSE and WLSE are determined by minimizing the following quantity

$$S(\mathbf{\Omega}) = \sum_{i=1}^{n} v_i \left[1 - \exp\left(-\alpha\rho_{i:n}{}^{\beta}\right) - \frac{i}{n+1}\right]^2,$$

where $\rho_{i:n} = e^{\theta x_{i:n}} - e^{-\theta x_{i:n}}, v_i = 1$ for the LSE, and $v_i = \frac{(n+1)^2(n+2)}{i(n-i+1)}$ for the WLSE method.

Also, the LSE and WLSE are obtained by solving the following equations simultaneously

$$\sum_{i=1}^{n} v_i \left[1 - \exp\left(-\alpha\rho_{i:n}{}^{\beta}\right) - \frac{i}{n+1}\right] \Delta_s(x_{i:n}|\mathbf{\Omega}) = 0, \quad s = 1,2,3,$$

where

$$\Delta_1(x_{i:n}|\mathbf{\Omega}) = \frac{\partial}{\partial\alpha} G(x_{i:n}|\mathbf{\Omega}) = \exp\left(-\alpha\rho_{i:n}{}^{\beta}\right)\rho_{i:n}{}^{\beta},$$

$$\Delta_2(x_{i:n}|\mathbf{\Omega}) = \frac{\partial}{\partial\beta} G(x_{i:n}|\mathbf{\Omega}) = \alpha\exp\left(-\alpha\rho_{i:n}{}^{\beta}\right) \rho_{i:n}{}^{\beta}\log(\rho_{i:n})$$

and

$$\Delta_3(x_{i:n}|\mathbf{\Omega}) = \frac{\partial}{\partial\theta} G(x_{i:n}|\mathbf{\Omega}) = \alpha\beta\exp\left(-\alpha\rho_{i:n}{}^{\beta}\right)\rho_{i:n}{}^{\beta-1} x_{i:n}(e^{\theta x_{i:n}} + e^{-\theta x_{i:n}}).$$

The uniform spacings of a rv from the NWEx distribution are defined be $D_i(\mathbf{\Omega}) = G(x_{i:n}|\mathbf{\Omega}) - G(x_{i-1:n}|\mathbf{\Omega}),\ i = 1,2,\dots,n+1,$ where $G(x_{0:n}|\mathbf{\Omega}) = 0$, $G(x_{n+1:n}|\mathbf{\Omega}) = 1$ and $D_0(\mathbf{\Omega}) + D_1(\mathbf{\Omega}) + .. + D_{n+1}(\mathbf{\Omega}) = 1$.

The MPS estimators (MPSE) can be obtained by maximizing the geometric mean of the spacings as follows

$$M(\mathbf{\Omega}) = \frac{1}{n+1}\sum_{i=1}^{n+1} \log D_i(\mathbf{\Omega}).$$

The MPSE can also be found by solving the nonlinear equations defined by

$$\frac{1}{n+1}\sum_{i=1}^{n+1} \frac{1}{D_i(\mathbf{\Omega})} [\Delta_s(x_{i:n}|\mathbf{\Omega}) - \Delta_s(x_{i-1:n}|\mathbf{\Omega})] = 0, \quad s = 1,2,3.$$

The CRVME of the NWEx parameters follow by minimizing the following function

$$C(\mathbf{\Omega}) = \frac{1}{12n} + \sum_{i=1}^{n} \left[1 - \exp\left(-\alpha\rho_{i:n}{}^{\beta}\right) - \frac{2i-1}{2n}\right]^2.$$

The ADE for the NWEx parameters are derived by minimizing the following expression:

$$A(\mathbf{\Omega}) = -n - \frac{1}{n} + \sum_{i=1}^{n} (2i-1) \ [\log G(x_{i:n}|\mathbf{\Omega}) + \log S(x_{n+1-i:n}|\mathbf{\Omega})].$$

The RADE of the NWEx parameters are determined by minimizing the following expression:

$$R(\mathbf{\Omega}) = \frac{n}{2} - 2\sum_{i=1}^{n} G(x_{i:n}|\mathbf{\Omega}) - \frac{1}{n}\sum_{i=1}^{n} (2i-1) \ \log S(x_{n+1-i:n}|\mathbf{\Omega}).$$

5.2 *Bayesian Estimators*

The NWEx parameters, α, β and θ, are assumed to be independent and to follow "gamma prior" distributions. Consequently, the "joint prior" density is given by

$$j(\mathbf{\Omega}) = \frac{b^a}{\Gamma(a)} \frac{d^c}{\Gamma(c)} \frac{v^h}{\Gamma(h)} \alpha^{a-1} \beta^{c-1} \theta^{h-1} \; e^{-(b\alpha+d\beta+v\theta)}, \alpha, \beta, \theta > 0, \tag{5.1}$$

where $\Gamma(a), \Gamma(c)$ and $\Gamma(h)$ are the standard gamma functions.

The likelihood function for $\underline{x}$ takes the form

$$\ell(\mathbf{\Omega}|\underline{x}) = \alpha^n \; \beta^n \; \theta^n e^{\theta \sum_{i=1}^{n} x_i} \sum_{i=1}^{n} \left[\rho_i^{\beta-1}\left(1 + e^{-2\theta x_i}\right)\exp\left(-\alpha\rho_i^{\beta}\right)\right]. \tag{5.2}$$

The joint pdf of α, β and θ and the sample $\underline{x}$ can be written as follows

$$\pi(\mathbf{\Omega}|\underline{x}) \propto \ell(\mathbf{\Omega}|\underline{x}) j(\mathbf{\Omega})$$

$$\propto \; \alpha^{n+a-1}\beta^{n+c-1}\theta^{n+h-1}e^{-(b\alpha+d\beta+v\theta)}\left\{e^{\theta \sum_{i=1}^{n} x_i} \sum_{i=1}^{n} \rho_i{}^{\beta-1}\left(1 + e^{-2\theta x_i}\right)\exp\left(-\alpha\rho_i{}^{\beta}\right)\right\}. \tag{5.3}$$

Based on Equation (5.3), the joint posterior density function of $\mathbf{\Omega}|\underline{x}$ is given by

$$\pi^*(\mathbf{\Omega}|\underline{x}) = \frac{\ell(\mathbf{\Omega}|\underline{x}) j(\mathbf{\Omega})}{\int_{\mathbf{\Omega}} \ell(\mathbf{\Omega}|\underline{x}) j(\mathbf{\Omega}) d\mathbf{\Omega}}.$$

Therefore, under the aforementioned three loss functions, the BEs for the parameters α, β and θ are expressed as:

$$\widehat{\mathbf{\Omega}}_{SE} = E(\mathbf{\Omega}|\underline{x}) = \frac{\mathbf{\Omega} \; \pi(\mathbf{\Omega}|\underline{x})}{\int_{\mathbf{\Omega}} \ell(\mathbf{\Omega}|\underline{x}) j(\mathbf{\Omega}) d\mathbf{\Omega}} \;, \tag{5.4}$$

$$\widehat{\mathbf{\Omega}}_{LN} \;\; = -\frac{1}{c} \log\{E_{\Omega}[\exp(-c\;\mathbf{\Omega})]\} = -\frac{1}{c}\left[\frac{\int_{\mathbf{\Omega}} e^{-c\;\mathbf{\Omega}} \; \pi(\mathbf{\Omega}|\underline{x}) d\mathbf{\Omega}}{\int_{\mathbf{\Omega}} \ell(\mathbf{\Omega}|\underline{x}) j(\mathbf{\Omega}) d\mathbf{\Omega}}\right] \tag{5.5}$$

and

$$\widehat{\mathbf{\Omega}}_{GE} \;\; = [E_{\Omega}\;(\mathbf{\Omega}^{-q})]^{\frac{-1}{q}} = \left[\frac{\int_{\mathbf{\Omega}} \mathbf{\Omega}^{-q} \; \pi(\mathbf{\Omega}|\underline{x}) d\mathbf{\Omega}}{\int_{\mathbf{\Omega}} \ell(\mathbf{\Omega}|\underline{x}) j(\mathbf{\Omega}) d\mathbf{\Omega}}\right]^{\frac{-1}{q}}. \tag{5.6}$$

The ratio of the integrals in Equations (5.4), (5.5)) and (5.6) are difficult to expressed in explicit forms. To approximate these integrals, we employ the Monte Carlo Markov Chain (MCMC) method, using the Metropolis-Hastings algorithm as an example to obtain the Bayesian estimates.

6. Numerical Simulations

In this section, we conduct a simulation study to evaluate the performance of various classical and BEs for estimating the parameters of the NWEx distribution. Random samples ($N = 5000$) from the NWEx distribution are generated for some parametric values, $\alpha = (0.1, 0.15, 0.3, 0.4, 0.5, 0.6, 1.5)$, $\beta = (0.4, 0.5, 0.6, 0.7, 1.0)$ and $\theta = (0.2, 0.5, 0.6, 0.8, 1.0, 1.5, 2.5, 3.0)$ and for a range of sample sizes, $n =$ 30, 50, 100, 250, and 500.

The performance of the estimators is assessed using the following measures: The average of absolute biases ($|BIAS|$) is defined by $|BIAS(\widehat{\boldsymbol{\Omega}})| = \frac{1}{N}\sum_{i=1}^{N} |\widehat{\boldsymbol{\Omega}} - \boldsymbol{\Omega}|$, mean-squared errors (MSE) is given by MSE$= \frac{1}{N}\sum_{i=1}^{N} (\widehat{\boldsymbol{\Omega}} - \boldsymbol{\Omega})^2$ and mean relative errors (MRE) is calculated by MRE$= \frac{\frac{1}{N}\sum_{i=1}^{N} |\widehat{\boldsymbol{\Omega}} - \boldsymbol{\Omega}|}{\boldsymbol{\Omega}}$.

All simulation results are conducted with R software (version 4.2.1). Samples are generated from the NWEx model based on its qf, as given by

$$Q(u) = \frac{-1}{\theta}\log\left\{1 - \frac{\left[\frac{-\log(1-u)}{\alpha}\right]^{\frac{1}{\beta}} + 2 \pm \sqrt{\left[\frac{\log(1-u)}{\alpha}\right]^{\frac{2}{\beta}} + 4}}{2}\right\}.$$

Tables 1–11 present the values of $|BIAS|$, MSE, and MRE for various estimators including MLE, WLSE, LSE, ADE, MPSE, RADE, CVME, LN, SE, and GE. Each table presents superscripts indicating the ranking of each estimator among those considered, along with ($\sum = \sum$ Ranks), which signifies the cumulative sum of ranks for each column at a specific sample size.

Table 12 provides a summary of both the partial and total rankings of the estimators. Our analysis demonstrates that the Bayesian approach consistently surpasses all classical methods across the three considered loss functions, securing total scores of 56, 127, and 147 for the SE, LN, and GE loss functions, respectively. In contrast, the ML method outperforms other classical estimation techniques, attaining a cumulative score of 237. Consequently, the ML method emerges as the most effective classical estimator for the NWEx parameters. This method will be employed in the next section to assess the performance of the NWEx model in comparison to other competing models. Ultimately, the Bayesian method with the SE loss function is determined to be the most reliable for estimating the NWEx parameters.

Table 1

Results of the numerical simulation for the NWEx distribution with α = 0.1, β = 0.5, θ = 2.5.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.10233 {5}	0.25016 {8}	0.16654 {6}	0.25951 {9}	0.06830 {4}	0.17551 {7}	0.50105 {10}	0.01000 {2.5}	0.01000 {2.5}	0.00992 {1}
	$\lvert BIAS\rvert$	β	0.25897 {5}	0.37091 {8}	0.32274 {7}	0.38234 {9}	0.23587 {4}	0.29888 {6}	0.46614 {10}	0.04599 {3}	0.04596 {2}	0.04591 {1}
		θ	1.14876 {4}	2.64632 {10}	2.51947 {9}	2.00570 {7}	2.11986 {8}	1.76300 {5}	1.97621 {6}	0.24997 {2}	0.24290 {1}	0.26701 {3}
		α	2.54195 {9}	1.28756 {6}	1.18845 {5}	1.57759 {7}	0.14832 {4}	1.67203 {8}	5.71164 {10}	0.00014 {2.5}	0.00014 {2.5}	0.00013 {1}
30	MSE	β	0.16781 {5}	0.28114 {8}	0.22236 {7}	0.34774 {9}	0.11610 {4}	0.22151 {6}	0.49371 {10}	0.00299 {3}	0.00293 {2}	0.00290 {1}
		θ	2.95020 {4}	19.07024 {8}	19.26486 {9}	9.89594 {6}	26.05138 {10}	10.22211 {7}	8.00664 {5}	0.09157 {2}	0.08497 {1}	0.10555 {3}
		α	1.02326 {5}	2.50163 {8}	1.66544 {6}	2.59511 {9}	0.68299 {4}	1.75508 {7}	5.01054 {10}	0.09782 {1}	0.10002 {3}	0.09917 {2}
	MRE	β	0.51795 {5}	0.74182 {8}	0.64548 {7}	0.76468 {9}	0.47174 {4}	0.59777 {6}	0.93227 {10}	0.08920 {1}	0.09191 {3}	0.09181 {2}
		θ	0.45951 {4}	1.05853 {10}	1.00779 {9}	0.80228 {7}	0.84794 {8}	0.70520 {5}	0.79049 {6}	0.09148 {1}	0.09716 {2}	0.10680 {3}
		$\sum$	46.0 {4}	74.0 {9}	65.0 {7}	72.0 {8}	50.0 {5}	57.0 {6}	77.0 {10}	18.0 {2}	19.0 {3}	17.0 {1}
		α	0.07921 {7}	0.12361 {9}	0.06153 {6}	0.12275 {8}	0.03515 {4}	0.04632 {5}	0.16209 {10}	0.00995 {2}	0.01000 {3}	0.00994 {1}
	$\lvert BIAS\rvert$	β	0.18076 {5}	0.29041 {8}	0.23166 {7}	0.29751 {9}	0.17909 {4}	0.20615 {6}	0.37389 {10}	0.04553 {1}	0.04568 {3}	0.04566 {2}
		θ	0.85653 {4}	1.76390 {10}	1.46881 {7}	1.52578 {8}	1.41461 {6}	1.11135 {5}	1.55125 {9}	0.24330 {2}	0.23800 {1}	0.25474 {3}
		α	1.53434 {10}	0.81247 {9}	0.15253 {6}	0.68721 {7}	0.00237 {4}	0.09844 {5}	0.69140 {8}	0.00014 {3}	0.00013 {1.5}	0.00013 {1.5}
50	MSE	β	0.07390 {5}	0.18568 {8}	0.11895 {7}	0.21012 {9}	0.05663 {4}	0.09639 {6}	0.31716 {10}	0.00290 {3}	0.00287 {1.5}	0.00287 {1.5}
		θ	1.52161 {4}	7.21798 {9}	4.70156 {7}	5.09125 {8}	7.28116 {10}	2.67899 {5}	4.50555 {6}	0.08536 {2}	0.08018 {1}	0.09395 {3}
		α	0.79209 {7}	1.23611 {9}	0.61528 {6}	1.22752 {8}	0.35152 {4}	0.46317 {5}	1.62088 {10}	0.09835 {1}	0.09999 {3}	0.09945 {2}
	MRE	β	0.36152 {5}	0.58082 {8}	0.46331 {7}	0.59502 {9}	0.35818 {4}	0.41229 {6}	0.74777 {10}	0.08937 {1}	0.09135 {3}	0.09131 {2}
		θ	0.34261 {4}	0.70556 {10}	0.58752 {7}	0.61031 {8}	0.56584 {6}	0.44454 {5}	0.62050 {9}	0.09108 {1}	0.09520 {2}	0.10190 {3}
		$\sum$	51.0 {6}	80.0 {9}	60.0 {7}	74.0 {8}	46.0 {4}	48.0 {5}	82.0 {10}	16.0 {1}	19.0 {2.5}	19.0 {2.5}
		α	0.02036 {4}	0.02954 {9}	0.02282 {7}	0.02808 {8}	0.02119 {6}	0.02077 {5}	0.04216 {10}	0.00953 {1}	0.00960 {3}	0.00957 {2}
	$\lvert BIAS\rvert$	β	0.11767 {4}	0.20422 {9}	0.15646 {7}	0.19713 {8}	0.12038 {5}	0.14221 {6}	0.28034 {10}	0.04472 {1}	0.04502 {3}	0.04501 {2}
		θ	0.59813 {4}	1.12759 {9}	0.85826 {7}	1.00219 {8}	0.78195 {6}	0.73332 {5}	1.17494 {10}	0.23359 {2}	0.23042 {1}	0.23781 {3}
		α	0.00070 {4.5}	0.00399 {9}	0.00093 {7}	0.00376 {8}	0.00077 {6}	0.00070 {4.5}	0.04376 {10}	0.00013 {2.5}	0.00013 {2.5}	0.00012 {1}
100	MSE	β	0.02670 {5}	0.08656 {9}	0.05011 {7}	0.08265 {8}	0.02368 {4}	0.03900 {6}	0.17934 {10}	0.00278 {2.5}	0.00278 {2.5}	0.00277 {1}
		θ	0.67963 {4}	2.51415 {10}	1.41226 {7}	1.91531 {8}	1.35240 {6}	0.93531 {5}	2.34210 {9}	0.07832 {2}	0.07497 {1}	0.08152 {3}
		α	0.20362 {4}	0.29543 {9}	0.22821 {7}	0.28076 {8}	0.21193 {6}	0.20770 {5}	0.42163 {10}	0.09489 {1}	0.09597 {3}	0.09572 {2}
	MRE	β	0.23533 {4}	0.40843 {9}	0.31292 {7}	0.39425 {8}	0.24076 {5}	0.28441 {6}	0.56069 {10}	0.08868 {1}	0.09003 {3}	0.09001 {2}

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		θ	0.23925 $^{\{4\}}$	0.45103 $^{\{9\}}$	0.34330 $^{\{7\}}$	0.40088 $^{\{8\}}$	0.31278 $^{\{6\}}$	0.29333 $^{\{5\}}$	0.46998 $^{\{10\}}$	0.09075 $^{\{1\}}$	0.09217 $^{\{2\}}$	0.09512 $^{\{3\}}$
		$\sum$	37.5 $^{\{4\}}$	82.0 $^{\{9\}}$	63.0 $^{\{7\}}$	72.0 $^{\{8\}}$	50.0 $^{\{6\}}$	47.5 $^{\{5\}}$	89.0 $^{\{10\}}$	14.0 $^{\{1\}}$	21.0 $^{\{3\}}$	19.0 $^{\{2\}}$
		α	0.01259 $^{\{4\}}$	0.01513 $^{\{9\}}$	0.01401 $^{\{7\}}$	0.01488 $^{\{8\}}$	0.01298 $^{\{5\}}$	0.01378 $^{\{6\}}$	0.01659 $^{\{10\}}$	0.00879 $^{\{1\}}$	0.00887 $^{\{3\}}$	0.00886 $^{\{2\}}$
	$\|BIAS\|$	β	0.07392 $^{\{5\}}$	0.12267 $^{\{9\}}$	0.08864 $^{\{7\}}$	0.12008 $^{\{8\}}$	0.07040 $^{\{4\}}$	0.08557 $^{\{6\}}$	0.16482 $^{\{10\}}$	0.04152 $^{\{1\}}$	0.04189 $^{\{2.5\}}$	0.04189 $^{\{2.5\}}$
		θ	0.37611 $^{\{4\}}$	0.65868 $^{\{9\}}$	0.47513 $^{\{7\}}$	0.62314 $^{\{8\}}$	0.39970 $^{\{5\}}$	0.45460 $^{\{6\}}$	0.74373 $^{\{10\}}$	0.21661 $^{\{1\}}$	0.21859 $^{\{3\}}$	0.21845 $^{\{2\}}$
		α	0.00026 $^{\{4\}}$	0.00039 $^{\{9\}}$	0.00031 $^{\{7\}}$	0.00037 $^{\{8\}}$	0.00027 $^{\{5\}}$	0.00030 $^{\{6\}}$	0.00055 $^{\{10\}}$	0.00011 $^{\{2\}}$	0.00011 $^{\{2\}}$	0.00011 $^{\{2\}}$
250	MSE	β	0.00934 $^{\{5\}}$	0.02821 $^{\{8\}}$	0.01388 $^{\{7\}}$	0.02949 $^{\{9\}}$	0.00813 $^{\{4\}}$	0.01272 $^{\{6\}}$	0.05934 $^{\{10\}}$	0.00243 $^{\{1\}}$	0.00245 $^{\{2.5\}}$	0.00245 $^{\{2.5\}}$
		θ	0.23576 $^{\{4\}}$	0.75623 $^{\{9\}}$	0.36528 $^{\{7\}}$	0.63440 $^{\{8\}}$	0.30739 $^{\{5\}}$	0.33667 $^{\{6\}}$	0.90542 $^{\{10\}}$	0.06601 $^{\{1\}}$	0.06648 $^{\{3\}}$	0.06635 $^{\{2\}}$
		α	0.12594 $^{\{4\}}$	0.15133 $^{\{9\}}$	0.14007 $^{\{7\}}$	0.14883 $^{\{8\}}$	0.12976 $^{\{5\}}$	0.13775 $^{\{6\}}$	0.16594 $^{\{10\}}$	0.08771 $^{\{1\}}$	0.08867 $^{\{3\}}$	0.08861 $^{\{2\}}$
	MRE	β	0.14784 $^{\{5\}}$	0.24535 $^{\{9\}}$	0.17729 $^{\{7\}}$	0.24017 $^{\{8\}}$	0.14081 $^{\{4\}}$	0.17115 $^{\{6\}}$	0.32963 $^{\{10\}}$	0.08274 $^{\{1\}}$	0.08379 $^{\{2.5\}}$	0.08379 $^{\{2.5\}}$
		θ	0.15044 $^{\{4\}}$	0.26347 $^{\{9\}}$	0.19005 $^{\{7\}}$	0.24926 $^{\{8\}}$	0.15988 $^{\{5\}}$	0.18184 $^{\{6\}}$	0.29749 $^{\{10\}}$	0.08637 $^{\{1\}}$	0.08744 $^{\{3\}}$	0.08738 $^{\{2\}}$
		$\sum$	39.0 $^{\{4\}}$	80.0 $^{\{9\}}$	63.0 $^{\{7\}}$	73.0 $^{\{8\}}$	42.0 $^{\{5\}}$	54.0 $^{\{6\}}$	90.0 $^{\{10\}}$	10.0 $^{\{1\}}$	24.5 $^{\{3\}}$	19.5 $^{\{2\}}$
		α	0.00940 $^{\{5\}}$	0.01047 $^{\{10\}}$	0.00968 $^{\{7\}}$	0.00994 $^{\{8\}}$	0.00913 $^{\{4\}}$	0.00957 $^{\{6\}}$	0.01035 $^{\{9\}}$	0.00778 $^{\{1\}}$	0.00785 $^{\{2.5\}}$	0.00785 $^{\{2.5\}}$
	$\|BIAS\|$	β	0.05033 $^{\{5\}}$	0.08127 $^{\{9\}}$	0.05652 $^{\{6\}}$	0.08085 $^{\{8\}}$	0.04901 $^{\{4\}}$	0.05811 $^{\{7\}}$	0.10084 $^{\{10\}}$	0.03693 $^{\{1\}}$	0.03728 $^{\{2.5\}}$	0.03728 $^{\{2.5\}}$
		θ	0.26610 $^{\{5\}}$	0.43517 $^{\{9\}}$	0.30864 $^{\{6\}}$	0.42089 $^{\{8\}}$	0.26002 $^{\{4\}}$	0.31230 $^{\{7\}}$	0.48698 $^{\{10\}}$	0.19449 $^{\{1\}}$	0.19631 $^{\{3\}}$	0.19628 $^{\{2\}}$
		α	0.00014 $^{\{5.5\}}$	0.00018 $^{\{9.5\}}$	0.00015 $^{\{7\}}$	0.00017 $^{\{8\}}$	0.00013 $^{\{4\}}$	0.00014 $^{\{5.5\}}$	0.00018 $^{\{9.5\}}$	0.00009 $^{\{2\}}$	0.00009 $^{\{2\}}$	0.00009 $^{\{2\}}$
500	MSE	β	0.00413 $^{\{5\}}$	0.01123 $^{\{8\}}$	0.00535 $^{\{6\}}$	0.01124 $^{\{9\}}$	0.00396 $^{\{4\}}$	0.00546 $^{\{7\}}$	0.01985 $^{\{10\}}$	0.00198 $^{\{1\}}$	0.00200 $^{\{2.5\}}$	0.00200 $^{\{2.5\}}$
		θ	0.11590 $^{\{4\}}$	0.29569 $^{\{9\}}$	0.15952 $^{\{7\}}$	0.27887 $^{\{8\}}$	0.13172 $^{\{5\}}$	0.15451 $^{\{6\}}$	0.37188 $^{\{10\}}$	0.05470 $^{\{1\}}$	0.05516 $^{\{3\}}$	0.05513 $^{\{2\}}$
		α	0.09400 $^{\{5\}}$	0.10473 $^{\{10\}}$	0.09675 $^{\{7\}}$	0.09936 $^{\{8\}}$	0.09131 $^{\{4\}}$	0.09569 $^{\{6\}}$	0.10351 $^{\{9\}}$	0.07774 $^{\{1\}}$	0.07855 $^{\{3\}}$	0.07851 $^{\{2\}}$
	MRE	β	0.10066 $^{\{5\}}$	0.16253 $^{\{9\}}$	0.11305 $^{\{6\}}$	0.16170 $^{\{8\}}$	0.09802 $^{\{4\}}$	0.11622 $^{\{7\}}$	0.20167 $^{\{10\}}$	0.07376 $^{\{1\}}$	0.07456 $^{\{2.5\}}$	0.07456 $^{\{2.5\}}$
		θ	0.10644 $^{\{5\}}$	0.17407 $^{\{9\}}$	0.12346 $^{\{6\}}$	0.16836 $^{\{8\}}$	0.10401 $^{\{4\}}$	0.12492 $^{\{7\}}$	0.19479 $^{\{10\}}$	0.07768 $^{\{1\}}$	0.07852 $^{\{3\}}$	0.07851 $^{\{2\}}$
		$\sum$	44.5 $^{\{5\}}$	82.5 $^{\{9\}}$	58.0 $^{\{6\}}$	73.0 $^{\{8\}}$	37.0 $^{\{4\}}$	58.5 $^{\{7\}}$	87.5 $^{\{10\}}$	10.0 $^{\{1\}}$	24.0 $^{\{3\}}$	20.0 $^{\{2\}}$

Table 2

Results of the numerical simulation for the NWEx distribution with α = 0.4, β = 0.5, θ = 1.5.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.13361 $^{\{4\}}$	0.33269 $^{\{8\}}$	0.20331 $^{\{6\}}$	0.35528 $^{\{9\}}$	0.16370 $^{\{5\}}$	0.27432 $^{\{7\}}$	0.38375 $^{\{10\}}$	0.03909 $^{\{2\}}$	0.03923 $^{\{3\}}$	0.03905 $^{\{1\}}$
	$\lvert BIAS\rvert$	β	0.11498 $^{\{4\}}$	0.13212 $^{\{8\}}$	0.12649 $^{\{7\}}$	0.13642 $^{\{9\}}$	0.11532 $^{\{5\}}$	0.12488 $^{\{6\}}$	0.15041 $^{\{10\}}$	0.04303 $^{\{1\}}$	0.04328 $^{\{3\}}$	0.04318 $^{\{2\}}$
		θ	0.51472 $^{\{4\}}$	0.69220 $^{\{10\}}$	0.67501 $^{\{8\}}$	0.68313 $^{\{9\}}$	0.60152 $^{\{6\}}$	0.58031 $^{\{5\}}$	0.63112 $^{\{7\}}$	0.13653 $^{\{2\}}$	0.13652 $^{\{1\}}$	0.13710 $^{\{3\}}$
		α	0.04808 $^{\{4\}}$	1.78654 $^{\{7\}}$	0.65590 $^{\{6\}}$	2.95897 $^{\{9\}}$	0.30767 $^{\{5\}}$	2.04960 $^{\{8\}}$	4.44653 $^{\{10\}}$	0.00211 $^{\{3\}}$	0.00209 $^{\{2\}}$	0.00208 $^{\{1\}}$
30	MSE	β	0.02325 $^{\{5\}}$	0.03039 $^{\{8\}}$	0.02631 $^{\{6\}}$	0.03581 $^{\{9\}}$	0.02098 $^{\{4\}}$	0.02754 $^{\{7\}}$	0.04413 $^{\{10\}}$	0.00263 $^{\{2.5\}}$	0.00263 $^{\{2.5\}}$	0.00262 $^{\{1\}}$
		θ	0.51928 $^{\{4\}}$	1.08461 $^{\{10\}}$	0.94496 $^{\{9\}}$	0.91928 $^{\{8\}}$	0.70360 $^{\{6\}}$	0.66138 $^{\{5\}}$	0.75770 $^{\{7\}}$	0.02646 $^{\{2\}}$	0.02604 $^{\{1\}}$	0.02648 $^{\{3\}}$
		α	0.33403 $^{\{4\}}$	0.83171 $^{\{8\}}$	0.50828 $^{\{6\}}$	0.88819 $^{\{9\}}$	0.40925 $^{\{5\}}$	0.68581 $^{\{7\}}$	0.95937 $^{\{10\}}$	0.09591 $^{\{1\}}$	0.09808 $^{\{3\}}$	0.09763 $^{\{2\}}$
	MRE	β	0.22997 $^{\{4\}}$	0.26423 $^{\{8\}}$	0.25297 $^{\{7\}}$	0.27284 $^{\{9\}}$	0.23064 $^{\{5\}}$	0.24977 $^{\{6\}}$	0.30082 $^{\{10\}}$	0.08502 $^{\{1\}}$	0.08656 $^{\{3\}}$	0.08637 $^{\{2\}}$
		θ	0.34314 $^{\{4\}}$	0.46147 $^{\{10\}}$	0.45001 $^{\{8\}}$	0.45542 $^{\{9\}}$	0.40101 $^{\{6\}}$	0.38688 $^{\{5\}}$	0.42075 $^{\{7\}}$	0.09077 $^{\{1\}}$	0.09101 $^{\{2\}}$	0.09140 $^{\{3\}}$
		$\sum$	37.0 $^{\{4\}}$	77.0 $^{\{8\}}$	63.0 $^{\{7\}}$	80.0 $^{\{9\}}$	47.0 $^{\{5\}}$	56.0 $^{\{6\}}$	81.0 $^{\{10\}}$	15.5 $^{\{1\}}$	20.5 $^{\{3\}}$	18.0 $^{\{2\}}$
		α	0.09822 $^{\{4\}}$	0.14995 $^{\{9\}}$	0.10800 $^{\{7\}}$	0.18773 $^{\{10\}}$	0.10772 $^{\{6\}}$	0.10089 $^{\{5\}}$	0.14890 $^{\{8\}}$	0.03842 $^{\{1\}}$	0.03865 $^{\{3\}}$	0.03858 $^{\{2\}}$
	$\lvert BIAS\rvert$	β	0.08429 $^{\{4\}}$	0.10631 $^{\{8\}}$	0.08908 $^{\{7\}}$	0.10682 $^{\{9\}}$	0.08769 $^{\{6\}}$	0.08753 $^{\{5\}}$	0.11465 $^{\{10\}}$	0.04127 $^{\{1\}}$	0.04157 $^{\{2\}}$	0.04160 $^{\{3\}}$
		θ	0.35958 $^{\{4\}}$	0.51068 $^{\{10\}}$	0.42444 $^{\{7\}}$	0.47985 $^{\{9\}}$	0.41545 $^{\{6\}}$	0.39032 $^{\{5\}}$	0.47446 $^{\{8\}}$	0.13446 $^{\{1\}}$	0.13494 $^{\{2\}}$	0.13525 $^{\{3\}}$
		α	0.02800 $^{\{7\}}$	0.37283 $^{\{9\}}$	0.02263 $^{\{5\}}$	0.64762 $^{\{10\}}$	0.02508 $^{\{6\}}$	0.01964 $^{\{4\}}$	0.27778 $^{\{8\}}$	0.00203 $^{\{2\}}$	0.00203 $^{\{2\}}$	0.00203 $^{\{2\}}$
50	MSE	β	0.01257 $^{\{5\}}$	0.01870 $^{\{8\}}$	0.01344 $^{\{7\}}$	0.02062 $^{\{9\}}$	0.01200 $^{\{4\}}$	0.01329 $^{\{6\}}$	0.02442 $^{\{10\}}$	0.00244 $^{\{1.5\}}$	0.00244 $^{\{1.5\}}$	0.00245 $^{\{3\}}$
		θ	0.22034 $^{\{4\}}$	0.47226 $^{\{10\}}$	0.31011 $^{\{7\}}$	0.41350 $^{\{9\}}$	0.28628 $^{\{6\}}$	0.26442 $^{\{5\}}$	0.40345 $^{\{8\}}$	0.02551 $^{\{2\}}$	0.02532 $^{\{1\}}$	0.02553 $^{\{3\}}$
		α	0.24554 $^{\{4\}}$	0.37486 $^{\{9\}}$	0.27000 $^{\{7\}}$	0.46932 $^{\{10\}}$	0.26930 $^{\{6\}}$	0.25223 $^{\{5\}}$	0.37226 $^{\{8\}}$	0.09511 $^{\{1\}}$	0.09664 $^{\{3\}}$	0.09644 $^{\{2\}}$
	MRE	β	0.16859 $^{\{4\}}$	0.21261 $^{\{8\}}$	0.17816 $^{\{7\}}$	0.21364 $^{\{9\}}$	0.17539 $^{\{6\}}$	0.17505 $^{\{5\}}$	0.22931 $^{\{10\}}$	0.08180 $^{\{1\}}$	0.08314 $^{\{2\}}$	0.08320 $^{\{3\}}$
		θ	0.23972 $^{\{4\}}$	0.34045 $^{\{10\}}$	0.28296 $^{\{7\}}$	0.31990 $^{\{9\}}$	0.27696 $^{\{6\}}$	0.26021 $^{\{5\}}$	0.31631 $^{\{8\}}$	0.09003 $^{\{2\}}$	0.08996 $^{\{1\}}$	0.09017 $^{\{3\}}$
		$\sum$	40.0 $^{\{4\}}$	81.0 $^{\{9\}}$	61.0 $^{\{7\}}$	84.0 $^{\{10\}}$	52.0 $^{\{6\}}$	45.0 $^{\{5\}}$	78.0 $^{\{8\}}$	12.5 $^{\{1\}}$	17.5 $^{\{2\}}$	24.0 $^{\{3\}}$
		α	0.06671 $^{\{5\}}$	0.07647 $^{\{10\}}$	0.07102 $^{\{7\}}$	0.07454 $^{\{9\}}$	0.06785 $^{\{6\}}$	0.06601 $^{\{4\}}$	0.07130 $^{\{8\}}$	0.03618 $^{\{1\}}$	0.03646 $^{\{3\}}$	0.03638 $^{\{2\}}$
	$\lvert BIAS\rvert$	β	0.06020 $^{\{4\}}$	0.07303 $^{\{9\}}$	0.06481 $^{\{6\}}$	0.06771 $^{\{8\}}$	0.06391 $^{\{5\}}$	0.06526 $^{\{7\}}$	0.08339 $^{\{10\}}$	0.03733 $^{\{1\}}$	0.03764 $^{\{2\}}$	0.03769 $^{\{3\}}$
		θ	0.26625 $^{\{4\}}$	0.33094 $^{\{9\}}$	0.29799 $^{\{7\}}$	0.31712 $^{\{8\}}$	0.28708 $^{\{6\}}$	0.27999 $^{\{5\}}$	0.33377 $^{\{10\}}$	0.12896 $^{\{1\}}$	0.12961 $^{\{2\}}$	0.12988 $^{\{3\}}$
		α	0.00732 $^{\{4\}}$	0.01050 $^{\{10\}}$	0.00868 $^{\{7\}}$	0.01027 $^{\{9\}}$	0.00798 $^{\{6\}}$	0.00746 $^{\{5\}}$	0.00931 $^{\{8\}}$	0.00183 $^{\{1.5\}}$	0.00184 $^{\{3\}}$	0.00183 $^{\{1.5\}}$
100	MSE	β	0.00589 $^{\{4\}}$	0.00845 $^{\{9\}}$	0.00676 $^{\{6\}}$	0.00741 $^{\{8\}}$	0.00636 $^{\{5\}}$	0.00686 $^{\{7\}}$	0.01147 $^{\{10\}}$	0.00201 $^{\{1\}}$	0.00202 $^{\{2\}}$	0.00203 $^{\{3\}}$
		θ	0.11741 $^{\{4\}}$	0.17922 $^{\{9\}}$	0.14569 $^{\{7\}}$	0.17204 $^{\{8\}}$	0.14212 $^{\{6\}}$	0.12923 $^{\{5\}}$	0.18573 $^{\{10\}}$	0.02365 $^{\{2\}}$	0.02360 $^{\{1\}}$	0.02372 $^{\{3\}}$
		α	0.16678 $^{\{5\}}$	0.19117 $^{\{10\}}$	0.17756 $^{\{7\}}$	0.18634 $^{\{9\}}$	0.16963 $^{\{6\}}$	0.16502 $^{\{4\}}$	0.17825 $^{\{8\}}$	0.09008 $^{\{1\}}$	0.09115 $^{\{3\}}$	0.09096 $^{\{2\}}$

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	MRE	β	0.12041 {4}	0.14605 {9}	0.12963 {6}	0.13542 {8}	0.12782 {5}	0.13051 {7}	0.16677 {10}	0.07410 {1}	0.07527 {2}	0.07538 {3}
		θ	0.17750 {4}	0.22063 {9}	0.19866 {7}	0.21141 {8}	0.19138 {6}	0.18666 {5}	0.22251 {10}	0.08609 {1}	0.08641 {2}	0.08658 {3}
		$\sum$	38.0 {4}	84.0 {9.5}	60.0 {7}	75.0 {8}	51.0 {6}	49.0 {5}	84.0 {9.5}	10.5 {1}	20.0 {2}	23.5 {3}
250	$\|BIAS\|$	α	0.04115 {4}	0.04744 {10}	0.04422 {7}	0.04734 {9}	0.04218 {5}	0.04405 {6}	0.04695 {8}	0.03146 {1}	0.03174 {3}	0.03172 {2}
		β	0.03906 {5}	0.04527 {9}	0.03963 {6}	0.04305 {8}	0.03698 {4}	0.04079 {7}	0.04990 {10}	0.03048 {1}	0.03074 {2}	0.03077 {3}
		θ	0.16180 {4}	0.20447 {9}	0.17582 {6}	0.20081 {8}	0.16378 {5}	0.17793 {7}	0.20690 {10}	0.11903 {1}	0.11985 {2}	0.11997 {3}
	MSE	α	0.00273 {4}	0.00369 {10}	0.00310 {6}	0.00357 {8}	0.00278 {5}	0.00312 {7}	0.00360 {9}	0.00144 {1}	0.00145 {2.5}	0.00145 {2.5}
		β	0.00243 {5}	0.00326 {9}	0.00248 {6}	0.00306 {8}	0.00217 {4}	0.00260 {7}	0.00407 {10}	0.00137 {1}	0.00138 {2.5}	0.00138 {2.5}
		θ	0.04113 {4}	0.06838 {9}	0.04985 {6}	0.06666 {8}	0.04449 {5}	0.05010 {7}	0.06851 {10}	0.02050 {1}	0.02055 {2}	0.02061 {3}
	MRE	α	0.10288 {4}	0.11860 {10}	0.11055 {7}	0.11834 {9}	0.10544 {5}	0.11012 {6}	0.11737 {8}	0.07835 {1}	0.07936 {3}	0.07929 {2}
		β	0.07811 {5}	0.09055 {9}	0.07925 {6}	0.08610 {8}	0.07396 {4}	0.08158 {7}	0.09981 {10}	0.06075 {1}	0.06149 {2}	0.06153 {3}
		θ	0.10787 {4}	0.13631 {9}	0.11721 {6}	0.13388 {8}	0.10919 {5}	0.11862 {7}	0.13793 {10}	0.07920 {1}	0.07990 {2}	0.07998 {3}
		$\sum$	39.0 {4}	84.0 {9}	56.0 {6}	74.0 {8}	42.0 {5}	61.0 {7}	85.0 {10}	9.0 {1}	21.0 {2}	24.0 {3}
500	$\|BIAS\|$	α	0.02931 {4}	0.03255 {10}	0.02993 {6}	0.03162 {9}	0.02976 {5}	0.03011 {8}	0.03010 {7}	0.02646 {1}	0.02672 {3}	0.02671 {2}
		β	0.02670 {4}	0.03008 {8}	0.02678 {5}	0.03103 {9}	0.02699 {6}	0.02839 {7}	0.03455 {10}	0.02421 {1}	0.02444 {2}	0.02445 {3}
		θ	0.11302 {4}	0.13570 {8}	0.11945 {6}	0.13968 {10}	0.11528 {5}	0.12320 {7}	0.13782 {9}	0.10310 {1}	0.10409 {3}	0.10407 {2}
	MSE	α	0.00134 {4}	0.00174 {10}	0.00143 {7}	0.00168 {9}	0.00138 {5.5}	0.00138 {5.5}	0.00151 {8}	0.00107 {1}	0.00108 {2.5}	0.00108 {2.5}
		β	0.00112 {4}	0.00144 {8}	0.00113 {5}	0.00149 {9}	0.00116 {6}	0.00123 {7}	0.00192 {10}	0.00090 {1}	0.00091 {2.5}	0.00091 {2.5}
		θ	0.01983 {4}	0.02861 {8}	0.02286 {6}	0.03063 {10}	0.02214 {5}	0.02319 {7}	0.03045 {9}	0.01614 {1}	0.01628 {3}	0.01627 {2}
	MRE	α	0.07328 {4}	0.08138 {10}	0.07481 {6}	0.07905 {9}	0.07440 {5}	0.07528 {8}	0.07524 {7}	0.06611 {1}	0.06680 {3}	0.06677 {2}
		β	0.05340 {4}	0.06015 {8}	0.05356 {5}	0.06206 {9}	0.05399 {6}	0.05678 {7}	0.06910 {10}	0.04839 {1}	0.04889 {2}	0.04891 {3}
		θ	0.07534 {4}	0.09047 {8}	0.07964 {6}	0.09312 {10}	0.07685 {5}	0.08214 {7}	0.09188 {9}	0.06848 {1}	0.06939 {3}	0.06938 {2}
		$\sum$	36.0 {4}	78.0 {8}	52.0 {6}	84.0 {10}	48.5 {5}	63.5 {7}	79.0 {9}	9.0 {1}	24.0 {3}	21.0 {2}

Table 3

Results of the numerical simulation for the NWEx distribution with α = 0.5, β = 0.6, θ = 0.8.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.23796$^{\{4\}}$	0.87258$^{\{10\}}$	0.53457$^{\{6\}}$	0.85157$^{\{9\}}$	0.34761$^{\{5\}}$	0.70644$^{\{7\}}$	0.81653$^{\{8\}}$	0.04874$^{\{1.5\}}$	0.04886$^{\{3\}}$	0.04874$^{\{1.5\}}$
	$\lvert BIAS\rvert$	β	0.12101$^{\{4\}}$	0.13654$^{\{8\}}$	0.13448$^{\{7\}}$	0.14012$^{\{9\}}$	0.12527$^{\{5\}}$	0.13312$^{\{6\}}$	0.15036$^{\{10\}}$	0.05177$^{\{1\}}$	0.05209$^{\{2.5\}}$	0.05209$^{\{2.5\}}$
		θ	0.30179$^{\{4\}}$	0.40646$^{\{9\}}$	0.39228$^{\{8\}}$	0.40766$^{\{10\}}$	0.34885$^{\{6\}}$	0.33722$^{\{5\}}$	0.34965$^{\{7\}}$	0.07471$^{\{1\}}$	0.07477$^{\{2\}}$	0.07512$^{\{3\}}$
		α	0.98025$^{\{4\}}$	7.83692$^{\{7\}}$	4.26055$^{\{6\}}$	10.22827$^{\{9\}}$	1.57797$^{\{5\}}$	10.06796$^{\{8\}}$	13.51018$^{\{10\}}$	0.00331$^{\{3\}}$	0.00327$^{\{2\}}$	0.00325$^{\{1\}}$
30	MSE	β	0.02481$^{\{5\}}$	0.03090$^{\{8\}}$	0.02939$^{\{6\}}$	0.03510$^{\{9\}}$	0.02421$^{\{4\}}$	0.02969$^{\{7\}}$	0.03961$^{\{10\}}$	0.00379$^{\{1.5\}}$	0.00380$^{\{3\}}$	0.00379$^{\{1.5\}}$
		θ	0.17855$^{\{4\}}$	0.34801$^{\{10\}}$	0.29088$^{\{8\}}$	0.30873$^{\{9\}}$	0.22372$^{\{6\}}$	0.21489$^{\{5\}}$	0.22497$^{\{7\}}$	0.00786$^{\{2\}}$	0.00774$^{\{1\}}$	0.00788$^{\{3\}}$
		α	0.47592$^{\{4\}}$	1.74517$^{\{10\}}$	1.06913$^{\{6\}}$	1.70314$^{\{9\}}$	0.69522$^{\{5\}}$	1.41287$^{\{7\}}$	1.63307$^{\{8\}}$	0.09489$^{\{1\}}$	0.09773$^{\{3\}}$	0.09747$^{\{2\}}$
	MRE	β	0.20169$^{\{4\}}$	0.22756$^{\{8\}}$	0.22413$^{\{7\}}$	0.23354$^{\{9\}}$	0.20879$^{\{5\}}$	0.22187$^{\{6\}}$	0.25060$^{\{10\}}$	0.08551$^{\{1\}}$	0.08682$^{\{2.5\}}$	0.08682$^{\{2.5\}}$
		θ	0.37724$^{\{4\}}$	0.50807$^{\{9\}}$	0.49035$^{\{8\}}$	0.50957$^{\{10\}}$	0.43606$^{\{6\}}$	0.42153$^{\{5\}}$	0.43706$^{\{7\}}$	0.09186$^{\{1\}}$	0.09346$^{\{2\}}$	0.09391$^{\{3\}}$
		$\sum$	37.0$^{\{4\}}$	79.0$^{\{9\}}$	62.0$^{\{7\}}$	83.0$^{\{10\}}$	47.0$^{\{5\}}$	56.0$^{\{6\}}$	77.0$^{\{8\}}$	13.0$^{\{1\}}$	21.0$^{\{3\}}$	20.0$^{\{2\}}$
		α	0.15850$^{\{4\}}$	0.40816$^{\{9\}}$	0.20726$^{\{7\}}$	0.45541$^{\{10\}}$	0.18132$^{\{5\}}$	0.18534$^{\{6\}}$	0.30601$^{\{8\}}$	0.04746$^{\{1\}}$	0.04768$^{\{3\}}$	0.04755$^{\{2\}}$
	$\lvert BIAS\rvert$	β	0.08983$^{\{4\}}$	0.11365$^{\{8\}}$	0.09666$^{\{7\}}$	0.11410$^{\{9\}}$	0.09512$^{\{6\}}$	0.09417$^{\{5\}}$	0.11790$^{\{10\}}$	0.04959$^{\{1\}}$	0.04998$^{\{2\}}$	0.05002$^{\{3\}}$
		θ	0.20423$^{\{4\}}$	0.31100$^{\{10\}}$	0.25109$^{\{7\}}$	0.29331$^{\{9\}}$	0.23399$^{\{6\}}$	0.22802$^{\{5\}}$	0.26051$^{\{8\}}$	0.07293$^{\{1\}}$	0.07317$^{\{2\}}$	0.07355$^{\{3\}}$
		α	0.42053$^{\{6\}}$	2.84565$^{\{9\}}$	0.36471$^{\{5\}}$	3.76079$^{\{10\}}$	0.21329$^{\{4\}}$	0.47456$^{\{7\}}$	2.04530$^{\{8\}}$	0.00316$^{\{3\}}$	0.00315$^{\{2\}}$	0.00313$^{\{1\}}$
50	MSE	β	0.01385$^{\{4\}}$	0.02051$^{\{8\}}$	0.01539$^{\{7\}}$	0.02213$^{\{9\}}$	0.01386$^{\{5\}}$	0.01509$^{\{6\}}$	0.02471$^{\{10\}}$	0.00349$^{\{1\}}$	0.00350$^{\{2.5\}}$	0.00350$^{\{2.5\}}$
		θ	0.07017$^{\{4\}}$	0.17071$^{\{10\}}$	0.10757$^{\{7\}}$	0.14866$^{\{9\}}$	0.09040$^{\{6\}}$	0.08827$^{\{5\}}$	0.11768$^{\{8\}}$	0.00748$^{\{2\}}$	0.00743$^{\{1\}}$	0.00752$^{\{3\}}$
		α	0.31699$^{\{4\}}$	0.81632$^{\{9\}}$	0.41452$^{\{7\}}$	0.91083$^{\{10\}}$	0.36263$^{\{5\}}$	0.37069$^{\{6\}}$	0.61203$^{\{8\}}$	0.09321$^{\{1\}}$	0.09537$^{\{3\}}$	0.09510$^{\{2\}}$
	MRE	β	0.14972$^{\{4\}}$	0.18941$^{\{8\}}$	0.16110$^{\{7\}}$	0.19016$^{\{9\}}$	0.15853$^{\{6\}}$	0.15695$^{\{5\}}$	0.19650$^{\{10\}}$	0.08235$^{\{1\}}$	0.08330$^{\{2\}}$	0.08337$^{\{3\}}$
		θ	0.25529$^{\{4\}}$	0.38875$^{\{10\}}$	0.31386$^{\{7\}}$	0.36664$^{\{9\}}$	0.29249$^{\{6\}}$	0.28502$^{\{5\}}$	0.32563$^{\{8\}}$	0.09030$^{\{1\}}$	0.09147$^{\{2\}}$	0.09194$^{\{3\}}$
		$\sum$	38.0$^{\{4\}}$	81.0$^{\{9\}}$	61.0$^{\{7\}}$	84.0$^{\{10\}}$	49.0$^{\{5\}}$	50.0$^{\{6\}}$	78.0$^{\{8\}}$	12.0$^{\{1\}}$	19.5$^{\{2\}}$	22.5$^{\{3\}}$
		α	0.09308$^{\{4\}}$	0.13319$^{\{10\}}$	0.10637$^{\{8\}}$	0.12773$^{\{9\}}$	0.10089$^{\{6\}}$	0.09843$^{\{5\}}$	0.10585$^{\{7\}}$	0.04459$^{\{1\}}$	0.04491$^{\{3\}}$	0.04482$^{\{2\}}$
	$\lvert BIAS\rvert$	β	0.06528$^{\{4\}}$	0.07871$^{\{9\}}$	0.06995$^{\{5\}}$	0.07224$^{\{8\}}$	0.07000$^{\{6\}}$	0.07067$^{\{7\}}$	0.08470$^{\{10\}}$	0.04466$^{\{1\}}$	0.04505$^{\{2\}}$	0.04510$^{\{3\}}$
		θ	0.15195$^{\{4\}}$	0.19986$^{\{10\}}$	0.17468$^{\{7\}}$	0.18994$^{\{9\}}$	0.16677$^{\{6\}}$	0.16375$^{\{5\}}$	0.18122$^{\{8\}}$	0.07076$^{\{1\}}$	0.07118$^{\{2\}}$	0.07144$^{\{3\}}$
		α	0.01482$^{\{4\}}$	0.14210$^{\{10\}}$	0.02245$^{\{7\}}$	0.11886$^{\{9\}}$	0.01950$^{\{6\}}$	0.01890$^{\{5\}}$	0.02648$^{\{8\}}$	0.00281$^{\{2.5\}}$	0.00281$^{\{2.5\}}$	0.00280$^{\{1\}}$
100	MSE	β	0.00674$^{\{4\}}$	0.00969$^{\{9\}}$	0.00786$^{\{6\}}$	0.00829$^{\{8\}}$	0.00764$^{\{5\}}$	0.00791$^{\{7\}}$	0.01163$^{\{10\}}$	0.00287$^{\{1\}}$	0.00289$^{\{2.5\}}$	0.00289$^{\{2.5\}}$
		θ	0.03805$^{\{4\}}$	0.06483$^{\{10\}}$	0.04941$^{\{7\}}$	0.06084$^{\{9\}}$	0.04648$^{\{6\}}$	0.04429$^{\{5\}}$	0.05450$^{\{8\}}$	0.00705$^{\{1.5\}}$	0.00705$^{\{1.5\}}$	0.00710$^{\{3\}}$
		α	0.18616$^{\{4\}}$	0.26638$^{\{10\}}$	0.21274$^{\{8\}}$	0.25546$^{\{9\}}$	0.20177$^{\{6\}}$	0.19686$^{\{5\}}$	0.21170$^{\{7\}}$	0.08833$^{\{1\}}$	0.08981$^{\{3\}}$	0.08964$^{\{2\}}$

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	MRE	β	$0.10880^{\{4\}}$	$0.13118^{\{9\}}$	$0.11658^{\{5\}}$	$0.12040^{\{8\}}$	$0.11666^{\{6\}}$	$0.11778^{\{7\}}$	$0.14117^{\{10\}}$	$0.07418^{\{1\}}$	$0.07508^{\{2\}}$	$0.07516^{\{3\}}$
		θ	$0.18993^{\{4\}}$	$0.24983^{\{10\}}$	$0.21834^{\{7\}}$	$0.23742^{\{9\}}$	$0.20847^{\{6\}}$	$0.20469^{\{5\}}$	$0.22652^{\{8\}}$	$0.08782^{\{1\}}$	$0.08897^{\{2\}}$	$0.08930^{\{3\}}$
		$\sum$	$36.0^{\{4\}}$	$87.0^{\{10\}}$	$60.0^{\{7\}}$	$78.0^{\{9\}}$	$53.0^{\{6\}}$	$51.0^{\{5\}}$	$76.0^{\{8\}}$	$11.0^{\{1\}}$	$20.5^{\{2\}}$	$22.5^{\{3\}}$
250	$\lvert BIAS\rvert$	α	$0.05728^{\{4\}}$	$0.07134^{\{9\}}$	$0.06331^{\{7\}}$	$0.07139^{\{10\}}$	$0.05870^{\{5\}}$	$0.06318^{\{6\}}$	$0.06634^{\{8\}}$	$0.03980^{\{1\}}$	$0.04014^{\{3\}}$	$0.04011^{\{2\}}$
		β	$0.04226^{\{5\}}$	$0.04885^{\{9\}}$	$0.04338^{\{6\}}$	$0.04638^{\{8\}}$	$0.04039^{\{4\}}$	$0.04435^{\{7\}}$	$0.05165^{\{10\}}$	$0.03473^{\{1\}}$	$0.03504^{\{2\}}$	$0.03506^{\{3\}}$
		θ	$0.09259^{\{4\}}$	$0.12192^{\{10\}}$	$0.10446^{\{7\}}$	$0.12129^{\{9\}}$	$0.09330^{\{5\}}$	$0.10408^{\{6\}}$	$0.11279^{\{8\}}$	$0.06478^{\{1\}}$	$0.06526^{\{2\}}$	$0.06540^{\{3\}}$
	MSE	α	$0.00531^{\{4\}}$	$0.00896^{\{10\}}$	$0.00668^{\{6\}}$	$0.00860^{\{9\}}$	$0.00573^{\{5\}}$	$0.00673^{\{7\}}$	$0.00751^{\{8\}}$	$0.00229^{\{1\}}$	$0.00231^{\{3\}}$	$0.00230^{\{2\}}$
		β	$0.00284^{\{5\}}$	$0.00374^{\{9\}}$	$0.00296^{\{6\}}$	$0.00347^{\{8\}}$	$0.00257^{\{4\}}$	$0.00305^{\{7\}}$	$0.00430^{\{10\}}$	$0.00178^{\{1\}}$	$0.00179^{\{2\}}$	$0.00180^{\{3\}}$
		θ	$0.01354^{\{4\}}$	$0.02389^{\{9\}}$	$0.01751^{\{7\}}$	$0.02418^{\{10\}}$	$0.01414^{\{5\}}$	$0.01715^{\{6\}}$	$0.02033^{\{8\}}$	$0.00603^{\{1\}}$	$0.00605^{\{2\}}$	$0.00608^{\{3\}}$
	MRE	α	$0.11455^{\{4\}}$	$0.14268^{\{9\}}$	$0.12662^{\{7\}}$	$0.14278^{\{10\}}$	$0.11740^{\{5\}}$	$0.12635^{\{6\}}$	$0.13268^{\{8\}}$	$0.07920^{\{1\}}$	$0.08028^{\{3\}}$	$0.08022^{\{2\}}$
		β	$0.07043^{\{5\}}$	$0.08142^{\{9\}}$	$0.07229^{\{6\}}$	$0.07730^{\{8\}}$	$0.06732^{\{4\}}$	$0.07392^{\{7\}}$	$0.08608^{\{10\}}$	$0.05774^{\{1\}}$	$0.05841^{\{2\}}$	$0.05843^{\{3\}}$
		θ	$0.11573^{\{4\}}$	$0.15240^{\{10\}}$	$0.13057^{\{7\}}$	$0.15162^{\{9\}}$	$0.11662^{\{5\}}$	$0.13010^{\{6\}}$	$0.14099^{\{8\}}$	$0.08056^{\{1\}}$	$0.08158^{\{2\}}$	$0.08175^{\{3\}}$
		$\sum$	$39.0^{\{4\}}$	$84.0^{\{10\}}$	$59.0^{\{7\}}$	$81.0^{\{9\}}$	$42.0^{\{5\}}$	$58.0^{\{6\}}$	$78.0^{\{8\}}$	$9.0^{\{1\}}$	$21.0^{\{2\}}$	$24.0^{\{3\}}$
500	$\lvert BIAS\rvert$	α	$0.03963^{\{4\}}$	$0.04755^{\{9\}}$	$0.04195^{\{6\}}$	$0.04769^{\{10\}}$	$0.04176^{\{5\}}$	$0.04259^{\{8\}}$	$0.04223^{\{7\}}$	$0.03485^{\{1\}}$	$0.03519^{\{3\}}$	$0.03518^{\{2\}}$
		β	$0.02899^{\{4\}}$	$0.03248^{\{8\}}$	$0.02937^{\{6\}}$	$0.03355^{\{9\}}$	$0.02930^{\{5\}}$	$0.03076^{\{7\}}$	$0.03652^{\{10\}}$	$0.02744^{\{1\}}$	$0.02770^{\{2\}}$	$0.02771^{\{3\}}$
		θ	$0.06403^{\{4\}}$	$0.08132^{\{9\}}$	$0.06961^{\{6\}}$	$0.08489^{\{10\}}$	$0.06611^{\{5\}}$	$0.07128^{\{7\}}$	$0.07648^{\{8\}}$	$0.05660^{\{1\}}$	$0.05714^{\{2\}}$	$0.05715^{\{3\}}$
	MSE	α	$0.00250^{\{4\}}$	$0.00382^{\{9\}}$	$0.00286^{\{7\}}$	$0.00384^{\{10\}}$	$0.00276^{\{5\}}$	$0.00278^{\{6\}}$	$0.00305^{\{8\}}$	$0.00182^{\{1\}}$	$0.00183^{\{2.5\}}$	$0.00183^{\{2.5\}}$
		β	$0.00134^{\{4.5\}}$	$0.00170^{\{8\}}$	$0.00134^{\{4.5\}}$	$0.00175^{\{9\}}$	$0.00136^{\{6\}}$	$0.00145^{\{7\}}$	$0.00213^{\{10\}}$	$0.00114^{\{1\}}$	$0.00115^{\{2.5\}}$	$0.00115^{\{2.5\}}$
		θ	$0.00643^{\{4\}}$	$0.01032^{\{9\}}$	$0.00782^{\{7\}}$	$0.01124^{\{10\}}$	$0.00708^{\{5\}}$	$0.00779^{\{6\}}$	$0.00920^{\{8\}}$	$0.00478^{\{1\}}$	$0.00482^{\{2.5\}}$	$0.00482^{\{2.5\}}$
	MRE	α	$0.07925^{\{4\}}$	$0.09510^{\{9\}}$	$0.08391^{\{6\}}$	$0.09538^{\{10\}}$	$0.08352^{\{5\}}$	$0.08519^{\{8\}}$	$0.08446^{\{7\}}$	$0.06964^{\{1\}}$	$0.07038^{\{3\}}$	$0.07035^{\{2\}}$
		β	$0.04832^{\{4\}}$	$0.05413^{\{8\}}$	$0.04894^{\{6\}}$	$0.05592^{\{9\}}$	$0.04884^{\{5\}}$	$0.05126^{\{7\}}$	$0.06087^{\{10\}}$	$0.04565^{\{1\}}$	$0.04617^{\{2\}}$	$0.04618^{\{3\}}$
		θ	$0.08004^{\{4\}}$	$0.10165^{\{9\}}$	$0.08701^{\{6\}}$	$0.10611^{\{10\}}$	$0.08264^{\{5\}}$	$0.08910^{\{7\}}$	$0.09560^{\{8\}}$	$0.07065^{\{1\}}$	$0.07143^{\{2\}}$	$0.07144^{\{3\}}$
		$\sum$	$36.5^{\{4\}}$	$78.0^{\{9\}}$	$54.5^{\{6\}}$	$87.0^{\{10\}}$	$46.0^{\{5\}}$	$63.0^{\{7\}}$	$76.0^{\{8\}}$	$9.0^{\{1\}}$	$21.5^{\{2\}}$	$23.5^{\{3\}}$

Table 4

Results of the numerical simulation for the NWEx distribution with $\alpha = 0.6, \beta = 1.0, \theta = 1.5$.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	1.99614 {4}	4.79638 {8}	4.47466 {6}	4.63926 {7}	2.40403 {5}	5.46333 {10}	5.17960 {9}	0.06047 {3}	0.05955 {1}	0.05967 {2}
	$\lvert BIAS\rvert$	β	0.18415 {4}	0.20862 {8}	0.20620 {7}	0.21264 {9}	0.19110 {5}	0.20198 {6}	0.21683 {10}	0.08412 {1}	0.08466 {3}	0.08449 {2}
		θ	0.82325 {4}	1.16472 {9}	1.10568 {8}	1.17779 {10}	0.94817 {6}	0.93751 {5}	0.96130 {7}	0.13773 {2}	0.13617 {1}	0.13888 {3}
		α	84.65044 {5}	98.16224 {6}	122.12386 {8}	112.47689 {7}	49.01497 {4}	226.39863 {10}	181.92373 {9}	0.00530 {3}	0.00501 {1}	0.00503 {2}
30	MSE	β	0.05469 {4}	0.06879 {8}	0.06540 {7}	0.07539 {9}	0.05481 {5}	0.06391 {6}	0.07550 {10}	0.01018 {2}	0.01019 {3}	0.01014 {1}
		θ	1.34048 {4}	3.04340 {10}	2.31486 {8}	2.55164 {9}	1.56577 {6}	1.56041 {5}	1.61894 {7}	0.02740 {2}	0.02631 {1}	0.02770 {3}
		α	3.32690 {4}	7.99396 {8}	7.45777 {6}	7.73211 {7}	4.00672 {5}	9.10555 {10}	8.63267 {9}	0.09444 {1}	0.09925 {2}	0.09945 {3}
	MRE	β	0.18415 {4}	0.20862 {8}	0.20620 {7}	0.21264 {9}	0.19110 {5}	0.20198 {6}	0.21683 {10}	0.08301 {1}	0.08466 {3}	0.08449 {2}
		θ	0.54883 {4}	0.77648 {9}	0.73712 {8}	0.78519 {10}	0.63212 {6}	0.62501 {5}	0.64087 {7}	0.08849 {1}	0.09078 {2}	0.09259 {3}
		$\sum$	37.0 {4}	74.0 {8}	65.0 {7}	77.0 {9}	47.0 {5}	63.0 {6}	78.0 {10}	16.0 {1}	17.0 {2}	21.0 {3}
		α	1.02878 {4}	3.02798 {9}	2.31778 {6}	3.40531 {10}	1.30686 {5}	2.41784 {7}	2.75106 {8}	0.05871 {3}	0.05843 {2}	0.05824 {1}
	$\lvert BIAS\rvert$	β	0.13747 {4}	0.17902 {10}	0.14988 {7}	0.17718 {9}	0.14691 {6}	0.14668 {5}	0.17469 {8}	0.07753 {1}	0.07811 {3}	0.07799 {2}
		θ	0.55181 {4}	0.92816 {10}	0.72683 {8}	0.88666 {9}	0.64293 {5}	0.66194 {6}	0.71840 {7}	0.13285 {2}	0.13231 {1}	0.13444 {3}
		α	38.13874 {5}	56.12371 {7}	46.78245 {6}	62.57074 {9}	17.77373 {4}	56.41774 {8}	68.14861 {10}	0.00491 {3}	0.00476 {2}	0.00473 {1}
50	MSE	β	0.03139 {4}	0.04927 {8}	0.03575 {7}	0.04949 {9}	0.03225 {5}	0.03502 {6}	0.04976 {10}	0.00887 {1.5}	0.00891 {3}	0.00887 {1.5}
		θ	0.51640 {4}	1.44831 {10}	0.88019 {8}	1.27607 {9}	0.67360 {5}	0.72002 {6}	0.85923 {7}	0.02530 {2}	0.02465 {1}	0.02555 {3}
		α	1.71463 {4}	5.04663 {9}	3.86296 {6}	5.67552 {10}	2.17810 {5}	4.02973 {7}	4.58509 {8}	0.09325 {1}	0.09739 {3}	0.09706 {2}
	MRE	β	0.13747 {4}	0.17902 {10}	0.14988 {7}	0.17718 {9}	0.14691 {6}	0.14668 {5}	0.17469 {8}	0.07703 {1}	0.07811 {3}	0.07799 {2}
		θ	0.36788 {4}	0.61877 {10}	0.48455 {8}	0.59111 {9}	0.42862 {5}	0.44129 {6}	0.47893 {7}	0.08617 {1}	0.08821 {2}	0.08963 {3}
		$\sum$	37.0 {4}	83.0 {9.5}	63.0 {7}	83.0 {9.5}	46.0 {5}	56.0 {6}	73.0 {8}	15.5 {1}	20.0 {3}	18.5 {2}
		α	0.27422 {4}	1.30639 {10}	0.74690 {7}	1.12422 {9}	0.48753 {5}	0.74506 {6}	0.84049 {8}	0.05607 {2}	0.05618 {3}	0.05599 {1}
	$\lvert BIAS\rvert$	β	0.10284 {4}	0.12765 {9}	0.11057 {5}	0.11838 {8}	0.11061 {6}	0.11366 {7}	0.13054 {10}	0.06517 {1}	0.06570 {3}	0.06563 {2}
		θ	0.41059 {4}	0.63295 {10}	0.50563 {7}	0.60389 {9}	0.46059 {5}	0.48572 {6}	0.50579 {8}	0.12356 {1}	0.12368 {2}	0.12451 {3}
		α	0.62460 {4}	14.80726 {10}	8.08225 {6}	11.14660 {9}	2.66735 {5}	9.00029 {7}	10.73493 {8}	0.00442 {3}	0.00436 {2}	0.00432 {1}
100	MSE	β	0.01634 {4}	0.02495 {9}	0.01940 {6}	0.02144 {8}	0.01906 {5}	0.02003 {7}	0.02679 {10}	0.00651 {1}	0.00655 {3}	0.00653 {2}
		θ	0.27681 {4}	0.63578 {10}	0.41508 {7}	0.59920 {9}	0.35723 {5}	0.38882 {6}	0.42682 {8}	0.02221 {2}	0.02194 {1}	0.02232 {3}
		α	0.45703 {4}	2.17732 {10}	1.24483 {7}	1.87370 {9}	0.81255 {5}	1.24177 {6}	1.40082 {8}	0.09110 {1}	0.09364 {3}	0.09331 {2}
	MRE	β	0.10284 {4}	0.12765 {9}	0.11057 {5}	0.11838 {8}	0.11061 {6}	0.11366 {7}	0.13054 {10}	0.06489 {1}	0.06570 {3}	0.06563 {2}

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		θ	0.27373 {4}	0.42197 {10}	0.33709 {7}	0.40260 {9}	0.30706 {5}	0.32381 {6}	0.33720 {8}	0.08167 {1}	0.08245 {2}	0.08301 {3}
		$\sum$	36.0 {4}	87.0 {10}	57.0 {6}	78.0 {8.5}	47.0 {5}	58.0 {7}	78.0 {8.5}	13.0 {1}	22.0 {3}	19.0 {2}
250	$\|BIAS\|$	α	0.13549 {4}	0.38633 {10}	0.19538 {6}	0.36809 {9}	0.15631 {5}	0.20895 {7}	0.22034 {8}	0.05496 {1}	0.05535 {3}	0.05518 {2}
		β	0.06634 {5}	0.08054 {9.5}	0.06956 {6}	0.07702 {8}	0.06401 {4}	0.07153 {7}	0.08054 {9.5}	0.04829 {1}	0.04871 {2.5}	0.04871 {2.5}
		θ	0.25015 {5}	0.38560 {9}	0.29728 {6}	0.38700 {10}	0.24637 {4}	0.29962 {7}	0.30665 {8}	0.11613 {1}	0.11670 {2}	0.11694 {3}
	MSE	α	0.03675 {4}	1.76087 {10}	0.28702 {6}	1.62622 {9}	0.19776 {5}	0.41439 {7}	0.58306 {8}	0.00421 {2}	0.00422 {3}	0.00418 {1}
		β	0.00695 {5}	0.01010 {9}	0.00756 {6}	0.00937 {8}	0.00649 {4}	0.00791 {7}	0.01037 {10}	0.00368 {1}	0.00370 {2.5}	0.00370 {2.5}
		θ	0.09911 {4}	0.24283 {9}	0.14205 {6}	0.24730 {10}	0.10462 {5}	0.14596 {7}	0.15371 {8}	0.01954 {2}	0.01948 {1}	0.01959 {3}
	MRE	α	0.22582 {4}	0.64388 {10}	0.32564 {6}	0.61348 {9}	0.26052 {5}	0.34825 {7}	0.36724 {8}	0.09106 {1}	0.09225 {3}	0.09196 {2}
		β	0.06634 {5}	0.08054 {9.5}	0.06956 {6}	0.07702 {8}	0.06401 {4}	0.07153 {7}	0.08054 {9.5}	0.04810 {1}	0.04871 {2.5}	0.04871 {2.5}
		θ	0.16676 {5}	0.25707 {9}	0.19818 {6}	0.25800 {10}	0.16425 {4}	0.19975 {7}	0.20443 {8}	0.07790 {2}	0.07780 {1}	0.07796 {3}
		$\sum$	41.0 {5}	85.0 {10}	54.0 {6}	81.0 {9}	40.0 {4}	63.0 {7}	77.0 {8}	12.0 {1}	20.5 {2}	21.5 {3}
500	$\|BIAS\|$	α	0.08662 {4}	0.14726 {9}	0.09973 {6}	0.14787 {10}	0.09583 {5}	0.10617 {7}	0.11000 {8}	0.05258 {1}	0.05301 {3}	0.05292 {2}
		β	0.04611 {5}	0.05363 {8}	0.04724 {6}	0.05539 {9}	0.04596 {4}	0.04954 {7}	0.05720 {10}	0.03759 {1}	0.03792 {2.5}	0.03792 {2.5}
		θ	0.17117 {5}	0.25182 {9}	0.19502 {6}	0.26425 {10}	0.16888 {4}	0.20187 {7}	0.20820 {8}	0.11059 {1}	0.11133 {2}	0.11146 {3}
	MSE	α	0.01319 {4}	0.11038 {10}	0.01904 {6}	0.06677 {9}	0.01788 {5}	0.02022 {7}	0.02579 {8}	0.00389 {1.5}	0.00391 {3}	0.00389 {1.5}
		β	0.00339 {5}	0.00467 {8}	0.00345 {6}	0.00480 {9}	0.00337 {4}	0.00378 {7}	0.00523 {10}	0.00221 {1}	0.00223 {2.5}	0.00223 {2.5}
		θ	0.04580 {4}	0.10155 {9}	0.06157 {6}	0.10999 {10}	0.05056 {5}	0.06319 {7}	0.06855 {8}	0.01756 {1}	0.01758 {2}	0.01761 {3}
	MRE	α	0.14437 {4}	0.24543 {9}	0.16622 {6}	0.24644 {10}	0.15972 {5}	0.17695 {7}	0.18333 {8}	0.08736 {1}	0.08836 {3}	0.08820 {2}
		β	0.04611 {5}	0.05363 {8}	0.04724 {6}	0.05539 {9}	0.04596 {4}	0.04954 {7}	0.05720 {10}	0.03744 {1}	0.03792 {2.5}	0.03792 {2.5}
		θ	0.11411 {5}	0.16788 {9}	0.13001 {6}	0.17617 {10}	0.11259 {4}	0.13458 {7}	0.13880 {8}	0.07438 {3}	0.07422 {1}	0.07430 {2}
		$\sum$	41.0 {5}	79.0 {9}	54.0 {6}	86.0 {10}	40.0 {4}	63.0 {7}	78.0 {8}	11.5 {1}	21.5 {3}	21.0 {2}

Table 5

Results of the numerical simulation for the NWEx distribution with α = 0.6, β = 0.5, θ = 0.6.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	$0.19140^{\{4\}}$	$0.63961^{\{7\}}$	$0.64496^{\{8\}}$	$0.70948^{\{10\}}$	$0.28622^{\{5\}}$	$0.38895^{\{6\}}$	$0.65765^{\{9\}}$	$0.05776^{\{1\}}$	$0.05801^{\{2\}}$	$0.05810^{\{3\}}$
	$\lvert BIAS\rvert$	β	$0.09781^{\{6\}}$	$0.10915^{\{8\}}$	$0.10762^{\{7\}}$	$0.10943^{\{9\}}$	$0.09757^{\{5\}}$	$0.09743^{\{4\}}$	$0.11572^{\{10\}}$	$0.04405^{\{1\}}$	$0.04436^{\{2\}}$	$0.04451^{\{3\}}$
		θ	$0.21320^{\{4\}}$	$0.29208^{\{9\}}$	$0.28422^{\{8\}}$	$0.30163^{\{10\}}$	$0.24055^{\{6\}}$	$0.23131^{\{5\}}$	$0.24579^{\{7\}}$	$0.05671^{\{1\}}$	$0.05676^{\{2\}}$	$0.05744^{\{3\}}$
		α	$0.07044^{\{4\}}$	$4.87845^{\{8\}}$	$8.46430^{\{10\}}$	$4.87643^{\{7\}}$	$0.90556^{\{5\}}$	$2.66790^{\{6\}}$	$7.72917^{\{9\}}$	$0.00463^{\{3\}}$	$0.00459^{\{1\}}$	$0.00461^{\{2\}}$
30	MSE	β	$0.01649^{\{6\}}$	$0.01962^{\{8\}}$	$0.01926^{\{7\}}$	$0.02047^{\{9\}}$	$0.01494^{\{4\}}$	$0.01595^{\{5\}}$	$0.02464^{\{10\}}$	$0.00272^{\{1\}}$	$0.00273^{\{2\}}$	$0.00274^{\{3\}}$
		θ	$0.08113^{\{4\}}$	$0.15899^{\{8\}}$	$0.17457^{\{9\}}$	$0.20814^{\{10\}}$	$0.10620^{\{6\}}$	$0.09929^{\{5\}}$	$0.11081^{\{7\}}$	$0.00452^{\{2\}}$	$0.00446^{\{1\}}$	$0.00458^{\{3\}}$
		α	$0.31900^{\{4\}}$	$1.06602^{\{7\}}$	$1.07493^{\{8\}}$	$1.18247^{\{10\}}$	$0.47703^{\{5\}}$	$0.64826^{\{6\}}$	$1.09609^{\{9\}}$	$0.09484^{\{1\}}$	$0.09669^{\{2\}}$	$0.09683^{\{3\}}$
	MRE	β	$0.19561^{\{6\}}$	$0.21831^{\{8\}}$	$0.21523^{\{7\}}$	$0.21887^{\{9\}}$	$0.19515^{\{5\}}$	$0.19485^{\{4\}}$	$0.23144^{\{10\}}$	$0.08707^{\{1\}}$	$0.08872^{\{2\}}$	$0.08902^{\{3\}}$
		θ	$0.35533^{\{4\}}$	$0.48680^{\{9\}}$	$0.47369^{\{8\}}$	$0.50271^{\{10\}}$	$0.40092^{\{6\}}$	$0.38552^{\{5\}}$	$0.40964^{\{7\}}$	$0.09236^{\{1\}}$	$0.09460^{\{2\}}$	$0.09574^{\{3\}}$
		$\sum$	$42.0^{\{4\}}$	$72.0^{\{7.5\}}$	$72.0^{\{7.5\}}$	$84.0^{\{10\}}$	$47.0^{\{6\}}$	$46.0^{\{5\}}$	$78.0^{\{9\}}$	$12.0^{\{1\}}$	$16.0^{\{2\}}$	$26.0^{\{3\}}$
		α	$0.13930^{\{4\}}$	$0.33414^{\{10\}}$	$0.20187^{\{7\}}$	$0.31366^{\{9\}}$	$0.16204^{\{5\}}$	$0.17455^{\{6\}}$	$0.21183^{\{8\}}$	$0.05564^{\{1\}}$	$0.05597^{\{2\}}$	$0.05607^{\{3\}}$
	$\lvert BIAS\rvert$	β	$0.07424^{\{4\}}$	$0.08920^{\{9\}}$	$0.07677^{\{6\}}$	$0.08405^{\{8\}}$	$0.07863^{\{7\}}$	$0.07566^{\{5\}}$	$0.09502^{\{10\}}$	$0.04083^{\{1\}}$	$0.04117^{\{2\}}$	$0.04130^{\{3\}}$
		θ	$0.15925^{\{4\}}$	$0.22838^{\{10\}}$	$0.18439^{\{7\}}$	$0.21316^{\{9\}}$	$0.17539^{\{6\}}$	$0.17227^{\{5\}}$	$0.19197^{\{8\}}$	$0.05529^{\{1\}}$	$0.05553^{\{2\}}$	$0.05587^{\{3\}}$
		α	$0.03694^{\{4\}}$	$1.20868^{\{9\}}$	$0.35348^{\{7\}}$	$1.24957^{\{10\}}$	$0.06996^{\{5\}}$	$0.11966^{\{6\}}$	$0.51531^{\{8\}}$	$0.00433^{\{1.5\}}$	$0.00433^{\{1.5\}}$	$0.00434^{\{3\}}$
50	MSE	β	$0.00906^{\{4\}}$	$0.01325^{\{9\}}$	$0.00923^{\{6\}}$	$0.01175^{\{8\}}$	$0.00958^{\{7\}}$	$0.00922^{\{5\}}$	$0.01603^{\{10\}}$	$0.00234^{\{1\}}$	$0.00235^{\{2\}}$	$0.00236^{\{3\}}$
		θ	$0.04603^{\{4\}}$	$0.08933^{\{10\}}$	$0.05963^{\{7\}}$	$0.07637^{\{9\}}$	$0.05108^{\{6\}}$	$0.04940^{\{5\}}$	$0.06454^{\{8\}}$	$0.00429^{\{2\}}$	$0.00426^{\{1\}}$	$0.00433^{\{3\}}$
		α	$0.23217^{\{4\}}$	$0.55689^{\{10\}}$	$0.33646^{\{7\}}$	$0.52276^{\{9\}}$	$0.27006^{\{5\}}$	$0.29092^{\{6\}}$	$0.35305^{\{8\}}$	$0.09187^{\{1\}}$	$0.09328^{\{2\}}$	$0.09345^{\{3\}}$
	MRE	β	$0.14847^{\{4\}}$	$0.17839^{\{9\}}$	$0.15354^{\{6\}}$	$0.16811^{\{8\}}$	$0.15727^{\{7\}}$	$0.15132^{\{5\}}$	$0.19004^{\{10\}}$	$0.08122^{\{1\}}$	$0.08233^{\{2\}}$	$0.08260^{\{3\}}$
		θ	$0.26542^{\{4\}}$	$0.38064^{\{10\}}$	$0.30731^{\{7\}}$	$0.35526^{\{9\}}$	$0.29232^{\{6\}}$	$0.28712^{\{5\}}$	$0.31995^{\{8\}}$	$0.09118^{\{1\}}$	$0.09255^{\{2\}}$	$0.09311^{\{3\}}$
		$\sum$	$36.0^{\{4\}}$	$86.0^{\{10\}}$	$60.0^{\{7\}}$	$79.0^{\{9\}}$	$54.0^{\{6\}}$	$48.0^{\{5\}}$	$78.0^{\{8\}}$	$10.5^{\{1\}}$	$16.5^{\{2\}}$	$27.0^{\{3\}}$
		α	$0.10170^{\{4\}}$	$0.12542^{\{10\}}$	$0.11280^{\{7\}}$	$0.12404^{\{9\}}$	$0.10890^{\{5\}}$	$0.11410^{\{8\}}$	$0.10922^{\{6\}}$	$0.05229^{\{1\}}$	$0.05270^{\{3\}}$	$0.05269^{\{2\}}$
	$\lvert BIAS\rvert$	β	$0.05050^{\{4\}}$	$0.06024^{\{9\}}$	$0.05624^{\{7\}}$	$0.05972^{\{8\}}$	$0.05196^{\{5\}}$	$0.05365^{\{6\}}$	$0.06289^{\{10\}}$	$0.03595^{\{1\}}$	$0.03626^{\{2\}}$	$0.03630^{\{3\}}$
		θ	$0.10883^{\{4\}}$	$0.14469^{\{10\}}$	$0.12529^{\{8\}}$	$0.13935^{\{9\}}$	$0.11664^{\{5\}}$	$0.11994^{\{6\}}$	$0.12517^{\{7\}}$	$0.05328^{\{1\}}$	$0.05359^{\{2\}}$	$0.05387^{\{3\}}$
		α	$0.01764^{\{4\}}$	$0.04360^{\{10\}}$	$0.02466^{\{7\}}$	$0.02992^{\{9\}}$	$0.02163^{\{5\}}$	$0.02514^{\{8\}}$	$0.02208^{\{6\}}$	$0.00387^{\{1\}}$	$0.00388^{\{2.5\}}$	$0.00388^{\{2.5\}}$
100	MSE	β	$0.00432^{\{5\}}$	$0.00578^{\{9\}}$	$0.00506^{\{7\}}$	$0.00567^{\{8\}}$	$0.00424^{\{4\}}$	$0.00446^{\{6\}}$	$0.00656^{\{10\}}$	$0.00186^{\{1\}}$	$0.00187^{\{2\}}$	$0.00188^{\{3\}}$
		θ	$0.01975^{\{4\}}$	$0.03474^{\{10\}}$	$0.02587^{\{7\}}$	$0.03182^{\{9\}}$	$0.02224^{\{5\}}$	$0.02309^{\{6\}}$	$0.02593^{\{8\}}$	$0.00403^{\{1.5\}}$	$0.00403^{\{1.5\}}$	$0.00407^{\{3\}}$

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	MRE	α	0.16951 $^{\{4\}}$	0.20903 $^{\{10\}}$	0.18800 $^{\{7\}}$	0.20674 $^{\{9\}}$	0.18150 $^{\{5\}}$	0.19016 $^{\{8\}}$	0.18203 $^{\{6\}}$	0.08649 $^{\{1\}}$	0.08783 $^{\{3\}}$	0.08782 $^{\{2\}}$
		β	0.10100 $^{\{4\}}$	0.12048 $^{\{9\}}$	0.11247 $^{\{7\}}$	0.11945 $^{\{8\}}$	0.10392 $^{\{5\}}$	0.10731 $^{\{6\}}$	0.12577 $^{\{10\}}$	0.07158 $^{\{1\}}$	0.07252 $^{\{2\}}$	0.07261 $^{\{3\}}$
		θ	0.18138 $^{\{4\}}$	0.24115 $^{\{10\}}$	0.20881 $^{\{8\}}$	0.23225 $^{\{9\}}$	0.19439 $^{\{5\}}$	0.19990 $^{\{6\}}$	0.20862 $^{\{7\}}$	0.08757 $^{\{1\}}$	0.08932 $^{\{2\}}$	0.08978 $^{\{3\}}$
		$\sum$	37.0 $^{\{4\}}$	87.0 $^{\{10\}}$	65.0 $^{\{7\}}$	78.0 $^{\{9\}}$	44.0 $^{\{5\}}$	60.0 $^{\{6\}}$	70.0 $^{\{8\}}$	9.5 $^{\{1\}}$	20.0 $^{\{2\}}$	24.5 $^{\{3\}}$
250	$\|BIAS\|$	α	0.06189 $^{\{4\}}$	0.07361 $^{\{10\}}$	0.06566 $^{\{6\}}$	0.07340 $^{\{9\}}$	0.06265 $^{\{5\}}$	0.06768 $^{\{8\}}$	0.06581 $^{\{7\}}$	0.04570 $^{\{1\}}$	0.04611 $^{\{3\}}$	0.04608 $^{\{2\}}$
		β	0.03350 $^{\{7\}}$	0.03705 $^{\{9\}}$	0.03185 $^{\{4\}}$	0.03629 $^{\{8\}}$	0.03271 $^{\{6\}}$	0.03236 $^{\{5\}}$	0.04004 $^{\{10\}}$	0.02765 $^{\{1\}}$	0.02790 $^{\{2\}}$	0.02791 $^{\{3\}}$
		θ	0.06887 $^{\{4\}}$	0.08934 $^{\{10\}}$	0.07274 $^{\{6\}}$	0.08686 $^{\{9\}}$	0.07023 $^{\{5\}}$	0.07370 $^{\{7\}}$	0.07893 $^{\{8\}}$	0.04825 $^{\{1\}}$	0.04862 $^{\{2\}}$	0.04870 $^{\{3\}}$
	MSE	α	0.00642 $^{\{4\}}$	0.00972 $^{\{10\}}$	0.00697 $^{\{6\}}$	0.00886 $^{\{9\}}$	0.00655 $^{\{5\}}$	0.00777 $^{\{8\}}$	0.00739 $^{\{7\}}$	0.00306 $^{\{1\}}$	0.00308 $^{\{2.5\}}$	0.00308 $^{\{2.5\}}$
		β	0.00218 $^{\{9\}}$	0.00215 $^{\{8\}}$	0.00165 $^{\{4\}}$	0.00206 $^{\{7\}}$	0.00167 $^{\{5\}}$	0.00175 $^{\{6\}}$	0.00248 $^{\{10\}}$	0.00114 $^{\{1\}}$	0.00115 $^{\{2.5\}}$	0.00115 $^{\{2.5\}}$
		θ	0.00799 $^{\{5\}}$	0.01287 $^{\{10\}}$	0.00819 $^{\{6\}}$	0.01247 $^{\{9\}}$	0.00763 $^{\{4\}}$	0.00894 $^{\{7\}}$	0.00990 $^{\{8\}}$	0.00336 $^{\{1\}}$	0.00337 $^{\{2\}}$	0.00338 $^{\{3\}}$
	MRE	α	0.10315 $^{\{4\}}$	0.12269 $^{\{10\}}$	0.10943 $^{\{6\}}$	0.12233 $^{\{9\}}$	0.10441 $^{\{5\}}$	0.11281 $^{\{8\}}$	0.10968 $^{\{7\}}$	0.07567 $^{\{1\}}$	0.07685 $^{\{3\}}$	0.07680 $^{\{2\}}$
		β	0.06700 $^{\{7\}}$	0.07409 $^{\{9\}}$	0.06369 $^{\{4\}}$	0.07258 $^{\{8\}}$	0.06543 $^{\{6\}}$	0.06473 $^{\{5\}}$	0.08008 $^{\{10\}}$	0.05521 $^{\{1\}}$	0.05579 $^{\{2\}}$	0.05583 $^{\{3\}}$
		θ	0.11478 $^{\{4\}}$	0.14890 $^{\{10\}}$	0.12124 $^{\{6\}}$	0.14477 $^{\{9\}}$	0.11706 $^{\{5\}}$	0.12283 $^{\{7\}}$	0.13156 $^{\{8\}}$	0.08011 $^{\{1\}}$	0.08103 $^{\{2\}}$	0.08116 $^{\{3\}}$
		$\sum$	48.0 $^{\{5.5\}}$	86.0 $^{\{10\}}$	48.0 $^{\{5.5\}}$	77.0 $^{\{9\}}$	46.0 $^{\{4\}}$	61.0 $^{\{7\}}$	75.0 $^{\{8\}}$	9.0 $^{\{1\}}$	21.0 $^{\{2\}}$	24.0 $^{\{3\}}$
500	$\|BIAS\|$	α	0.04773 $^{\{8\}}$	0.05025 $^{\{9\}}$	0.04543 $^{\{5\}}$	0.05133 $^{\{10\}}$	0.04622 $^{\{6\}}$	0.04520 $^{\{4\}}$	0.04658 $^{\{7\}}$	0.03826 $^{\{1\}}$	0.03863 $^{\{3\}}$	0.03862 $^{\{2\}}$
		β	0.02740 $^{\{9\}}$	0.02532 $^{\{7\}}$	0.02405 $^{\{6\}}$	0.02608 $^{\{8\}}$	0.02255 $^{\{4\}}$	0.02386 $^{\{5\}}$	0.02847 $^{\{10\}}$	0.02134 $^{\{1\}}$	0.02154 $^{\{2\}}$	0.02155 $^{\{3\}}$
		θ	0.05243 $^{\{6\}}$	0.06027 $^{\{10\}}$	0.05304 $^{\{7\}}$	0.06008 $^{\{9\}}$	0.04798 $^{\{4\}}$	0.05009 $^{\{5\}}$	0.05624 $^{\{8\}}$	0.04198 $^{\{1\}}$	0.04239 $^{\{2.5\}}$	0.04239 $^{\{2.5\}}$
	MSE	α	0.00508 $^{\{10\}}$	0.00406 $^{\{8\}}$	0.00334 $^{\{5\}}$	0.00426 $^{\{9\}}$	0.00338 $^{\{6\}}$	0.00329 $^{\{4\}}$	0.00346 $^{\{7\}}$	0.00225 $^{\{1\}}$	0.00227 $^{\{3\}}$	0.00226 $^{\{2\}}$
		β	0.00355 $^{\{10\}}$	0.00099 $^{\{7\}}$	0.00092 $^{\{6\}}$	0.00107 $^{\{8\}}$	0.00080 $^{\{4\}}$	0.00090 $^{\{5\}}$	0.00125 $^{\{9\}}$	0.00069 $^{\{1\}}$	0.00070 $^{\{2.5\}}$	0.00070 $^{\{2.5\}}$
		θ	0.00747 $^{\{10\}}$	0.00592 $^{\{9\}}$	0.00456 $^{\{6\}}$	0.00574 $^{\{8\}}$	0.00369 $^{\{4\}}$	0.00402 $^{\{5\}}$	0.00491 $^{\{7\}}$	0.00264 $^{\{1\}}$	0.00266 $^{\{2.5\}}$	0.00266 $^{\{2.5\}}$
	MRE	α	0.07954 $^{\{8\}}$	0.08376 $^{\{9\}}$	0.07571 $^{\{5\}}$	0.08555 $^{\{10\}}$	0.07703 $^{\{6\}}$	0.07533 $^{\{4\}}$	0.07764 $^{\{7\}}$	0.06363 $^{\{1\}}$	0.06438 $^{\{3\}}$	0.06436 $^{\{2\}}$
		β	0.05480 $^{\{9\}}$	0.05063 $^{\{7\}}$	0.04809 $^{\{6\}}$	0.05216 $^{\{8\}}$	0.04509 $^{\{4\}}$	0.04772 $^{\{5\}}$	0.05693 $^{\{10\}}$	0.04265 $^{\{1\}}$	0.04309 $^{\{2\}}$	0.04310 $^{\{3\}}$
		θ	0.08738 $^{\{6\}}$	0.10045 $^{\{10\}}$	0.08840 $^{\{7\}}$	0.10013 $^{\{9\}}$	0.07996 $^{\{4\}}$	0.08349 $^{\{5\}}$	0.09373 $^{\{8\}}$	0.06986 $^{\{1\}}$	0.07064 $^{\{2\}}$	0.07065 $^{\{3\}}$
		$\sum$	76.0 $^{\{8.5\}}$	76.0 $^{\{8.5\}}$	53.0 $^{\{6\}}$	79.0 $^{\{10\}}$	42.0 $^{\{4.5\}}$	42.0 $^{\{4.5\}}$	73.0 $^{\{7\}}$	9.0 $^{\{1\}}$	22.5 $^{\{2.5\}}$	22.5 $^{\{2.5\}}$

Table 6

Results of the numerical simulation for the NWEx distribution with α = 1.5, β = 0.5, θ = 0.2.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.69176 $^{\{4\}}$	3.28646 $^{\{10\}}$	3.06875 $^{\{8\}}$	2.91192 $^{\{6\}}$	1.51318 $^{\{5\}}$	3.02409 $^{\{7\}}$	3.09922 $^{\{9\}}$	0.14281 $^{\{2\}}$	0.14279 $^{\{1\}}$	0.14444 $^{\{3\}}$
	$\lvert BIAS\rvert$	β	0.07064 $^{\{5\}}$	0.07977 $^{\{9\}}$	0.07683 $^{\{7\}}$	0.08036 $^{\{10\}}$	0.07060 $^{\{4\}}$	0.07642 $^{\{6\}}$	0.07699 $^{\{8\}}$	0.04188 $^{\{1\}}$	0.04223 $^{\{2\}}$	0.04235 $^{\{3\}}$
		θ	0.15402 $^{\{5\}}$	0.22642 $^{\{9\}}$	0.20270 $^{\{8\}}$	0.25548 $^{\{10\}}$	0.15434 $^{\{6\}}$	0.15994 $^{\{7\}}$	0.15373 $^{\{4\}}$	0.02000 $^{\{2\}}$	0.01983 $^{\{1\}}$	0.02039 $^{\{3\}}$
		α	3.48999 $^{\{4\}}$	28.43173 $^{\{7\}}$	32.32993 $^{\{8\}}$	27.24710 $^{\{6\}}$	8.09837 $^{\{5\}}$	38.99114 $^{\{9\}}$	41.44856 $^{\{10\}}$	0.02862 $^{\{2\}}$	0.02814 $^{\{1\}}$	0.02878 $^{\{3\}}$
30	MSE	β	0.00788 $^{\{5\}}$	0.01014 $^{\{9\}}$	0.00904 $^{\{6\}}$	0.01091 $^{\{10\}}$	0.00750 $^{\{4\}}$	0.00929 $^{\{7\}}$	0.01005 $^{\{8\}}$	0.00251 $^{\{1\}}$	0.00252 $^{\{2\}}$	0.00253 $^{\{3\}}$
		θ	0.06278 $^{\{7\}}$	0.14480 $^{\{9\}}$	0.09405 $^{\{8\}}$	0.16883 $^{\{10\}}$	0.04406 $^{\{4\}}$	0.04832 $^{\{6\}}$	0.04588 $^{\{5\}}$	0.00057 $^{\{2\}}$	0.00055 $^{\{1\}}$	0.00058 $^{\{3\}}$
		α	0.46117 $^{\{4\}}$	2.19097 $^{\{10\}}$	2.04583 $^{\{8\}}$	1.94128 $^{\{6\}}$	1.00879 $^{\{5\}}$	2.01606 $^{\{7\}}$	2.06614 $^{\{9\}}$	0.09193 $^{\{1\}}$	0.09520 $^{\{2\}}$	0.09629 $^{\{3\}}$
	MRE	β	0.14128 $^{\{5\}}$	0.15954 $^{\{9\}}$	0.15367 $^{\{7\}}$	0.16072 $^{\{10\}}$	0.14119 $^{\{4\}}$	0.15284 $^{\{6\}}$	0.15397 $^{\{8\}}$	0.08319 $^{\{1\}}$	0.08447 $^{\{2\}}$	0.08470 $^{\{3\}}$
		θ	0.77009 $^{\{5\}}$	1.13209 $^{\{9\}}$	1.01352 $^{\{8\}}$	1.27738 $^{\{10\}}$	0.77171 $^{\{6\}}$	0.79969 $^{\{7\}}$	0.76864 $^{\{4\}}$	0.09516 $^{\{1\}}$	0.09915 $^{\{2\}}$	0.10197 $^{\{3\}}$
		$\sum$	44.0 $^{\{5\}}$	81.0 $^{\{10\}}$	68.0 $^{\{8\}}$	78.0 $^{\{9\}}$	43.0 $^{\{4\}}$	62.0 $^{\{6\}}$	65.0 $^{\{7\}}$	13.0 $^{\{1\}}$	14.0 $^{\{2\}}$	27.0 $^{\{3\}}$
		α	0.51373 $^{\{4\}}$	2.50271 $^{\{9\}}$	2.06752 $^{\{7\}}$	2.66407 $^{\{10\}}$	1.03332 $^{\{5\}}$	1.81970 $^{\{6\}}$	2.07121 $^{\{8\}}$	0.13418 $^{\{1\}}$	0.13458 $^{\{2\}}$	0.13568 $^{\{3\}}$
	$\lvert BIAS\rvert$	β	0.05290 $^{\{4\}}$	0.06461 $^{\{9\}}$	0.05612 $^{\{7\}}$	0.06556 $^{\{10\}}$	0.05514 $^{\{6\}}$	0.05489 $^{\{5\}}$	0.06283 $^{\{8\}}$	0.03670 $^{\{1\}}$	0.03701 $^{\{2\}}$	0.03707 $^{\{3\}}$
		θ	0.09576 $^{\{4\}}$	0.18999 $^{\{10\}}$	0.12917 $^{\{8\}}$	0.18307 $^{\{9\}}$	0.10490 $^{\{5\}}$	0.11208 $^{\{6\}}$	0.11210 $^{\{7\}}$	0.01959 $^{\{2\}}$	0.01955 $^{\{1\}}$	0.01990 $^{\{3\}}$
		α	1.70709 $^{\{4\}}$	19.06632 $^{\{8\}}$	18.10077 $^{\{7\}}$	21.73350 $^{\{9\}}$	4.46865 $^{\{5\}}$	17.26301 $^{\{6\}}$	22.52022 $^{\{10\}}$	0.02531 $^{\{2\}}$	0.02510 $^{\{1\}}$	0.02553 $^{\{3\}}$
50	MSE	β	0.00464 $^{\{5\}}$	0.00651 $^{\{8\}}$	0.00490 $^{\{7\}}$	0.00680 $^{\{10\}}$	0.00462 $^{\{4\}}$	0.00487 $^{\{6\}}$	0.00652 $^{\{9\}}$	0.00201 $^{\{1\}}$	0.00202 $^{\{2\}}$	0.00203 $^{\{3\}}$
		θ	0.01844 $^{\{4\}}$	0.07193 $^{\{10\}}$	0.03121 $^{\{8\}}$	0.06944 $^{\{9\}}$	0.01902 $^{\{5\}}$	0.02110 $^{\{6\}}$	0.02151 $^{\{7\}}$	0.00054 $^{\{2\}}$	0.00052 $^{\{1\}}$	0.00055 $^{\{3\}}$
		α	0.34249 $^{\{4\}}$	1.66848 $^{\{9\}}$	1.37835 $^{\{7\}}$	1.77605 $^{\{10\}}$	0.68888 $^{\{5\}}$	1.21313 $^{\{6\}}$	1.38081 $^{\{8\}}$	0.08786 $^{\{1\}}$	0.08972 $^{\{2\}}$	0.09046 $^{\{3\}}$
	MRE	β	0.10580 $^{\{4\}}$	0.12923 $^{\{9\}}$	0.11224 $^{\{7\}}$	0.13112 $^{\{10\}}$	0.11027 $^{\{6\}}$	0.10979 $^{\{5\}}$	0.12566 $^{\{8\}}$	0.07305 $^{\{1\}}$	0.07402 $^{\{2\}}$	0.07415 $^{\{3\}}$
		θ	0.47881 $^{\{4\}}$	0.94995 $^{\{10\}}$	0.64584 $^{\{8\}}$	0.91533 $^{\{9\}}$	0.52449 $^{\{5\}}$	0.56038 $^{\{6\}}$	0.56051 $^{\{7\}}$	0.09411 $^{\{1\}}$	0.09777 $^{\{2\}}$	0.09951 $^{\{3\}}$
		$\sum$	37.0 $^{\{4\}}$	82.0 $^{\{9\}}$	66.0 $^{\{7\}}$	86.0 $^{\{10\}}$	46.0 $^{\{5\}}$	52.0 $^{\{6\}}$	72.0 $^{\{8\}}$	12.0 $^{\{1\}}$	15.0 $^{\{2\}}$	27.0 $^{\{3\}}$
		α	0.33974 $^{\{4\}}$	1.58068 $^{\{10\}}$	0.96812 $^{\{8\}}$	1.43858 $^{\{9\}}$	0.57737 $^{\{5\}}$	0.88474 $^{\{7\}}$	0.83135 $^{\{6\}}$	0.12622 $^{\{1\}}$	0.12692 $^{\{2\}}$	0.12752 $^{\{3\}}$
	$\lvert BIAS\rvert$	β	0.04034 $^{\{6\}}$	0.04545 $^{\{10\}}$	0.03967 $^{\{4\}}$	0.04239 $^{\{8\}}$	0.03973 $^{\{5\}}$	0.04090 $^{\{7\}}$	0.04391 $^{\{9\}}$	0.03028 $^{\{1\}}$	0.03054 $^{\{2\}}$	0.03056 $^{\{3\}}$
		θ	0.06502 $^{\{4\}}$	0.12881 $^{\{10\}}$	0.08759 $^{\{8\}}$	0.12494 $^{\{9\}}$	0.07112 $^{\{5\}}$	0.08244 $^{\{7\}}$	0.07614 $^{\{6\}}$	0.01931 $^{\{1\}}$	0.01938 $^{\{2\}}$	0.01959 $^{\{3\}}$
		α	0.24019 $^{\{4\}}$	9.39936 $^{\{10\}}$	5.43142 $^{\{8\}}$	8.00459 $^{\{9\}}$	1.57107 $^{\{5\}}$	4.88776 $^{\{7\}}$	4.67519 $^{\{6\}}$	0.02277 $^{\{2\}}$	0.02276 $^{\{1\}}$	0.02298 $^{\{3\}}$
100	MSE	β	0.00321 $^{\{9\}}$	0.00330 $^{\{10\}}$	0.00257 $^{\{5\}}$	0.00285 $^{\{7\}}$	0.00251 $^{\{4\}}$	0.00259 $^{\{6\}}$	0.00312 $^{\{8\}}$	0.00142 $^{\{1\}}$	0.00143 $^{\{2.5\}}$	0.00143 $^{\{2.5\}}$
		θ	0.00725 $^{\{4\}}$	0.02700 $^{\{9\}}$	0.01334 $^{\{8\}}$	0.02733 $^{\{10\}}$	0.00849 $^{\{5\}}$	0.01122 $^{\{7\}}$	0.00992 $^{\{6\}}$	0.00052 $^{\{1.5\}}$	0.00052 $^{\{1.5\}}$	0.00053 $^{\{3\}}$
		α	0.22649 $^{\{4\}}$	1.05379 $^{\{10\}}$	0.64541 $^{\{8\}}$	0.95906 $^{\{9\}}$	0.38491 $^{\{5\}}$	0.58982 $^{\{7\}}$	0.55424 $^{\{6\}}$	0.08293 $^{\{1\}}$	0.08461 $^{\{2\}}$	0.08501 $^{\{3\}}$

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	MRE	β	0.08067 $^{\{6\}}$	0.09089 $^{\{10\}}$	0.07935 $^{\{4\}}$	0.08479 $^{\{8\}}$	0.07947 $^{\{5\}}$	0.08181 $^{\{7\}}$	0.08781 $^{\{9\}}$	0.06035 $^{\{1\}}$	0.06109 $^{\{2\}}$	0.06112 $^{\{3\}}$
		θ	0.32508 $^{\{4\}}$	0.64403 $^{\{10\}}$	0.43796 $^{\{8\}}$	0.62472 $^{\{9\}}$	0.35558 $^{\{5\}}$	0.41221 $^{\{7\}}$	0.38069 $^{\{6\}}$	0.09410 $^{\{1\}}$	0.09689 $^{\{2\}}$	0.09796 $^{\{3\}}$
		$\sum$	45.0 $^{\{5\}}$	89.0 $^{\{10\}}$	61.0 $^{\{6\}}$	78.0 $^{\{9\}}$	44.0 $^{\{4\}}$	62.0 $^{\{7.5\}}$	62.0 $^{\{7.5\}}$	10.5 $^{\{1\}}$	17.0 $^{\{2\}}$	26.5 $^{\{3\}}$
250	$\lvert BIAS\rvert$	α	0.21107 $^{\{4\}}$	0.70128 $^{\{10\}}$	0.31080 $^{\{7\}}$	0.65437 $^{\{9\}}$	0.23818 $^{\{5\}}$	0.34242 $^{\{8\}}$	0.29632 $^{\{6\}}$	0.11156 $^{\{1\}}$	0.11244 $^{\{2\}}$	0.11252 $^{\{3\}}$
		β	0.02826 $^{\{9\}}$	0.02843 $^{\{10\}}$	0.02572 $^{\{5\}}$	0.02714 $^{\{7\}}$	0.02427 $^{\{4\}}$	0.02634 $^{\{6\}}$	0.02789 $^{\{8\}}$	0.02082 $^{\{1\}}$	0.02100 $^{\{2.5\}}$	0.02100 $^{\{2.5\}}$
		θ	0.04008 $^{\{4\}}$	0.07691 $^{\{9\}}$	0.04946 $^{\{8\}}$	0.07922 $^{\{10\}}$	0.04014 $^{\{5\}}$	0.04831 $^{\{7\}}$	0.04421 $^{\{6\}}$	0.01837 $^{\{1\}}$	0.01849 $^{\{2\}}$	0.01862 $^{\{3\}}$
	MSE	α	0.07987 $^{\{4\}}$	2.53119 $^{\{10\}}$	0.45739 $^{\{6\}}$	2.05334 $^{\{9\}}$	0.15541 $^{\{5\}}$	0.71264 $^{\{8\}}$	0.45929 $^{\{7\}}$	0.01790 $^{\{1\}}$	0.01798 $^{\{2\}}$	0.01801 $^{\{3\}}$
		β	0.00294 $^{\{10\}}$	0.00129 $^{\{9\}}$	0.00105 $^{\{5\}}$	0.00114 $^{\{7\}}$	0.00092 $^{\{4\}}$	0.00108 $^{\{6\}}$	0.00124 $^{\{8\}}$	0.00069 $^{\{1\}}$	0.00070 $^{\{2.5\}}$	0.00070 $^{\{2.5\}}$
		θ	0.00278 $^{\{5\}}$	0.00959 $^{\{9\}}$	0.00434 $^{\{8\}}$	0.01051 $^{\{10\}}$	0.00251 $^{\{4\}}$	0.00401 $^{\{7\}}$	0.00325 $^{\{6\}}$	0.00047 $^{\{1.5\}}$	0.00047 $^{\{1.5\}}$	0.00048 $^{\{3\}}$
	MRE	α	0.14071 $^{\{4\}}$	0.46752 $^{\{10\}}$	0.20720 $^{\{7\}}$	0.43625 $^{\{9\}}$	0.15879 $^{\{5\}}$	0.22828 $^{\{8\}}$	0.19755 $^{\{6\}}$	0.07393 $^{\{1\}}$	0.07496 $^{\{2\}}$	0.07501 $^{\{3\}}$
		β	0.05653 $^{\{9\}}$	0.05687 $^{\{10\}}$	0.05145 $^{\{5\}}$	0.05428 $^{\{7\}}$	0.04855 $^{\{4\}}$	0.05269 $^{\{6\}}$	0.05577 $^{\{8\}}$	0.04150 $^{\{1\}}$	0.04200 $^{\{2\}}$	0.04201 $^{\{3\}}$
		θ	0.20038 $^{\{4\}}$	0.38455 $^{\{9\}}$	0.24730 $^{\{8\}}$	0.39612 $^{\{10\}}$	0.20072 $^{\{5\}}$	0.24154 $^{\{7\}}$	0.22106 $^{\{6\}}$	0.09024 $^{\{1\}}$	0.09247 $^{\{2\}}$	0.09311 $^{\{3\}}$
		$\sum$	53.0 $^{\{5\}}$	86.0 $^{\{10\}}$	59.0 $^{\{6\}}$	78.0 $^{\{9\}}$	41.0 $^{\{4\}}$	63.0 $^{\{8\}}$	61.0 $^{\{7\}}$	9.5 $^{\{1\}}$	18.5 $^{\{2\}}$	26.0 $^{\{3\}}$
500	$\lvert BIAS\rvert$	α	0.14233 $^{\{4\}}$	0.31862 $^{\{9\}}$	0.16600 $^{\{6\}}$	0.33095 $^{\{10\}}$	0.15532 $^{\{5\}}$	0.17965 $^{\{8\}}$	0.17259 $^{\{7\}}$	0.10134 $^{\{1\}}$	0.10229 $^{\{3\}}$	0.10227 $^{\{2\}}$
		β	0.02293 $^{\{10\}}$	0.01949 $^{\{8\}}$	0.01768 $^{\{5\}}$	0.01942 $^{\{7\}}$	0.01703 $^{\{4\}}$	0.01846 $^{\{6\}}$	0.02009 $^{\{9\}}$	0.01543 $^{\{1\}}$	0.01558 $^{\{2.5\}}$	0.01558 $^{\{2.5\}}$
		θ	0.02772 $^{\{5\}}$	0.04820 $^{\{9\}}$	0.03211 $^{\{7\}}$	0.05262 $^{\{10\}}$	0.02736 $^{\{4\}}$	0.03264 $^{\{8\}}$	0.03095 $^{\{6\}}$	0.01755 $^{\{1\}}$	0.01771 $^{\{2\}}$	0.01773 $^{\{3\}}$
	MSE	α	0.03464 $^{\{4\}}$	0.50484 $^{\{9\}}$	0.05060 $^{\{6\}}$	0.52385 $^{\{10\}}$	0.04419 $^{\{5\}}$	0.06115 $^{\{8\}}$	0.05789 $^{\{7\}}$	0.01486 $^{\{1\}}$	0.01498 $^{\{2.5\}}$	0.01498 $^{\{2.5\}}$
		β	0.00304 $^{\{10\}}$	0.00063 $^{\{8\}}$	0.00049 $^{\{5\}}$	0.00060 $^{\{7\}}$	0.00047 $^{\{4\}}$	0.00052 $^{\{6\}}$	0.00064 $^{\{9\}}$	0.00037 $^{\{1\}}$	0.00038 $^{\{2.5\}}$	0.00038 $^{\{2.5\}}$
		θ	0.00151 $^{\{5\}}$	0.00404 $^{\{9\}}$	0.00175 $^{\{7\}}$	0.00471 $^{\{10\}}$	0.00117 $^{\{4\}}$	0.00177 $^{\{8\}}$	0.00153 $^{\{6\}}$	0.00043 $^{\{1\}}$	0.00044 $^{\{2.5\}}$	0.00044 $^{\{2.5\}}$
	MRE	α	0.09489 $^{\{4\}}$	0.21241 $^{\{9\}}$	0.11067 $^{\{6\}}$	0.22063 $^{\{10\}}$	0.10355 $^{\{5\}}$	0.11977 $^{\{8\}}$	0.11506 $^{\{7\}}$	0.06750 $^{\{1\}}$	0.06819 $^{\{3\}}$	0.06818 $^{\{2\}}$
		β	0.04586 $^{\{10\}}$	0.03898 $^{\{8\}}$	0.03537 $^{\{5\}}$	0.03884 $^{\{7\}}$	0.03407 $^{\{4\}}$	0.03692 $^{\{6\}}$	0.04018 $^{\{9\}}$	0.03081 $^{\{1\}}$	0.03115 $^{\{2.5\}}$	0.03115 $^{\{2.5\}}$
		θ	0.13861 $^{\{5\}}$	0.24100 $^{\{9\}}$	0.16057 $^{\{7\}}$	0.26309 $^{\{10\}}$	0.13681 $^{\{4\}}$	0.16318 $^{\{8\}}$	0.15476 $^{\{6\}}$	0.08748 $^{\{1\}}$	0.08855 $^{\{2\}}$	0.08866 $^{\{3\}}$
		$\sum$	57.0 $^{\{6\}}$	78.0 $^{\{9\}}$	54.0 $^{\{5\}}$	81.0 $^{\{10\}}$	39.0 $^{\{4\}}$	66.0 $^{\{7.5\}}$	66.0 $^{\{7.5\}}$	9.0 $^{\{1\}}$	22.5 $^{\{2.5\}}$	22.5 $^{\{2.5\}}$

Table 7

Results of the numerical simulation for the NWEx distribution with α = 0.6, β = 0.5, θ = 3.0.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.21006 $^{\{4\}}$	0.67312 $^{\{10\}}$	0.36111 $^{\{6\}}$	0.65768 $^{\{9\}}$	0.30143 $^{\{5\}}$	0.49936 $^{\{7\}}$	0.52914 $^{\{8\}}$	0.05673 $^{\{2\}}$	0.05689 $^{\{3\}}$	0.05671 $^{\{1\}}$
	$\|BIAS\|$	β	0.09425 $^{\{4\}}$	0.10537 $^{\{8\}}$	0.10173 $^{\{6\}}$	0.10715 $^{\{9\}}$	0.09818 $^{\{5\}}$	0.10298 $^{\{7\}}$	0.11592 $^{\{10\}}$	0.04206 $^{\{1\}}$	0.04232 $^{\{3\}}$	0.04223 $^{\{2\}}$
		θ	1.09630 $^{\{4\}}$	1.45980 $^{\{9\}}$	1.32006 $^{\{8\}}$	1.47753 $^{\{10\}}$	1.24840 $^{\{7\}}$	1.18880 $^{\{5\}}$	1.22738 $^{\{6\}}$	0.27160 $^{\{1\}}$	0.27163 $^{\{2\}}$	0.27272 $^{\{3\}}$
		α	0.24781 $^{\{4\}}$	4.43993 $^{\{8\}}$	1.70574 $^{\{6\}}$	6.18234 $^{\{10\}}$	0.75411 $^{\{5\}}$	4.43094 $^{\{7\}}$	5.59946 $^{\{9\}}$	0.00451 $^{\{3\}}$	0.00447 $^{\{2\}}$	0.00445 $^{\{1\}}$
30	MSE	β	0.01512 $^{\{5\}}$	0.01825 $^{\{8\}}$	0.01699 $^{\{6\}}$	0.02074 $^{\{9\}}$	0.01482 $^{\{4\}}$	0.01788 $^{\{7\}}$	0.02363 $^{\{10\}}$	0.00257 $^{\{2.5\}}$	0.00257 $^{\{2.5\}}$	0.00256 $^{\{1\}}$
		θ	2.36976 $^{\{4\}}$	4.40149 $^{\{10\}}$	3.52777 $^{\{8\}}$	4.02479 $^{\{9\}}$	2.77013 $^{\{7\}}$	2.67190 $^{\{5\}}$	2.74711 $^{\{6\}}$	0.10490 $^{\{3\}}$	0.10328 $^{\{1\}}$	0.10456 $^{\{2\}}$
		α	0.35010 $^{\{4\}}$	1.12187 $^{\{10\}}$	0.60186 $^{\{6\}}$	1.09614 $^{\{9\}}$	0.50238 $^{\{5\}}$	0.83227 $^{\{7\}}$	0.88190 $^{\{8\}}$	0.09220 $^{\{1\}}$	0.09481 $^{\{3\}}$	0.09451 $^{\{2\}}$
	MRE	β	0.18850 $^{\{4\}}$	0.21074 $^{\{8\}}$	0.20346 $^{\{6\}}$	0.21429 $^{\{9\}}$	0.19637 $^{\{5\}}$	0.20595 $^{\{7\}}$	0.23185 $^{\{10\}}$	0.08342 $^{\{1\}}$	0.08464 $^{\{3\}}$	0.08446 $^{\{2\}}$
		θ	0.36543 $^{\{4\}}$	0.48660 $^{\{9\}}$	0.44002 $^{\{8\}}$	0.49251 $^{\{10\}}$	0.41613 $^{\{7\}}$	0.39627 $^{\{5\}}$	0.40913 $^{\{6\}}$	0.08949 $^{\{1\}}$	0.09054 $^{\{2\}}$	0.09091 $^{\{3\}}$
		$\sum$	37.0 $^{\{4\}}$	80.0 $^{\{9\}}$	60.0 $^{\{7\}}$	84.0 $^{\{10\}}$	50.0 $^{\{5\}}$	57.0 $^{\{6\}}$	73.0 $^{\{8\}}$	15.5 $^{\{1\}}$	21.5 $^{\{3\}}$	17.0 $^{\{2\}}$
		α	0.13808 $^{\{4\}}$	0.32434 $^{\{9\}}$	0.16606 $^{\{5\}}$	0.36801 $^{\{10\}}$	0.17235 $^{\{7\}}$	0.16933 $^{\{6\}}$	0.22000 $^{\{8\}}$	0.05457 $^{\{1\}}$	0.05486 $^{\{3\}}$	0.05474 $^{\{2\}}$
	$\|BIAS\|$	β	0.07019 $^{\{4\}}$	0.08751 $^{\{8\}}$	0.07218 $^{\{5\}}$	0.08841 $^{\{9\}}$	0.07411 $^{\{7\}}$	0.07267 $^{\{6\}}$	0.08992 $^{\{10\}}$	0.04022 $^{\{1\}}$	0.04052 $^{\{3\}}$	0.04040 $^{\{2\}}$
		θ	0.73287 $^{\{4\}}$	1.12326 $^{\{10\}}$	0.84685 $^{\{7\}}$	1.05901 $^{\{9\}}$	0.83021 $^{\{6\}}$	0.80365 $^{\{5\}}$	0.90527 $^{\{8\}}$	0.26805 $^{\{1\}}$	0.26913 $^{\{2\}}$	0.26984 $^{\{3\}}$
		α	0.03597 $^{\{4\}}$	1.36779 $^{\{9\}}$	0.07092 $^{\{5\}}$	1.79817 $^{\{10\}}$	0.11116 $^{\{6\}}$	0.18847 $^{\{7\}}$	0.46500 $^{\{8\}}$	0.00419 $^{\{2.5\}}$	0.00419 $^{\{2.5\}}$	0.00417 $^{\{1\}}$
50	MSE	β	0.00837 $^{\{4\}}$	0.01217 $^{\{8\}}$	0.00872 $^{\{6\}}$	0.01314 $^{\{9\}}$	0.00846 $^{\{5\}}$	0.00904 $^{\{7\}}$	0.01441 $^{\{10\}}$	0.00236 $^{\{2\}}$	0.00237 $^{\{3\}}$	0.00235 $^{\{1\}}$
		θ	0.90115 $^{\{4\}}$	2.22226 $^{\{10\}}$	1.30571 $^{\{7\}}$	1.93022 $^{\{9\}}$	1.15604 $^{\{6\}}$	1.10777 $^{\{5\}}$	1.41586 $^{\{8\}}$	0.10154 $^{\{2\}}$	0.10087 $^{\{1\}}$	0.10176 $^{\{3\}}$
		α	0.23013 $^{\{4\}}$	0.54056 $^{\{9\}}$	0.27677 $^{\{5\}}$	0.61336 $^{\{10\}}$	0.28725 $^{\{7\}}$	0.28221 $^{\{6\}}$	0.36667 $^{\{8\}}$	0.09004 $^{\{1\}}$	0.09143 $^{\{3\}}$	0.09124 $^{\{2\}}$
	MRE	β	0.14038 $^{\{4\}}$	0.17503 $^{\{8\}}$	0.14435 $^{\{5\}}$	0.17682 $^{\{9\}}$	0.14822 $^{\{7\}}$	0.14533 $^{\{6\}}$	0.17985 $^{\{10\}}$	0.08000 $^{\{1\}}$	0.08104 $^{\{3\}}$	0.08080 $^{\{2\}}$
		θ	0.24429 $^{\{4\}}$	0.37442 $^{\{10\}}$	0.28228 $^{\{7\}}$	0.35300 $^{\{9\}}$	0.27674 $^{\{6\}}$	0.26788 $^{\{5\}}$	0.30176 $^{\{8\}}$	0.08881 $^{\{1\}}$	0.08971 $^{\{2\}}$	0.08995 $^{\{3\}}$
		$\sum$	36.0 $^{\{4\}}$	81.0 $^{\{9\}}$	52.0 $^{\{5\}}$	84.0 $^{\{10\}}$	57.0 $^{\{7\}}$	53.0 $^{\{6\}}$	78.0 $^{\{8\}}$	12.5 $^{\{1\}}$	22.5 $^{\{3\}}$	19.0 $^{\{2\}}$
		α	0.09761 $^{\{4\}}$	0.13156 $^{\{10\}}$	0.10575 $^{\{6\}}$	0.12134 $^{\{9\}}$	0.10730 $^{\{8\}}$	0.09900 $^{\{5\}}$	0.10658 $^{\{7\}}$	0.05179 $^{\{1\}}$	0.05218 $^{\{3\}}$	0.05211 $^{\{2\}}$
	$\|BIAS\|$	β	0.05049 $^{\{4\}}$	0.06076 $^{\{9\}}$	0.05240 $^{\{5\}}$	0.05539 $^{\{8\}}$	0.05464 $^{\{7\}}$	0.05362 $^{\{6\}}$	0.06436 $^{\{10\}}$	0.03463 $^{\{1\}}$	0.03493 $^{\{3\}}$	0.03489 $^{\{2\}}$
		θ	0.53793 $^{\{4\}}$	0.71459 $^{\{10\}}$	0.58133 $^{\{6\}}$	0.67658 $^{\{9\}}$	0.59569 $^{\{7\}}$	0.55952 $^{\{5\}}$	0.63139 $^{\{8\}}$	0.25885 $^{\{1\}}$	0.26040 $^{\{2\}}$	0.26061 $^{\{3\}}$
		α	0.01581 $^{\{4\}}$	0.09700 $^{\{10\}}$	0.02150 $^{\{7\}}$	0.03488 $^{\{9\}}$	0.02078 $^{\{6\}}$	0.01805 $^{\{5\}}$	0.02167 $^{\{8\}}$	0.00382 $^{\{1\}}$	0.00383 $^{\{2.5\}}$	0.00383 $^{\{2.5\}}$
100	MSE	β	0.00399 $^{\{4\}}$	0.00577 $^{\{9\}}$	0.00443 $^{\{5\}}$	0.00487 $^{\{8\}}$	0.00467 $^{\{7\}}$	0.00457 $^{\{6\}}$	0.00673 $^{\{10\}}$	0.00181 $^{\{1\}}$	0.00182 $^{\{2.5\}}$	0.00182 $^{\{2.5\}}$
		θ	0.48014 $^{\{4\}}$	0.83179 $^{\{10\}}$	0.56955 $^{\{6\}}$	0.77340 $^{\{9\}}$	0.60140 $^{\{7\}}$	0.52635 $^{\{5\}}$	0.66258 $^{\{8\}}$	0.09470 $^{\{2\}}$	0.09468 $^{\{1\}}$	0.09501 $^{\{3\}}$

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	MRE	α	0.16268 {4}	0.21927 {10}	0.17625 {6}	0.20224 {9}	0.17883 {8}	0.16499 {5}	0.17763 {7}	0.08568 {1}	0.08697 {3}	0.08685 {2}
		β	0.10097 {4}	0.12153 {9}	0.10481 {5}	0.11078 {8}	0.10928 {7}	0.10724 {6}	0.12871 {10}	0.06915 {1}	0.06986 {3}	0.06978 {2}
		θ	0.17931 {4}	0.23820 {10}	0.19378 {6}	0.22553 {9}	0.19856 {7}	0.18651 {5}	0.21046 {8}	0.08681 {2}	0.08680 {1}	0.08687 {3}
		$\sum$	36.0 {4}	87.0 {10}	52.0 {6}	78.0 {9}	64.0 {7}	48.0 {5}	76.0 {8}	11.0 {1}	21.0 {2}	22.0 {3}
250	$\|BIAS\|$	α	0.06152 {4}	0.07249 {9}	0.06501 {7}	0.07367 {10}	0.06278 {5}	0.06467 {6}	0.06766 {8}	0.04516 {1}	0.04555 {3}	0.04553 {2}
		β	0.03276 {5}	0.03704 {9}	0.03324 {6}	0.03536 {8}	0.03153 {4}	0.03387 {7}	0.03820 {10}	0.02674 {1}	0.02698 {2.5}	0.02698 {2.5}
		θ	0.33167 {5}	0.42571 {9}	0.35842 {7}	0.42859 {10}	0.31966 {4}	0.35621 {6}	0.38232 {8}	0.23826 {1}	0.24003 {3}	0.23990 {2}
	MSE	α	0.00608 {4}	0.00883 {9}	0.00691 {6}	0.00884 {10}	0.00651 {5}	0.00703 {7}	0.00768 {8}	0.00301 {1}	0.00303 {3}	0.00302 {2}
		β	0.00171 {5}	0.00218 {9}	0.00174 {6}	0.00201 {8}	0.00157 {4}	0.00176 {7}	0.00237 {10}	0.00112 {2}	0.00112 {2}	0.00112 {2}
		θ	0.17439 {4}	0.29483 {9}	0.21233 {7}	0.30307 {10}	0.18177 {5}	0.20642 {6}	0.23513 {8}	0.08212 {1}	0.08245 {3}	0.08238 {2}
	MRE	α	0.10253 {4}	0.12081 {9}	0.10835 {7}	0.12278 {10}	0.10464 {5}	0.10778 {6}	0.11277 {8}	0.07497 {1}	0.07592 {3}	0.07588 {2}
		β	0.06552 {5}	0.07407 {9}	0.06648 {6}	0.07073 {8}	0.06306 {4}	0.06775 {7}	0.07641 {10}	0.05341 {1}	0.05396 {2.5}	0.05396 {2.5}
		θ	0.11056 {5}	0.14190 {9}	0.11947 {7}	0.14286 {10}	0.10655 {4}	0.11874 {6}	0.12744 {8}	0.08020 {3}	0.08001 {2}	0.07997 {1}
		$\sum$	41.0 {5}	81.0 {9}	59.0 {7}	84.0 {10}	40.0 {4}	58.0 {6}	78.0 {8}	12.0 {1}	24.0 {3}	18.0 {2}
500	$\|BIAS\|$	α	0.04237 {4}	0.04825 {9}	0.04357 {6}	0.04885 {10}	0.04458 {8}	0.04402 {7}	0.04346 {5}	0.03929 {1}	0.03965 {3}	0.03964 {2}
		β	0.02255 {5}	0.02435 {8}	0.02234 {4}	0.02515 {9}	0.02273 {6}	0.02338 {7}	0.02707 {10}	0.02051 {1}	0.02069 {2}	0.02070 {3}
		θ	0.22735 {5}	0.27779 {9}	0.23441 {6}	0.29308 {10}	0.21808 {4}	0.24006 {7}	0.25732 {8}	0.20796 {1}	0.20956 {2.5}	0.20956 {2.5}
	MSE	α	0.00287 {4}	0.00387 {9}	0.00306 {6}	0.00398 {10}	0.00314 {7}	0.00301 {5}	0.00319 {8}	0.00234 {1}	0.00236 {2.5}	0.00236 {2.5}
		β	0.00082 {5.5}	0.00098 {8}	0.00077 {4}	0.00101 {9}	0.00082 {5.5}	0.00084 {7}	0.00117 {10}	0.00065 {1}	0.00066 {2.5}	0.00066 {2.5}
		θ	0.08244 {4}	0.12462 {9}	0.09190 {7}	0.13806 {10}	0.09077 {5}	0.09084 {6}	0.10440 {8}	0.06430 {1}	0.06462 {3}	0.06459 {2}
	MRE	α	0.07062 {4}	0.08042 {9}	0.07262 {6}	0.08141 {10}	0.07430 {8}	0.07337 {7}	0.07244 {5}	0.06528 {1}	0.06609 {3}	0.06607 {2}
		β	0.04510 {5}	0.04869 {8}	0.04468 {4}	0.05031 {9}	0.04546 {6}	0.04677 {7}	0.05415 {10}	0.04089 {1}	0.04138 {2}	0.04139 {3}
		θ	0.07578 {5}	0.09260 {9}	0.07814 {6}	0.09769 {10}	0.07269 {4}	0.08002 {7}	0.08577 {8}	0.06983 {1}	0.06985 {2.5}	0.06985 {2.5}
		$\sum$	41.5 {4}	78.0 {9}	49.0 {5}	87.0 {10}	53.5 {6}	60.0 {7}	72.0 {8}	9.0 {1}	23.0 {3}	22.0 {2}

Table 8

Results of the numerical simulation for the NWEx distribution with $\alpha = 0.6, \beta = 0.6, \theta = 0.5$.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.23196 {4}	1.07663 {7}	1.12314 {8}	1.26256 {10}	0.46641 {5}	0.67570 {6}	1.13751 {9}	0.05925 {1}	0.05939 {2}	0.05961 {3}
	$\|BIAS\|$	β	0.11517 {4}	0.13057 {9}	0.12827 {7}	0.13032 {8}	0.11579 {5}	0.11621 {6}	0.13534 {10}	0.05226 {1}	0.05263 {2}	0.05284 {3}
		θ	0.19094 {4}	0.27307 {9}	0.26361 {8}	0.28202 {10}	0.21752 {6}	0.21260 {5}	0.22358 {7}	0.04771 {1}	0.04773 {2}	0.04834 {3}
		α	0.14495 {4}	10.96398 {7}	16.18018 {9}	13.45127 {8}	2.70810 {5}	8.27126 {6}	19.50336 {10}	0.00486 {3}	0.00481 {1}	0.00484 {2}
30	MSE	β	0.02265 {6}	0.02749 {8}	0.02698 {7}	0.02843 {9}	0.02081 {4}	0.02238 {5}	0.03207 {10}	0.00380 {1}	0.00381 {2}	0.00384 {3}
		θ	0.06535 {4}	0.13856 {8}	0.15286 {9}	0.18605 {10}	0.08703 {6}	0.08337 {5}	0.09159 {7}	0.00319 {2}	0.00314 {1}	0.00326 {3}
		α	0.38660 {4}	1.79438 {7}	1.87191 {8}	2.10426 {10}	0.77736 {5}	1.12617 {6}	1.89584 {9}	0.09638 {1}	0.09899 {2}	0.09934 {3}
	MRE	β	0.19196 {4}	0.21762 {9}	0.21379 {7}	0.21719 {8}	0.19298 {5}	0.19369 {6}	0.22556 {10}	0.08621 {1}	0.08772 {2}	0.08807 {3}
		θ	0.38187 {4}	0.54615 {9}	0.52721 {8}	0.56405 {10}	0.43504 {6}	0.42520 {5}	0.44716 {7}	0.09271 {1}	0.09545 {2}	0.09668 {3}
		$\sum$	38.0 {4}	73.0 {8}	71.0 {7}	83.0 {10}	47.0 {5}	50.0 {6}	79.0 {9}	12.0 {1}	16.0 {2}	26.0 {3}
		α	0.16189 {4}	0.68259 {10}	0.31624 {7}	0.63477 {9}	0.22302 {5}	0.29028 {6}	0.42419 {8}	0.05685 {1}	0.05715 {3}	0.05712 {2}
	$\|BIAS\|$	β	0.08919 {4}	0.10690 {9}	0.09196 {6}	0.10074 {8}	0.09383 {7}	0.09059 {5}	0.11233 {10}	0.04814 {1}	0.04851 {2}	0.04867 {3}
		θ	0.14368 {4}	0.21520 {10}	0.16976 {7}	0.20075 {9}	0.15931 {6}	0.15797 {5}	0.17512 {8}	0.04550 {1}	0.04566 {2}	0.04605 {3}
		α	0.05613 {4}	5.18531 {10}	1.35030 {7}	4.64129 {9}	0.30060 {5}	1.18664 {6}	2.96293 {8}	0.00449 {3}	0.00447 {1.5}	0.00447 {1.5}
50	MSE	β	0.01341 {6}	0.01861 {9}	0.01320 {5}	0.01666 {8}	0.01362 {7}	0.01319 {4}	0.02193 {10}	0.00329 {1}	0.00331 {2}	0.00332 {3}
		θ	0.03752 {4}	0.07921 {10}	0.05063 {7}	0.06793 {9}	0.04207 {6}	0.04148 {5}	0.05390 {8}	0.00292 {2}	0.00289 {1}	0.00296 {3}
		α	0.26981 {4}	1.13766 {10}	0.52707 {7}	1.05795 {9}	0.37170 {5}	0.48381 {6}	0.70698 {8}	0.09301 {1}	0.09524 {3}	0.09520 {2}
	MRE	β	0.14864 {4}	0.17816 {9}	0.15326 {6}	0.16790 {8}	0.15638 {7}	0.15098 {5}	0.18721 {10}	0.07965 {1}	0.08086 {2}	0.08112 {3}
		θ	0.28737 {4}	0.43039 {10}	0.33952 {7}	0.40151 {9}	0.31862 {6}	0.31593 {5}	0.35023 {8}	0.08921 {1}	0.09133 {2}	0.09211 {3}
		$\sum$	38.0 {4}	87.0 {10}	59.0 {7}	78.0 {8.5}	54.0 {6}	47.0 {5}	78.0 {8.5}	12.0 {1}	18.5 {2}	23.5 {3}
		α	0.11662 {4}	0.19184 {10}	0.14885 {8}	0.16829 {9}	0.12977 {5}	0.14140 {7}	0.13442 {6}	0.05366 {1}	0.05406 {3}	0.05403 {2}
	$\|BIAS\|$	β	0.05994 {4}	0.07260 {9}	0.06739 {7}	0.07204 {8}	0.06219 {5}	0.06431 {6}	0.07518 {10}	0.04322 {1}	0.04360 {2}	0.04364 {3}
		θ	0.09709 {4}	0.13547 {10}	0.11504 {8}	0.13063 {9}	0.10534 {5}	0.10970 {6}	0.11320 {7}	0.04409 {1}	0.04434 {2}	0.04454 {3}
		α	0.02539 {4}	0.39166 {10}	0.21723 {9}	0.13185 {8}	0.03500 {5}	0.05638 {7}	0.05113 {6}	0.00403 {1.5}	0.00404 {3}	0.00403 {1.5}
100	MSE	β	0.00606 {4}	0.00836 {9}	0.00725 {7}	0.00824 {8}	0.00607 {5}	0.00638 {6}	0.00930 {10}	0.00270 {1}	0.00272 {2}	0.00273 {3}
		θ	0.01578 {4}	0.03053 {10}	0.02178 {8}	0.02814 {9}	0.01807 {5}	0.01937 {6}	0.02118 {7}	0.00276 {1.5}	0.00276 {1.5}	0.00279 {3}

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		α	0.19436 $^{\{4\}}$	0.31974 $^{\{10\}}$	0.24808 $^{\{8\}}$	0.28049 $^{\{9\}}$	0.21628 $^{\{5\}}$	0.23567 $^{\{7\}}$	0.22404 $^{\{6\}}$	0.08841 $^{\{1\}}$	0.09010 $^{\{3\}}$	0.09005 $^{\{2\}}$
	MRE	β	0.09990 $^{\{4\}}$	0.12100 $^{\{9\}}$	0.11232 $^{\{7\}}$	0.12007 $^{\{8\}}$	0.10364 $^{\{5\}}$	0.10718 $^{\{6\}}$	0.12530 $^{\{10\}}$	0.07194 $^{\{1\}}$	0.07267 $^{\{2\}}$	0.07274 $^{\{3\}}$
		θ	0.19419 $^{\{4\}}$	0.27095 $^{\{10\}}$	0.23007 $^{\{8\}}$	0.26125 $^{\{9\}}$	0.21067 $^{\{5\}}$	0.21939 $^{\{6\}}$	0.22640 $^{\{7\}}$	0.08749 $^{\{1\}}$	0.08869 $^{\{2\}}$	0.08907 $^{\{3\}}$
		$\sum$	36.0 $^{\{4\}}$	87.0 $^{\{10\}}$	70.0 $^{\{8\}}$	77.0 $^{\{9\}}$	45.0 $^{\{5\}}$	57.0 $^{\{6\}}$	69.0 $^{\{7\}}$	10.0 $^{\{1\}}$	20.5 $^{\{2\}}$	23.5 $^{\{3\}}$
		α	0.06954 $^{\{4\}}$	0.09015 $^{\{10\}}$	0.07646 $^{\{6\}}$	0.08814 $^{\{9\}}$	0.07260 $^{\{5\}}$	0.07909 $^{\{8\}}$	0.07708 $^{\{7\}}$	0.04702 $^{\{1\}}$	0.04741 $^{\{3\}}$	0.04738 $^{\{2\}}$
	$\|BIAS\|$	β	0.03884 $^{\{6\}}$	0.04467 $^{\{9\}}$	0.03815 $^{\{4\}}$	0.04377 $^{\{8\}}$	0.03896 $^{\{7\}}$	0.03876 $^{\{5\}}$	0.04773 $^{\{10\}}$	0.03307 $^{\{1\}}$	0.03337 $^{\{2\}}$	0.03339 $^{\{3\}}$
		θ	0.06072 $^{\{4\}}$	0.08318 $^{\{10\}}$	0.06619 $^{\{6\}}$	0.08066 $^{\{9\}}$	0.06313 $^{\{5\}}$	0.06707 $^{\{7\}}$	0.07133 $^{\{8\}}$	0.04114 $^{\{1\}}$	0.04144 $^{\{2\}}$	0.04152 $^{\{3\}}$
		α	0.00799 $^{\{4\}}$	0.01604 $^{\{10\}}$	0.00987 $^{\{6\}}$	0.01366 $^{\{9\}}$	0.00901 $^{\{5\}}$	0.01108 $^{\{8\}}$	0.01054 $^{\{7\}}$	0.00323 $^{\{1\}}$	0.00325 $^{\{3\}}$	0.00324 $^{\{2\}}$
250	MSE	β	0.00239 $^{\{6\}}$	0.00313 $^{\{9\}}$	0.00237 $^{\{4\}}$	0.00300 $^{\{8\}}$	0.00238 $^{\{5\}}$	0.00251 $^{\{7\}}$	0.00353 $^{\{10\}}$	0.00162 $^{\{1\}}$	0.00164 $^{\{2.5\}}$	0.00164 $^{\{2.5\}}$
		θ	0.00584 $^{\{4\}}$	0.01118 $^{\{10\}}$	0.00679 $^{\{6\}}$	0.01077 $^{\{9\}}$	0.00614 $^{\{5\}}$	0.00740 $^{\{7\}}$	0.00805 $^{\{8\}}$	0.00244 $^{\{1\}}$	0.00245 $^{\{2\}}$	0.00246 $^{\{3\}}$
		α	0.11590 $^{\{4\}}$	0.15026 $^{\{10\}}$	0.12744 $^{\{6\}}$	0.14689 $^{\{9\}}$	0.12100 $^{\{5\}}$	0.13181 $^{\{8\}}$	0.12846 $^{\{7\}}$	0.07801 $^{\{1\}}$	0.07902 $^{\{3\}}$	0.07896 $^{\{2\}}$
	MRE	β	0.06473 $^{\{6\}}$	0.07444 $^{\{9\}}$	0.06359 $^{\{4\}}$	0.07295 $^{\{8\}}$	0.06493 $^{\{7\}}$	0.06460 $^{\{5\}}$	0.07956 $^{\{10\}}$	0.05508 $^{\{1\}}$	0.05562 $^{\{2\}}$	0.05565 $^{\{3\}}$
		θ	0.12144 $^{\{4\}}$	0.16637 $^{\{10\}}$	0.13239 $^{\{6\}}$	0.16133 $^{\{9\}}$	0.12626 $^{\{5\}}$	0.13414 $^{\{7\}}$	0.14267 $^{\{8\}}$	0.08192 $^{\{1\}}$	0.08289 $^{\{2\}}$	0.08305 $^{\{3\}}$
		$\sum$	42.0 $^{\{4\}}$	87.0 $^{\{10\}}$	48.0 $^{\{5\}}$	78.0 $^{\{9\}}$	49.0 $^{\{6\}}$	62.0 $^{\{7\}}$	75.0 $^{\{8\}}$	9.0 $^{\{1\}}$	21.5 $^{\{2\}}$	23.5 $^{\{3\}}$
		α	0.04886 $^{\{4\}}$	0.05982 $^{\{9\}}$	0.05273 $^{\{6\}}$	0.06163 $^{\{10\}}$	0.05280 $^{\{7\}}$	0.05215 $^{\{5\}}$	0.05406 $^{\{8\}}$	0.04180 $^{\{1\}}$	0.04219 $^{\{3\}}$	0.04218 $^{\{2\}}$
	$\|BIAS\|$	β	0.02649 $^{\{3.5\}}$	0.03053 $^{\{8\}}$	0.02880 $^{\{7\}}$	0.03144 $^{\{9\}}$	0.02697 $^{\{5\}}$	0.02859 $^{\{6\}}$	0.03393 $^{\{10\}}$	0.02623 $^{\{1\}}$	0.02648 $^{\{2\}}$	0.02649 $^{\{3.5\}}$
		θ	0.04136 $^{\{4\}}$	0.05595 $^{\{10\}}$	0.04814 $^{\{7\}}$	0.05586 $^{\{9\}}$	0.04322 $^{\{5\}}$	0.04555 $^{\{6\}}$	0.05067 $^{\{8\}}$	0.03663 $^{\{1\}}$	0.03698 $^{\{2\}}$	0.03699 $^{\{3\}}$
		α	0.00387 $^{\{4\}}$	0.00583 $^{\{9\}}$	0.00458 $^{\{7\}}$	0.00634 $^{\{10\}}$	0.00448 $^{\{6\}}$	0.00445 $^{\{5\}}$	0.00473 $^{\{8\}}$	0.00263 $^{\{1\}}$	0.00265 $^{\{2.5\}}$	0.00265 $^{\{2.5\}}$
500	MSE	β	0.00117 $^{\{5\}}$	0.00145 $^{\{8\}}$	0.00132 $^{\{7\}}$	0.00156 $^{\{9\}}$	0.00114 $^{\{4\}}$	0.00129 $^{\{6\}}$	0.00178 $^{\{10\}}$	0.00103 $^{\{1\}}$	0.00104 $^{\{2.5\}}$	0.00104 $^{\{2.5\}}$
		θ	0.00283 $^{\{4\}}$	0.00509 $^{\{10\}}$	0.00376 $^{\{7\}}$	0.00497 $^{\{9\}}$	0.00299 $^{\{5\}}$	0.00332 $^{\{6\}}$	0.00399 $^{\{8\}}$	0.00197 $^{\{1\}}$	0.00199 $^{\{2.5\}}$	0.00199 $^{\{2.5\}}$
		α	0.08143 $^{\{4\}}$	0.09970 $^{\{9\}}$	0.08789 $^{\{6\}}$	0.10272 $^{\{10\}}$	0.08800 $^{\{7\}}$	0.08692 $^{\{5\}}$	0.09010 $^{\{8\}}$	0.06956 $^{\{1\}}$	0.07032 $^{\{3\}}$	0.07030 $^{\{2\}}$
	MRE	β	0.04416 $^{\{4\}}$	0.05088 $^{\{8\}}$	0.04800 $^{\{7\}}$	0.05240 $^{\{9\}}$	0.04495 $^{\{5\}}$	0.04765 $^{\{6\}}$	0.05655 $^{\{10\}}$	0.04364 $^{\{1\}}$	0.04414 $^{\{2\}}$	0.04415 $^{\{3\}}$
		θ	0.08273 $^{\{4\}}$	0.11191 $^{\{10\}}$	0.09628 $^{\{7\}}$	0.11172 $^{\{9\}}$	0.08644 $^{\{5\}}$	0.09110 $^{\{6\}}$	0.10135 $^{\{8\}}$	0.07321 $^{\{1\}}$	0.07397 $^{\{2\}}$	0.07398 $^{\{3\}}$
		$\sum$	36.5 $^{\{4\}}$	81.0 $^{\{9\}}$	61.0 $^{\{7\}}$	84.0 $^{\{10\}}$	49.0 $^{\{5\}}$	51.0 $^{\{6\}}$	78.0 $^{\{8\}}$	9.0 $^{\{1\}}$	21.5 $^{\{2\}}$	24.0 $^{\{3\}}$

Table 9

Results of the numerical simulation for the NWEx distribution with α = 0.5, β = 0.5, θ = 1.0.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.16008 {4}	0.44274 {7}	0.44610 {8}	0.47664 {9}	0.22481 {5}	0.27885 {6}	0.52449 {10}	0.04862 {2}	0.04879 {3}	0.04860 {1}
	$\|BIAS\|$	β	0.10654 {6}	0.11947 {9}	0.11669 {7}	0.11932 {8}	0.10609 {5}	0.10517 {4}	0.12846 {10}	0.04353 {1}	0.04382 {3}	0.04379 {2}
		θ	0.34301 {4}	0.46312 {9}	0.44523 {8}	0.47873 {10}	0.39423 {6}	0.36939 {5}	0.40920 {7}	0.09112 {2}	0.09111 {1}	0.09148 {3}
		α	0.04902 {4}	3.06591 {8}	5.86499 {9}	2.63509 {7}	0.77334 {5}	1.75504 {6}	6.10045 {10}	0.00329 {3}	0.00326 {2}	0.00324 {1}
30	MSE	β	0.01992 {6}	0.02405 {8}	0.02347 {7}	0.02507 {9}	0.01776 {4}	0.01884 {5}	0.03096 {10}	0.00269 {2.5}	0.00269 {2.5}	0.00268 {1}
		θ	0.20747 {4}	0.41305 {8}	0.42329 {9}	0.54883 {10}	0.28928 {6}	0.25592 {5}	0.30830 {7}	0.01176 {2.5}	0.01159 {1}	0.01176 {2.5}
		α	0.32015 {4}	0.88547 {7}	0.89221 {8}	0.95328 {9}	0.44962 {5}	0.55770 {6}	1.04897 {10}	0.09555 {1}	0.09758 {3}	0.09720 {2}
	MRE	β	0.21308 {6}	0.23893 {9}	0.23338 {7}	0.23865 {8}	0.21218 {5}	0.21035 {4}	0.25692 {10}	0.08593 {1}	0.08765 {3}	0.08758 {2}
		θ	0.34301 {4}	0.46312 {9}	0.44523 {8}	0.47873 {10}	0.39423 {6}	0.36939 {5}	0.40920 {7}	0.09017 {1}	0.09111 {2}	0.09148 {3}
		$\sum$	42.0 {4}	74.0 {8}	71.0 {7}	80.0 {9}	47.0 {6}	46.0 {5}	81.0 {10}	16.0 {1}	20.5 {3}	17.5 {2}
		α	0.11598 {4}	0.20980 {10}	0.14395 {7}	0.20566 {9}	0.12705 {5}	0.13289 {6}	0.17107 {8}	0.04727 {1}	0.04755 {3}	0.04746 {2}
	$\|BIAS\|$	β	0.07974 {4}	0.09734 {9}	0.08215 {6}	0.09137 {8}	0.08474 {7}	0.08130 {5}	0.10697 {10}	0.04095 {1}	0.04125 {2}	0.04128 {3}
		θ	0.25819 {4}	0.35750 {10}	0.29302 {7}	0.33036 {9}	0.28522 {6}	0.27523 {5}	0.32106 {8}	0.08999 {1}	0.09028 {2}	0.09070 {3}
		α	0.02509 {4}	0.45097 {9}	0.08375 {7}	0.51338 {10}	0.03345 {5}	0.04054 {6}	0.42536 {8}	0.00310 {2.5}	0.00310 {2.5}	0.00309 {1}
50	MSE	β	0.01056 {4}	0.01607 {9}	0.01070 {6}	0.01422 {8}	0.01120 {7}	0.01068 {5}	0.02079 {10}	0.00241 {1}	0.00242 {2.5}	0.00242 {2.5}
		θ	0.12127 {4}	0.21756 {10}	0.14533 {7}	0.18365 {9}	0.13752 {6}	0.12750 {5}	0.18360 {8}	0.01150 {2}	0.01142 {1}	0.01156 {3}
		α	0.23196 {4}	0.41960 {10}	0.28790 {7}	0.41132 {9}	0.25411 {5}	0.26578 {6}	0.34214 {8}	0.09333 {1}	0.09511 {3}	0.09493 {2}
	MRE	β	0.15947 {4}	0.19469 {9}	0.16429 {6}	0.18273 {8}	0.16948 {7}	0.16260 {5}	0.21394 {10}	0.08119 {1}	0.08250 {2}	0.08255 {3}
		θ	0.25819 {4}	0.35750 {10}	0.29302 {7}	0.33036 {9}	0.28522 {6}	0.27523 {5}	0.32106 {8}	0.08956 {1}	0.09028 {2}	0.09070 {3}
		$\sum$	36.0 {4}	86.0 {10}	60.0 {7}	79.0 {9}	54.0 {6}	48.0 {5}	78.0 {8}	11.5 {1}	20.0 {2}	22.5 {3}
		α	0.08409 {4}	0.09721 {9}	0.09111 {7}	0.09763 {10}	0.08879 {5}	0.09209 {8}	0.08997 {6}	0.04350 {1}	0.04382 {3}	0.04378 {2}
	$\|BIAS\|$	β	0.05453 {4}	0.06527 {9}	0.06053 {7}	0.06409 {8}	0.05577 {5}	0.05740 {6}	0.06896 {10}	0.03673 {1}	0.03705 {2}	0.03709 {3}
		θ	0.17694 {4}	0.22800 {10}	0.20106 {7}	0.21806 {9}	0.18970 {5}	0.19080 {6}	0.20940 {8}	0.08625 {1}	0.08678 {2}	0.08702 {3}
		α	0.01178 {4}	0.01869 {10}	0.01493 {7}	0.01711 {9}	0.01418 {5}	0.01575 {8}	0.01452 {6}	0.00271 {1.5}	0.00272 {3}	0.00271 {1.5}
100	MSE	β	0.00503 {5}	0.00682 {9}	0.00588 {7}	0.00663 {8}	0.00489 {4}	0.00517 {6}	0.00807 {10}	0.00195 {1}	0.00197 {2.5}	0.00197 {2.5}
		θ	0.05213 {4}	0.08678 {10}	0.06618 {7}	0.07827 {9}	0.05878 {6}	0.05864 {5}	0.07294 {8}	0.01063 {1.5}	0.01063 {1.5}	0.01071 {3}

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	MRE	α	0.16818 {4}	0.19442 {9}	0.18221 {7}	0.19525 {10}	0.17759 {5}	0.18417 {8}	0.17994 {6}	0.08623 {1}	0.08765 {3}	0.08755 {2}
		β	0.10906 {4}	0.13053 {9}	0.12106 {7}	0.12818 {8}	0.11155 {5}	0.11480 {6}	0.13793 {10}	0.07317 {1}	0.07409 {2}	0.07418 {3}
		θ	0.17694 {4}	0.22800 {10}	0.20106 {7}	0.21806 {9}	0.18970 {5}	0.19080 {6}	0.20940 {8}	0.08574 {1}	0.08678 {2}	0.08702 {3}
		$\sum$	37.0 {4}	85.0 {10}	63.0 {7}	80.0 {9}	45.0 {5}	59.0 {6}	72.0 {8}	10.0 {1}	21.0 {2}	23.0 {3}
250	$\lvert BIAS \rvert$	α	0.05071 {4}	0.05935 {10}	0.05401 {6}	0.05931 {9}	0.05132 {5}	0.05564 {8}	0.05434 {7}	0.03848 {1}	0.03881 {3}	0.03878 {2}
		β	0.03474 {5}	0.04020 {9}	0.03436 {4}	0.03928 {8}	0.03535 {7}	0.03485 {6}	0.04413 {10}	0.02904 {1}	0.02930 {2}	0.02933 {3}
		θ	0.10960 {4}	0.13970 {10}	0.11761 {6}	0.13699 {9}	0.11428 {5}	0.11811 {7}	0.13111 {8}	0.07939 {1}	0.07998 {2}	0.08005 {3}
	MSE	α	0.00423 {4}	0.00616 {10}	0.00465 {6}	0.00563 {9}	0.00438 {5}	0.00515 {8}	0.00500 {7}	0.00217 {1}	0.00219 {3}	0.00218 {2}
		β	0.00193 {5}	0.00252 {9}	0.00193 {5}	0.00242 {8}	0.00193 {5}	0.00205 {7}	0.00303 {10}	0.00126 {1}	0.00127 {2.5}	0.00127 {2.5}
		θ	0.01910 {4}	0.03160 {10}	0.02127 {6}	0.03119 {9}	0.02031 {5}	0.02298 {7}	0.02753 {8}	0.00917 {1}	0.00921 {2}	0.00922 {3}
	MRE	α	0.10143 {4}	0.11870 {10}	0.10802 {6}	0.11862 {9}	0.10264 {5}	0.11127 {8}	0.10867 {7}	0.07665 {1}	0.07762 {3}	0.07756 {2}
		β	0.06949 {5}	0.08041 {9}	0.06872 {4}	0.07856 {8}	0.07069 {7}	0.06971 {6}	0.08826 {10}	0.05793 {1}	0.05861 {2}	0.05865 {3}
		θ	0.10960 {4}	0.13970 {10}	0.11761 {6}	0.13699 {9}	0.11428 {5}	0.11811 {7}	0.13111 {8}	0.07942 {1}	0.07998 {2}	0.08005 {3}
		$\sum$	39.0 {4}	87.0 {10}	49.0 {5.5}	78.0 {9}	49.0 {5.5}	64.0 {7}	75.0 {8}	9.0 {1}	21.5 {2}	23.5 {3}
500	$\lvert BIAS \rvert$	α	0.03638 {4}	0.04088 {9}	0.03736 {6}	0.04114 {10}	0.03811 {7}	0.03730 {5}	0.03850 {8}	0.03246 {1}	0.03277 {3}	0.03276 {2}
		β	0.02373 {4}	0.02723 {8}	0.02590 {7}	0.02823 {9}	0.02406 {5}	0.02552 {6}	0.03133 {10}	0.02280 {1}	0.02302 {2}	0.02303 {3}
		θ	0.07547 {4}	0.09519 {10}	0.08544 {7}	0.09488 {9}	0.07622 {5}	0.08041 {6}	0.09362 {8}	0.06927 {1}	0.06993 {2.5}	0.06993 {2.5}
	MSE	α	0.00209 {4}	0.00267 {9}	0.00223 {6}	0.00271 {10}	0.00229 {7}	0.00222 {5}	0.00235 {8}	0.00161 {1}	0.00163 {2.5}	0.00163 {2.5}
		β	0.00094 {5}	0.00115 {8}	0.00107 {7}	0.00126 {9}	0.00091 {4}	0.00103 {6}	0.00152 {10}	0.00078 {1}	0.00079 {2.5}	0.00079 {2.5}
		θ	0.00934 {4}	0.01476 {10}	0.01178 {7}	0.01428 {9}	0.00960 {5}	0.01039 {6}	0.01360 {8}	0.00718 {1}	0.00724 {2.5}	0.00724 {2.5}
	MRE	α	0.07275 {4}	0.08177 {9}	0.07472 {6}	0.08228 {10}	0.07623 {7}	0.07459 {5}	0.07699 {8}	0.06491 {1}	0.06555 {3}	0.06553 {2}
		β	0.04746 {4}	0.05445 {8}	0.05181 {7}	0.05647 {9}	0.04811 {5}	0.05104 {6}	0.06267 {10}	0.04554 {1}	0.04604 {2}	0.04605 {3}
		θ	0.07547 {4}	0.09519 {10}	0.08544 {7}	0.09488 {9}	0.07622 {5}	0.08041 {6}	0.09362 {8}	0.06910 {1}	0.06993 {2.5}	0.06993 {2.5}
		$\sum$	37.0 {4}	81.0 {9}	60.0 {7}	84.0 {10}	50.0 {5}	51.0 {6}	78.0 {8}	9.0 {1}	22.5 {2.5}	22.5 {2.5}

Table 10

Results of the numerical simulation for the NWEx distribution with α = 0.3, β = 0.7, θ = 0.5.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.13585 {4}	0.58000 {8}	0.51864 {7}	0.75263 {9}	0.29428 {5}	0.36985 {6}	1.03765 {10}	0.02920 {1}	0.02923 {2.5}	0.02923 {2.5}
	$\|BIAS\|$	β	0.18020 {5}	0.21101 {9}	0.20256 {7}	0.20929 {8}	0.18145 {6}	0.17913 {4}	0.23223 {10}	0.06254 {1}	0.06289 {2}	0.06322 {3}
		θ	0.18121 {4}	0.27860 {10}	0.25741 {8}	0.27790 {9}	0.23470 {6}	0.20840 {5}	0.24786 {7}	0.04747 {2}	0.04745 {1}	0.04824 {3}
		α	0.05720 {4}	5.87035 {7}	7.29718 {8}	8.99718 {9}	2.99661 {5}	5.65616 {6}	25.74219 {10}	0.00119 {3}	0.00118 {1.5}	0.00118 {1.5}
30	MSE	β	0.05852 {6}	0.07502 {8}	0.06883 {7}	0.07682 {9}	0.05254 {4}	0.05515 {5}	0.10113 {10}	0.00549 {2}	0.00548 {1}	0.00553 {3}
		θ	0.05791 {4}	0.18367 {9}	0.15620 {8}	0.21850 {10}	0.11836 {6}	0.08589 {5}	0.12419 {7}	0.00318 {2}	0.00312 {1}	0.00326 {3}
		α	0.45284 {4}	1.93333 {8}	1.72881 {7}	2.50875 {9}	0.98092 {5}	1.23283 {6}	3.45882 {10}	0.09461 {1}	0.09745 {3}	0.09743 {2}
	MRE	β	0.25743 {5}	0.30144 {9}	0.28937 {7}	0.29898 {8}	0.25922 {6}	0.25591 {4}	0.33175 {10}	0.08829 {1}	0.08984 {2}	0.09032 {3}
		θ	0.36241 {4}	0.55720 {10}	0.51482 {8}	0.55580 {9}	0.46940 {6}	0.41680 {5}	0.49571 {7}	0.09213 {1}	0.09489 {2}	0.09649 {3}
		$\sum$	40.0 {4}	78.0 {8}	67.0 {7}	80.0 {9}	49.0 {6}	46.0 {5}	81.0 {10}	14.0 {1}	16.0 {2}	24.0 {3}
		α	0.08901 {4}	0.27785 {9}	0.13427 {7}	0.23813 {8}	0.10299 {5}	0.12362 {6}	0.29168 {10}	0.02839 {1}	0.02851 {2.5}	0.02851 {2.5}
	$\|BIAS\|$	β	0.13405 {4}	0.17331 {9}	0.14042 {6}	0.16339 {8}	0.14473 {7}	0.13767 {5}	0.19657 {10}	0.05993 {1}	0.06034 {2}	0.06053 {3}
		θ	0.14198 {4}	0.20757 {10}	0.16544 {7}	0.18711 {8}	0.16402 {6}	0.15235 {5}	0.19604 {9}	0.04567 {1}	0.04575 {2}	0.04629 {3}
		α	0.01709 {4}	1.78094 {9}	0.23948 {6}	1.16372 {8}	0.03934 {5}	0.28282 {7}	3.13934 {10}	0.00112 {2}	0.00112 {2}	0.00112 {2}
50	MSE	β	0.03028 {4}	0.05089 {9}	0.03165 {6}	0.04644 {8}	0.03314 {7}	0.03096 {5}	0.07153 {10}	0.00505 {1}	0.00506 {2}	0.00509 {3}
		θ	0.03707 {4}	0.07646 {10}	0.04816 {6}	0.06162 {8}	0.05055 {7}	0.04051 {5}	0.07302 {9}	0.00294 {2}	0.00291 {1}	0.00298 {3}
		α	0.29671 {4}	0.92616 {9}	0.44757 {7}	0.79377 {8}	0.34332 {5}	0.41208 {6}	0.97227 {10}	0.09287 {1}	0.09504 {2}	0.09505 {3}
	MRE	β	0.19150 {4}	0.24759 {9}	0.20060 {6}	0.23341 {8}	0.20675 {7}	0.19668 {5}	0.28081 {10}	0.08468 {1}	0.08620 {2}	0.08647 {3}
		θ	0.28396 {4}	0.41513 {10}	0.33088 {7}	0.37422 {8}	0.32804 {6}	0.30470 {5}	0.39209 {9}	0.08924 {1}	0.09151 {2}	0.09258 {3}
		$\sum$	36.0 {4}	84.0 {9}	58.0 {7}	72.0 {8}	55.0 {6}	49.0 {5}	87.0 {10}	11.0 {1}	17.5 {2}	25.5 {3}
		α	0.06219 {4}	0.08075 {10}	0.07057 {7}	0.07728 {9}	0.06705 {5}	0.07020 {6}	0.07489 {8}	0.02724 {1}	0.02744 {3}	0.02742 {2}
	$\|BIAS\|$	β	0.09068 {4}	0.11514 {9}	0.10485 {7}	0.11270 {8}	0.09410 {5}	0.09676 {6}	0.12501 {10}	0.05599 {1}	0.05645 {2}	0.05649 {3}
		θ	0.09515 {4}	0.12926 {10}	0.11267 {7}	0.12271 {8}	0.10569 {6}	0.10375 {5}	0.12682 {9}	0.04433 {1}	0.04454 {2}	0.04476 {3}
		α	0.00687 {4}	0.01899 {10}	0.01041 {6}	0.01214 {8}	0.00912 {5}	0.01084 {7}	0.01478 {9}	0.00104 {1.5}	0.00105 {3}	0.00104 {1.5}
100	MSE	β	0.01400 {5}	0.02167 {9}	0.01766 {7}	0.02094 {8}	0.01398 {4}	0.01519 {6}	0.02717 {10}	0.00442 {1}	0.00444 {2}	0.00445 {3}
		θ	0.01532 {4}	0.02880 {10}	0.02084 {7}	0.02550 {8}	0.01850 {6}	0.01762 {5}	0.02716 {9}	0.00276 {2}	0.00275 {1}	0.00279 {3}
		α	0.20729 {4}	0.26916 {10}	0.23524 {7}	0.25758 {9}	0.22349 {5}	0.23399 {6}	0.24964 {8}	0.09001 {1}	0.09147 {3}	0.09141 {2}

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	MRE	β	0.12955 $^{\{4\}}$	0.16449 $^{\{9\}}$	0.14979 $^{\{7\}}$	0.16100 $^{\{8\}}$	0.13443 $^{\{5\}}$	0.13823 $^{\{6\}}$	0.17858 $^{\{10\}}$	0.07964 $^{\{1\}}$	0.08064 $^{\{2\}}$	0.08071 $^{\{3\}}$
		θ	0.19030 $^{\{4\}}$	0.25851 $^{\{10\}}$	0.22535 $^{\{7\}}$	0.24542 $^{\{8\}}$	0.21137 $^{\{6\}}$	0.20751 $^{\{5\}}$	0.25365 $^{\{9\}}$	0.08779 $^{\{1\}}$	0.08908 $^{\{2\}}$	0.08952 $^{\{3\}}$
		$\sum$	37.0 $^{\{4\}}$	87.0 $^{\{10\}}$	62.0 $^{\{7\}}$	74.0 $^{\{8\}}$	47.0 $^{\{5\}}$	52.0 $^{\{6\}}$	82.0 $^{\{9\}}$	10.5 $^{\{1\}}$	20.0 $^{\{2\}}$	23.5 $^{\{3\}}$
250	$\|BIAS\|$	α	0.03673 $^{\{4\}}$	0.04565 $^{\{10\}}$	0.04029 $^{\{6\}}$	0.04482 $^{\{9\}}$	0.03759 $^{\{5\}}$	0.04122 $^{\{7\}}$	0.04178 $^{\{8\}}$	0.02485 $^{\{1\}}$	0.02508 $^{\{3\}}$	0.02507 $^{\{2\}}$
		β	0.05718 $^{\{4\}}$	0.07099 $^{\{9\}}$	0.05945 $^{\{5\}}$	0.06887 $^{\{8\}}$	0.05948 $^{\{6\}}$	0.05975 $^{\{7\}}$	0.08002 $^{\{10\}}$	0.04545 $^{\{1\}}$	0.04589 $^{\{2\}}$	0.04590 $^{\{3\}}$
		θ	0.05894 $^{\{4\}}$	0.07709 $^{\{9\}}$	0.06500 $^{\{7\}}$	0.07627 $^{\{8\}}$	0.06375 $^{\{5\}}$	0.06465 $^{\{6\}}$	0.07806 $^{\{10\}}$	0.03995 $^{\{1\}}$	0.04032 $^{\{2\}}$	0.04033 $^{\{3\}}$
	MSE	α	0.00230 $^{\{4\}}$	0.00396 $^{\{10\}}$	0.00266 $^{\{6\}}$	0.00335 $^{\{9\}}$	0.00240 $^{\{5\}}$	0.00289 $^{\{7\}}$	0.00313 $^{\{8\}}$	0.00089 $^{\{1\}}$	0.00090 $^{\{2.5\}}$	0.00090 $^{\{2.5\}}$
		β	0.00532 $^{\{4\}}$	0.00792 $^{\{9\}}$	0.00572 $^{\{6\}}$	0.00754 $^{\{8\}}$	0.00546 $^{\{5\}}$	0.00604 $^{\{7\}}$	0.01005 $^{\{10\}}$	0.00304 $^{\{1\}}$	0.00307 $^{\{2.5\}}$	0.00307 $^{\{2.5\}}$
		θ	0.00555 $^{\{4\}}$	0.00976 $^{\{8\}}$	0.00651 $^{\{6\}}$	0.00983 $^{\{10\}}$	0.00624 $^{\{5\}}$	0.00690 $^{\{7\}}$	0.00981 $^{\{9\}}$	0.00231 $^{\{1\}}$	0.00233 $^{\{2.5\}}$	0.00233 $^{\{2.5\}}$
	MRE	α	0.12245 $^{\{4\}}$	0.15217 $^{\{10\}}$	0.13428 $^{\{6\}}$	0.14940 $^{\{9\}}$	0.12529 $^{\{5\}}$	0.13739 $^{\{7\}}$	0.13925 $^{\{8\}}$	0.08247 $^{\{1\}}$	0.08361 $^{\{3\}}$	0.08357 $^{\{2\}}$
		β	0.08168 $^{\{4\}}$	0.10142 $^{\{9\}}$	0.08493 $^{\{5\}}$	0.09838 $^{\{8\}}$	0.08498 $^{\{6\}}$	0.08536 $^{\{7\}}$	0.11432 $^{\{10\}}$	0.06480 $^{\{1\}}$	0.06556 $^{\{2.5\}}$	0.06556 $^{\{2.5\}}$
		θ	0.11787 $^{\{4\}}$	0.15419 $^{\{9\}}$	0.13001 $^{\{7\}}$	0.15253 $^{\{8\}}$	0.12751 $^{\{5\}}$	0.12931 $^{\{6\}}$	0.15611 $^{\{10\}}$	0.07960 $^{\{1\}}$	0.08065 $^{\{2\}}$	0.08066 $^{\{3\}}$
		$\sum$	36.0 $^{\{4\}}$	83.0 $^{\{9.5\}}$	54.0 $^{\{6\}}$	77.0 $^{\{8\}}$	47.0 $^{\{5\}}$	61.0 $^{\{7\}}$	83.0 $^{\{9.5\}}$	9.0 $^{\{1\}}$	22.0 $^{\{2\}}$	23.0 $^{\{3\}}$
500	$\|BIAS\|$	α	0.02623 $^{\{4\}}$	0.03081 $^{\{10\}}$	0.02748 $^{\{5\}}$	0.03063 $^{\{9\}}$	0.02756 $^{\{6\}}$	0.02773 $^{\{7\}}$	0.02917 $^{\{8\}}$	0.02209 $^{\{1\}}$	0.02230 $^{\{3\}}$	0.02229 $^{\{2\}}$
		β	0.03971 $^{\{4\}}$	0.04786 $^{\{8\}}$	0.04415 $^{\{7\}}$	0.04957 $^{\{9\}}$	0.03974 $^{\{5\}}$	0.04303 $^{\{6\}}$	0.05671 $^{\{10\}}$	0.03700 $^{\{1\}}$	0.03736 $^{\{2.5\}}$	0.03736 $^{\{2.5\}}$
		θ	0.04152 $^{\{5\}}$	0.05308 $^{\{8\}}$	0.04699 $^{\{7\}}$	0.05336 $^{\{9\}}$	0.04124 $^{\{4\}}$	0.04426 $^{\{6\}}$	0.05559 $^{\{10\}}$	0.03492 $^{\{1\}}$	0.03525 $^{\{2\}}$	0.03526 $^{\{3\}}$
	MSE	α	0.00110 $^{\{4\}}$	0.00152 $^{\{9\}}$	0.00121 $^{\{5\}}$	0.00153 $^{\{10\}}$	0.00122 $^{\{6.5\}}$	0.00122 $^{\{6.5\}}$	0.00138 $^{\{8\}}$	0.00072 $^{\{1\}}$	0.00073 $^{\{2.5\}}$	0.00073 $^{\{2.5\}}$
		β	0.00261 $^{\{5\}}$	0.00353 $^{\{8\}}$	0.00316 $^{\{7\}}$	0.00397 $^{\{9\}}$	0.00253 $^{\{4\}}$	0.00297 $^{\{6\}}$	0.00507 $^{\{10\}}$	0.00210 $^{\{1\}}$	0.00212 $^{\{2.5\}}$	0.00212 $^{\{2.5\}}$
		θ	0.00280 $^{\{4\}}$	0.00463 $^{\{9\}}$	0.00355 $^{\{7\}}$	0.00451 $^{\{8\}}$	0.00282 $^{\{5\}}$	0.00316 $^{\{6\}}$	0.00483 $^{\{10\}}$	0.00182 $^{\{1\}}$	0.00184 $^{\{2.5\}}$	0.00184 $^{\{2.5\}}$
	MRE	α	0.08745 $^{\{4\}}$	0.10269 $^{\{10\}}$	0.09160 $^{\{5\}}$	0.10209 $^{\{9\}}$	0.09188 $^{\{6\}}$	0.09243 $^{\{7\}}$	0.09725 $^{\{8\}}$	0.07360 $^{\{1\}}$	0.07433 $^{\{3\}}$	0.07431 $^{\{2\}}$
		β	0.05673 $^{\{4\}}$	0.06838 $^{\{8\}}$	0.06307 $^{\{7\}}$	0.07082 $^{\{9\}}$	0.05677 $^{\{5\}}$	0.06147 $^{\{6\}}$	0.08102 $^{\{10\}}$	0.05278 $^{\{1\}}$	0.05336 $^{\{2\}}$	0.05337 $^{\{3\}}$
		θ	0.08305 $^{\{5\}}$	0.10616 $^{\{8\}}$	0.09397 $^{\{7\}}$	0.10671 $^{\{9\}}$	0.08249 $^{\{4\}}$	0.08851 $^{\{6\}}$	0.11118 $^{\{10\}}$	0.06970 $^{\{1\}}$	0.07050 $^{\{2\}}$	0.07051 $^{\{3\}}$
		$\sum$	39.0 $^{\{4\}}$	78.0 $^{\{8\}}$	57.0 $^{\{7\}}$	81.0 $^{\{9\}}$	45.5 $^{\{5\}}$	56.5 $^{\{6\}}$	84.0 $^{\{10\}}$	9.0 $^{\{1\}}$	22.0 $^{\{2\}}$	23.0 $^{\{3\}}$

Table 11

Results of the numerical simulation for the NWEx distribution with α = 0.15, β = 0.4, θ = 2.5.

n	Est.	Param.	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
		α	0.05284 {4}	0.14744 {8}	0.09404 {7}	0.20459 {9}	0.06352 {5}	0.08275 {6}	0.31616 {10}	0.01476 {1}	0.01479 {2}	0.01482 {3}
	$\|BIAS\|$	β	0.17451 {5}	0.24275 {8}	0.20006 {7}	0.26154 {9}	0.16103 {4}	0.19553 {6}	0.32138 {10}	0.03664 {1}	0.03669 {2}	0.03688 {3}
		θ	1.00327 {4}	2.21473 {10}	1.77823 {9}	1.75513 {8}	1.61668 {6}	1.36649 {5}	1.74055 {7}	0.24664 {2}	0.24168 {1}	0.25966 {3}
		α	0.00484 {4}	0.57352 {8}	0.29606 {7}	1.30531 {9}	0.05845 {5}	0.13621 {6}	2.78205 {10}	0.00030 {2}	0.00030 {2}	0.00030 {2}
30	MSE	β	0.07757 {5}	0.13974 {8}	0.09604 {7}	0.17166 {9}	0.04944 {4}	0.09060 {6}	0.25961 {10}	0.00189 {3}	0.00187 {1}	0.00188 {2}
		θ	2.02879 {4}	13.82800 {10}	9.98074 {9}	8.10914 {7}	9.61959 {8}	5.68855 {5}	6.67301 {6}	0.08766 {2}	0.08294 {1}	0.09720 {3}
		α	0.35224 {4}	0.98294 {8}	0.62693 {7}	1.36396 {9}	0.42347 {5}	0.55170 {6}	2.10774 {10}	0.09627 {1}	0.09862 {2}	0.09877 {3}
	MRE	β	0.43628 {5}	0.60688 {8}	0.50015 {7}	0.65386 {9}	0.40257 {4}	0.48882 {6}	0.80344 {10}	0.08936 {1}	0.09173 {2}	0.09220 {3}
		θ	0.40131 {4}	0.88589 {10}	0.71129 {9}	0.70205 {8}	0.64667 {6}	0.54660 {5}	0.69622 {7}	0.09105 {1}	0.09667 {2}	0.10386 {3}
		$\sum$	39.0 {4}	78.0 {9}	69.0 {7}	77.0 {8}	47.0 {5}	51.0 {6}	80.0 {10}	14.0 {1}	15.0 {2}	25.0 {3}
		α	0.03941 {4}	0.06572 {8}	0.05004 {7}	0.09172 {9}	0.04536 {6}	0.04158 {5}	0.11601 {10}	0.01444 {1}	0.01451 {3}	0.01448 {2}
	$\|BIAS\|$	β	0.12147 {5}	0.18483 {8}	0.14341 {7}	0.19549 {9}	0.12050 {4}	0.13687 {6}	0.23557 {10}	0.03596 {1}	0.03612 {2}	0.03620 {3}
		θ	0.75355 {4}	1.46444 {10}	1.11970 {7}	1.21550 {8}	0.98686 {6}	0.91545 {5}	1.32279 {9}	0.23886 {2}	0.23530 {1}	0.24591 {3}
		α	0.00252 {4}	0.06044 {8}	0.02266 {7}	0.41895 {10}	0.00475 {6}	0.00299 {5}	0.41038 {9}	0.00029 {2}	0.00029 {2}	0.00029 {2}
50	MSE	β	0.02973 {5}	0.07880 {8}	0.04584 {7}	0.08924 {9}	0.02701 {4}	0.03685 {6}	0.14236 {10}	0.00181 {3}	0.00180 {1.5}	0.00180 {1.5}
		θ	1.05453 {4}	5.86596 {10}	2.82792 {7}	2.87393 {8}	2.16510 {6}	1.71448 {5}	3.45940 {9}	0.08176 {2}	0.07819 {1}	0.08716 {3}
		α	0.26276 {4}	0.43814 {8}	0.33361 {7}	0.61145 {9}	0.30240 {6}	0.27719 {5}	0.77340 {10}	0.09498 {1}	0.09676 {3}	0.09653 {2}
	MRE	β	0.30368 {5}	0.46208 {8}	0.35853 {7}	0.48873 {9}	0.30124 {4}	0.34219 {6}	0.58893 {10}	0.08903 {1}	0.09031 {2}	0.09051 {3}
		θ	0.30142 {4}	0.58578 {10}	0.44788 {7}	0.48620 {8}	0.39474 {6}	0.36618 {5}	0.52911 {9}	0.09095 {1}	0.09412 {2}	0.09836 {3}
		$\sum$	39.0 {4}	78.0 {8}	63.0 {7}	79.0 {9}	48.0 {5.5}	48.0 {5.5}	86.0 {10}	14.0 {1}	17.5 {2}	22.5 {3}
		α	0.02739 {4}	0.03157 {8}	0.02880 {5}	0.03310 {9}	0.02929 {7}	0.02919 {6}	0.04289 {10}	0.01400 {1}	0.01411 {3}	0.01407 {2}
	$\|BIAS\|$	β	0.07783 {4}	0.12122 {8}	0.09927 {7}	0.13135 {9}	0.08129 {5}	0.08726 {6}	0.16963 {10}	0.03524 {1}	0.03550 {2.5}	0.03550 {2.5}
		θ	0.50532 {4}	0.79287 {8}	0.64296 {7}	0.86192 {9}	0.63999 {6}	0.61174 {5}	0.92957 {10}	0.22856 {2}	0.22700 {1}	0.23076 {3}
		α	0.00125 {4}	0.00168 {8}	0.00144 {7}	0.00186 {9}	0.00141 {6}	0.00139 {5}	0.03362 {10}	0.00027 {2}	0.00027 {2}	0.00027 {2}
100	MSE	β	0.01207 {5}	0.03211 {8}	0.02326 {7}	0.03571 {9}	0.01090 {4}	0.01364 {6}	0.07074 {10}	0.00173 {1.5}	0.00173 {1.5}	0.00174 {3}
		θ	0.45375 {4}	1.07872 {8}	0.73064 {6}	1.33707 {9}	0.86621 {7}	0.67312 {5}	1.45147 {10}	0.07397 {2}	0.07182 {1}	0.07539 {3}
		α	0.18262 {4}	0.21045 {8}	0.19198 {5}	0.22067 {9}	0.19529 {7}	0.19460 {6}	0.28595 {10}	0.09256 {1}	0.09404 {3}	0.09383 {2}

Contd..

	MRE	β	0.19458 $^{\{4\}}$	0.30305 $^{\{8\}}$	0.24817 $^{\{7\}}$	0.32838 $^{\{9\}}$	0.20323 $^{\{5\}}$	0.21816 $^{\{6\}}$	0.42407 $^{\{10\}}$	0.08722 $^{\{1\}}$	0.08875 $^{\{2\}}$	0.08876 $^{\{3\}}$
		θ	0.20213 $^{\{4\}}$	0.31715 $^{\{8\}}$	0.25718 $^{\{7\}}$	0.34477 $^{\{9\}}$	0.25600 $^{\{6\}}$	0.24470 $^{\{5\}}$	0.37183 $^{\{10\}}$	0.09006 $^{\{1\}}$	0.09080 $^{\{2\}}$	0.09230 $^{\{3\}}$
		$\sum$	37.0 $^{\{4\}}$	72.0 $^{\{8\}}$	58.0 $^{\{7\}}$	81.0 $^{\{9\}}$	53.0 $^{\{6\}}$	50.0 $^{\{5\}}$	90.0 $^{\{10\}}$	12.5 $^{\{1\}}$	18.0 $^{\{2\}}$	23.5 $^{\{3\}}$
250	$\|BIAS\|$	α	0.01743 $^{\{4\}}$	0.01959 $^{\{10\}}$	0.01786 $^{\{5\}}$	0.01951 $^{\{9\}}$	0.01791 $^{\{6\}}$	0.01903 $^{\{8\}}$	0.01898 $^{\{7\}}$	0.01237 $^{\{1\}}$	0.01248 $^{\{3\}}$	0.01247 $^{\{2\}}$
		β	0.04893 $^{\{5\}}$	0.07371 $^{\{9\}}$	0.05697 $^{\{7\}}$	0.07268 $^{\{8\}}$	0.04749 $^{\{4\}}$	0.05306 $^{\{6\}}$	0.09432 $^{\{10\}}$	0.03167 $^{\{1\}}$	0.03197 $^{\{2.5\}}$	0.03197 $^{\{2.5\}}$
		θ	0.31793 $^{\{4\}}$	0.49442 $^{\{8\}}$	0.39239 $^{\{7\}}$	0.49645 $^{\{9\}}$	0.33575 $^{\{5\}}$	0.37486 $^{\{6\}}$	0.58711 $^{\{10\}}$	0.21289 $^{\{1\}}$	0.21484 $^{\{3\}}$	0.21478 $^{\{2\}}$
	MSE	α	0.00049 $^{\{4\}}$	0.00061 $^{\{8.5\}}$	0.00050 $^{\{5\}}$	0.00061 $^{\{8.5\}}$	0.00052 $^{\{6\}}$	0.00058 $^{\{7\}}$	0.00062 $^{\{10\}}$	0.00022 $^{\{2\}}$	0.00022 $^{\{2\}}$	0.00022 $^{\{2\}}$
		β	0.00407 $^{\{5\}}$	0.00964 $^{\{9\}}$	0.00551 $^{\{7\}}$	0.00921 $^{\{8\}}$	0.00345 $^{\{4\}}$	0.00469 $^{\{6\}}$	0.01840 $^{\{10\}}$	0.00143 $^{\{1\}}$	0.00144 $^{\{2.5\}}$	0.00144 $^{\{2.5\}}$
		θ	0.16783 $^{\{4\}}$	0.39661 $^{\{8\}}$	0.25719 $^{\{7\}}$	0.39662 $^{\{9\}}$	0.20060 $^{\{5\}}$	0.22679 $^{\{6\}}$	0.58754 $^{\{10\}}$	0.06411 $^{\{1\}}$	0.06461 $^{\{3\}}$	0.06455 $^{\{2\}}$
	MRE	α	0.11618 $^{\{4\}}$	0.13061 $^{\{10\}}$	0.11906 $^{\{5\}}$	0.13010 $^{\{9\}}$	0.11941 $^{\{6\}}$	0.12689 $^{\{8\}}$	0.12657 $^{\{7\}}$	0.08221 $^{\{1\}}$	0.08321 $^{\{3\}}$	0.08316 $^{\{2\}}$
		β	0.12232 $^{\{5\}}$	0.18426 $^{\{9\}}$	0.14244 $^{\{7\}}$	0.18171 $^{\{8\}}$	0.11874 $^{\{4\}}$	0.13265 $^{\{6\}}$	0.23579 $^{\{10\}}$	0.07919 $^{\{1\}}$	0.07991 $^{\{2\}}$	0.07993 $^{\{3\}}$
		θ	0.12717 $^{\{4\}}$	0.19777 $^{\{8\}}$	0.15696 $^{\{7\}}$	0.19858 $^{\{9\}}$	0.13430 $^{\{5\}}$	0.14994 $^{\{6\}}$	0.23485 $^{\{10\}}$	0.08480 $^{\{1\}}$	0.08594 $^{\{3\}}$	0.08591 $^{\{2\}}$
		$\sum$	39.0 $^{\{4\}}$	79.5 $^{\{9\}}$	57.0 $^{\{6\}}$	77.5 $^{\{8\}}$	45.0 $^{\{5\}}$	59.0 $^{\{7\}}$	84.0 $^{\{10\}}$	10.0 $^{\{1\}}$	24.0 $^{\{3\}}$	20.0 $^{\{2\}}$
500	$\|BIAS\|$	α	0.01223 $^{\{5\}}$	0.01322 $^{\{8\}}$	0.01249 $^{\{6\}}$	0.01335 $^{\{10\}}$	0.01180 $^{\{4\}}$	0.01304 $^{\{7\}}$	0.01334 $^{\{9\}}$	0.01113 $^{\{1\}}$	0.01124 $^{\{3\}}$	0.01123 $^{\{2\}}$
		β	0.03033 $^{\{4\}}$	0.05013 $^{\{9\}}$	0.03678 $^{\{6\}}$	0.04927 $^{\{8\}}$	0.03235 $^{\{5\}}$	0.03787 $^{\{7\}}$	0.06393 $^{\{10\}}$	0.02792 $^{\{1\}}$	0.02819 $^{\{2.5\}}$	0.02819 $^{\{2.5\}}$
		θ	0.20798 $^{\{4\}}$	0.34674 $^{\{9\}}$	0.25140 $^{\{6\}}$	0.33234 $^{\{8\}}$	0.21381 $^{\{5\}}$	0.26968 $^{\{7\}}$	0.41856 $^{\{10\}}$	0.19005 $^{\{1\}}$	0.19186 $^{\{3\}}$	0.19182 $^{\{2\}}$
	MSE	α	0.00024 $^{\{5.5\}}$	0.00027 $^{\{7.5\}}$	0.00024 $^{\{5.5\}}$	0.00028 $^{\{9.5\}}$	0.00023 $^{\{4\}}$	0.00027 $^{\{7.5\}}$	0.00028 $^{\{9.5\}}$	0.00018 $^{\{1\}}$	0.00019 $^{\{2.5\}}$	0.00019 $^{\{2.5\}}$
		β	0.00164 $^{\{4\}}$	0.00414 $^{\{8\}}$	0.00229 $^{\{6\}}$	0.00423 $^{\{9\}}$	0.00170 $^{\{5\}}$	0.00233 $^{\{7\}}$	0.00717 $^{\{10\}}$	0.00114 $^{\{1\}}$	0.00115 $^{\{2.5\}}$	0.00115 $^{\{2.5\}}$
		θ	0.07629 $^{\{4\}}$	0.19096 $^{\{9\}}$	0.10854 $^{\{6\}}$	0.18406 $^{\{8\}}$	0.08678 $^{\{5\}}$	0.11560 $^{\{7\}}$	0.27592 $^{\{10\}}$	0.05279 $^{\{1\}}$	0.05325 $^{\{3\}}$	0.05322 $^{\{2\}}$
	MRE	α	0.08154 $^{\{5\}}$	0.08811 $^{\{8\}}$	0.08326 $^{\{6\}}$	0.08898 $^{\{10\}}$	0.07868 $^{\{4\}}$	0.08693 $^{\{7\}}$	0.08896 $^{\{9\}}$	0.07415 $^{\{1\}}$	0.07491 $^{\{3\}}$	0.07489 $^{\{2\}}$
		β	0.07583 $^{\{4\}}$	0.12533 $^{\{9\}}$	0.09194 $^{\{6\}}$	0.12318 $^{\{8\}}$	0.08088 $^{\{5\}}$	0.09466 $^{\{7\}}$	0.15982 $^{\{10\}}$	0.06976 $^{\{1\}}$	0.07047 $^{\{2.5\}}$	0.07047 $^{\{2.5\}}$
		θ	0.08319 $^{\{4\}}$	0.13870 $^{\{9\}}$	0.10056 $^{\{6\}}$	0.13293 $^{\{8\}}$	0.08552 $^{\{5\}}$	0.10787 $^{\{7\}}$	0.16743 $^{\{10\}}$	0.07602 $^{\{1\}}$	0.07675 $^{\{3\}}$	0.07673 $^{\{2\}}$
		$\sum$	39.5 $^{\{4\}}$	76.5 $^{\{8\}}$	53.5 $^{\{6\}}$	78.5 $^{\{9\}}$	42.0 $^{\{5\}}$	63.5 $^{\{7\}}$	87.5 $^{\{10\}}$	9.0 $^{\{1\}}$	25.0 $^{\{3\}}$	20.0 $^{\{2\}}$

Table 12

Ranking results for all estimation methods across different combinations of the parameters.

Parameters	n	MLE	LSE	WLSE	CRVME	MPSE	ADE	RADE	SE	LN	GE
	30	4	9	7	8	5	6	10	2	3	1
α=0.1	50	6	9	7	8	4	5	10	1	2.5	2.5
β=0.5	100	4	9	7	8	6	5	10	1	3	2
θ=2.5	250	4	9	7	8	5	6	10	1	3	2
	500	5	9	6	8	4	7	10	1	3	2
	30	4	8	7	9	5	6	10	1	3	2
α=0.4	50	4	9	7	10	6	5	8	1	2	3
β=0.5	100	4	9.5	7	8	6	5	9.5	1	2	3
θ=1.5	250	4	9	6	8	5	7	10	1	2	3
	500	4	8	6	10	5	7	9	1	3	2
	30	4	9	7	10	5	6	8	1	3	2
α=0.5	50	4	9	7	10	5	6	8	1	2	3
β=0.6	100	4	10	7	9	6	5	8	1	2	3
θ=0.8	250	4	10	7	9	5	6	8	1	2	3
	500	4	9	6	10	5	7	8	1	2	3
	30	4	8	7	9	5	6	10	1	2	3
α=0.6	50	4	9.5	7	9.5	5	6	8	1	3	2
β=1.0	100	4	10	6	8.5	5	7	8.5	1	3	2
θ=1.5	250	5	10	6	9	4	7	8	1	2	3
	500	5	9	6	10	4	7	8	1	3	2
	30	4	7.5	7.5	10	6	5	9	1	2	3
α=0.6	50	4	10	7	9	6	5	8	1	2	3
β=0.5	100	4	10	7	9	5	6	8	1	2	3
θ=0.6	250	5.5	10	5.5	9	4	7	8	1	2	3
	500	8.5	8.5	6	10	4.5	4.5	7	1	2.5	2.5
	30	5	10	8	9	4	6	7	1	2	3
α=1.5	50	4	9	7	10	5	6	8	1	2	3
β=0.5	100	5	10	6	9	4	7.5	7.5	1	2	3
θ=0.2	250	5	10	6	9	4	8	7	1	2	3
	500	6	9	5	10	4	7.5	7.5	1	2.5	2.5
	30	4	9	7	10	5	6	8	1	3	2
α=0.6	50	4	9	5	10	7	6	8	1	3	2

Contd..

β=0.5	100	4	10	6	9	7	5	8	1	2	3
θ=3.0	250	5	9	7	10	4	6	8	1	3	2
	500	4	9	5	10	6	7	8	1	3	2
	30	4	8	7	10	5	6	9	1	2	3
α=0.6	50	4	10	7	8.5	6	5	8.5	1	2	3
β=0.6	100	4	10	8	9	5	6	7	1	2	3
θ=0.5	250	4	10	5	9	6	7	8	1	2	3
	500	4	9	7	10	5	6	8	1	2	3
	30	4	8	7	9	6	5	10	1	3	2
α=0.5	50	4	10	7	9	6	5	8	1	2	3
β=0.5	100	4	10	7	9	5	6	8	1	2	3
θ=1.0	250	4	10	5.5	9	5.5	7	8	1	2	3
	500	4	9	7	10	5	6	8	1	2.5	2.5
	30	4	8	7	9	6	5	10	1	2	3
α=0.3	50	4	9	7	8	6	5	10	1	2	3
β=0.7	100	4	10	7	8	5	6	9	1	2	3
θ=0.5	250	4	9.5	6	8	5	7	9.5	1	2	3
	500	4	8	7	9	5	6	10	1	2	3
	30	4	9	7	8	5	6	10	1	2	3
α=0.15	50	4	8	7	9	5.5	5.5	10	1	2	3
β=0.4	100	4	8	7	9	6	5	10	1	2	3
θ=0.5	250	4	9	6	8	5	7	10	1	2	3
	500	4	8	6	9	5	7	10	1	2	3
Σ		237	502.5	362.5	499.5	283.5	333	4777	56	127	147
Overallrank		4	10	7	9	5	6	8	1	2	3

7. Real Data Applications

This section explores three practical applications of the NWEx distribution to demonstrate its relevance. We compare the NWEx distribution with several alternative distributions, including the Ex, "exponentiated Ex" (EEx) by Gupta, Gupta, and Gupta, [21], "beta generalized Ex" (BGEx) by Barreto-Souza, Santos, and Cordeiro, [22], "alpha-power Ex" (APEx) by Mahdavi and Kundu [23], "transmuted-generalized Ex" (TGEx) by Khan, King, and Hudson [24], "alpha-power exponentiated Ex" (APEEx) by Afify, Gemeay, and Ibrahim, [25], "beta Ex" (BEx) by Nadarajah and Kotz [26], "logistic Ex" (LEx) by Lan and Leemis [27], and "generalized odd log–logistic Ex" (GOLLEx) by Afify et al. [28].

The ML estimates are calculated using the R software. To assess "goodness-of-fit", several information criteria (IC) are employed, including the minus log-likelihood (M_1), Akaike IC (M_2), consistent AIC (M_3), "Bayesian" IC (M_4), "Hannan–Quinn" IC (M_5), as well as the "Cramér–von Mises" (M_6),"Anderson–Darling" (M_7), and "Kolmogorov–Smirnov" (M_8) statistics, along with the KS p-value (M_9), for all distributions under consideration.

The first dataset consists of 40 observations representing time to failure (103h) of the turbocharger from one type of engine as given by Xu et al., [29]. These data were also utilized by Afify, Al-Mofleh, and Dey, [30]. The data values are: 7.7, 1.6, 7.7, 6.5, 2.0, 7.1, 8.3, 2.6, 8.1, 3.0, 6.7, 8.4, 7.8, 3.5, 7.9, 8.4, 8.5, 3.9, 8.7, 4.5, 8.8, 4.6, 9.0, 6.0, 4.8, 8.0, 5.0, 7.3, 5.1, 7.3, 5.3, 7.3, 5.4, 6.1, 5.6, 6.0, 5.8, 6.3, 7.0, 6.5.

The second dataset pertains to the failure times of 50 items during the design phase of product reliability (Lai Murthy, and Xie [31]). The data values are: 51.17, 0.12, 49.44, 0.43, 58.62, 0.92, 60.29, 1.14, 72.13, 1.24, 1.61, 1.93, 2.38, 4.51, 5.09, 6.79, 7.64, 8.45, 11.9, 11.94, 13.01, 13.25, 14.32, 17.47, 18.1, 18.66, 19.23, 24.39, 25.01, 26.41, 26.8, 27.75, 29.69, 29.84, 31.65, 32.64, 35.00, 40.70, 42.34, 43.05, 43.40, 44.36, 45.40, 48.14, 49.10, 72.22, 72.25, 72.29, 85.20, 89.52.

The third dataset refers to the tensile strength for single carbon fibers (in GPA) for 69 samples (Raqab Madi, and Kundu [32]). The data values are: Here is the list of numbers rearranged randomly: 0.312, 0.314, 0.479, 0.552, 0.7, 0.803, 0.861, 0.865, 0.944, 0.958, 0.966, 0.997, 1.006, 1.021, 1.027, 1.055, 1.063, 1.098, 1.114, 1.14, 1.179, 1.224, 1.253, 1.272, 1.274, 1.301, 1.301, 1.359, 1.382, 1.382, 1.426, 1.434, 1.478, 1.49, 1.511, 1.514, 1.535, 1.554, 1.566, 1.57, 1.586, 1.629, 1.633, 1.648, 1.684, 1.697, 1.726, 1.77, 1.773, 1.778, 1.809, 1.818, 1.821, 1.848, 1.954, 1.954, 1.954, 2.012, 2.067, 2.084, 2.096, 2.128, 2.233, 2.433, 2.585, 2.585, 2.585, 2.09.

The ML estimates for each fitted distribution are provided in Tables 13-15 along with the standard errors (SEs), shown in parentheses, for the three datasets. Furthermore, the "goodness-of-fit" statistics for these datasets are summarized in Tables 16-18. The findings highlight that the proposed NWEx distribution delivers a better fit to the analyzed data than eleven alternative exponential model extensions.

Furthermore, Figures 5-7 present plots of the fitted functions, including the pdf, cdf, sf, and "probability-probability" (pp) plots of the NWEx distribution, for the three datasets. These visualizations corroborate the numerical results shown in Tables 16-18 and demonstrate that the NWEx distribution provides a better fit for the three datasets compared to other competing exponential models.

Table 13

The ML estimates and SEs for the models considered with the first dataset.

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$
NWEx	0.0113 (0.0076)	1.7486 (0.9791)	0.3607 (0.2214)	
APEEx	35.6620 (54.3293)	0.5798 (0.0712)	7.1395 (3.7016)	
BEx	7.7275 (1.6946)	65.4834 (51.0220)	0.0179 (0.0129)	
GOLLEx	3.9344 (2.7138)	0.8825 (0.9201)	0.0973 (0.1100)	
TGEx	8.5470 (3.1709)	0.5000 (0.0621)	-0.6507 (0.2560)	
BGEx	0.1037 (0.0831)	1320.7166 (201.6799)	0.2195 (0.1096)	49.0520 (54.7401)
EEx			9.5145 (2.8960)	0.4498 (0.0577)
APEx			210785.5 (16786.68)	0.4790 (0.03105)
LEx			0.1108 (0.0059)	3.6322 (0.4954)
Ex				0.1599 (0.0252)

Table 14

The ML estimates and SEs for the models considered with the second dataset.

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$
NWEx	0.3786 (0.1330)	0.7014 (0.1507)	0.0392 (0.0134)	
APEEx	7.5230 (10.5428)	0.0386 (0.0068)	0.6191 (0.2632)	
BEx	0.7081 (0.7669)	0.0056 (0.0031)	5.8917 (3.4498)	
GOLLEx	0.6983 (0.3764)	1.4251 (0.9967)	0.0510 (0.0358)	
TGEx	0.7587 (0.2160)	0.0344 (0.0061)	-0.4648 (0.3559)	
BGEx	0.0494 (0.2561)	102.9887 (335.8097)	0.0149 (0.0533)	15.4643 (89.6663)
EEx			0.9081 (0.1621)	0.0311 (0.0058)

Contd..

APEx			2.1200 (1.6608)	0.0392 (0.0078)
LEx			0.0353 (0.0063)	0.9101 (0.1295)
Ex				0.0331 (0.0046)

Table 15

The ML estimates and SEs for the models considered with the third dataset.

Model	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{\lambda}$
NWEx	1.0921 (5.6291)	3.0314 (0.6866)	0.2879 (0.4500)	
APEEx	58.4201 (82.4618)	2.4421 (0.2365)	5.5577 (2.4417)	
BEx	7.0024 (1.1682)	44.6392 (62.1647)	0.1014 (0.1329)	
GOLLEx	3.8009 (2.5096)	0.8494 (0.8436)	0.4109 (0.4535)	
TGEx	7.4544 (2.1207)	2.1100 (0.2049)	-0.7206 (0.1872)	
BGEx	0.6192 (0.4796)	405.2820 (2151.7182)	0.1996 (0.4208)	5.1460 (4.1819)
EEx			8.8283 (2.0341)	1.8965 (0.1885)
APEx			129938.3 (23859.06)	2.0474 (0.1027)
LEx			0.4881 (0.0212)	3.4224 (0.3518)
Ex				0.6890 (0.0829)

Table 16

Results of the analytical measures for the first dataset.

Model	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
NWEx	79.8039	165.6070	166.2745	170.6744	167.4397	0.0307	0.2270	0.0892	0.9072
APEE	86.9504	179.9009	180.5676	184.9676	181.7329	0.1832	1.2261	0.1307	0.5007

Contd..

BEx	87.4150	180.8302	181.4969	185.8968	182.6621	0.2053	1.3623	0.1278	0.5303
GOLLEx	86.6442	179.2885	179.9552	184.3551	181.1204	0.1638	1.1116	0.1242	0.5674
TGEx	88.6702	183.3405	184.0072	188.4072	185.1725	0.2333	1.5194	0.1448	0.3707
BGEx	79.8859	167.7720	168.9148	174.5275	170.2146	0.0373	0.3012	0.1068	0.7512
EEx	90.1426	184.2853	184.6097	187.6631	185.5066	0.2757	1.7600	0.1541	0.2975
APEx	88.9920	181.9841	182.3084	185.3618	183.2054	0.2340	1.5271	0.1516	0.3165
LEx	86.6504	177.3009	177.6252	180.6787	178.5222	0.1644	1.1151	0.1241	0.5688
Ex	113.3193	228.6385	228.7438	230.3274	229.2492	0.2065	1.3689	0.3631	0.0000

Table 17

Results of the analytical measures for the second dataset.

Model	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
NWEx	216.1370	438.2739	438.7957	444.0100	440.4582	0.0233	0.1844	0.0567	0.9942
APEE	218.8173	443.6347	444.1564	449.3707	445.8190	0.0703	0.4755	0.0871	0.8108
BEx	220.3055	446.6110	447.1327	452.3471	448.7953	2.7494	13.8788	0.8399	0.0000
GOLLEx	220.0070	446.0140	446.5358	451.7501	448.1983	0.1228	0.7658	0.1418	0.2427
TGEx	219.4770	444.9541	445.4758	450.6901	447.1384	0.0959	0.6222	0.1056	0.5943
BGEx	216.4137	440.8270	441.7160	448.4755	443.7398	0.0376	0.2511	0.0803	0.8774
EEx	220.2089	444.4178	444.6731	448.2418	445.8740	0.1231	0.7767	0.1242	0.3904
APEx	219.9456	443.8911	444.1464	447.7152	445.3473	0.0964	0.6232	0.0981	0.6842
LEx	220.1255	444.2511	444.5064	448.0751	445.7073	0.1219	0.7677	0.1305	0.3319
Ex	220.3567	442.7133	442.7966	444.6253	443.4414	0.1241	0.7831	0.1143	0.4947

Table 18

Results of the analytical measures for the third dataset.

Model	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9
NWEx	48.9364	103.8729	104.2421	110.5752	106.5319	0.0268	0.2306	0.0476	0.9975
APEE	51.6968	109.3937	109.7630	116.0961	112.0528	0.0924	0.6303	0.0656	0.9278
BEx	53.0932	112.1865	112.5557	118.8888	114.8455	0.1225	0.8621	0.088	0.6581
GOLLEx	51.6397	109.2794	109.6487	115.9818	111.9385	0.0817	0.5640	0.0552	0.9843
TGEx	54.1933	114.3867	114.7560	121.0890	117.0458	0.1504	1.0248	0.0889	0.6455
BGEx	48.9274	105.8548	106.4798	114.7912	109.4002	0.0260	0.2254	0.0487	0.9966
EEx	56.6685	117.3371	117.5190	121.8054	119.1198	0.2057	1.3957	0.1119	0.3531
APEx	54.4328	112.8658	113.0476	117.3340	114.6385	0.1455	0.9933	0.0927	0.5923
LEx	51.6534	107.3069	107.4888	111.7752	109.0796	0.0823	0.5686	0.0557	0.9826
Ex	94.7013	191.4026	191.4623	193.6367	192.2890	0.1238	0.8712	0.3622	0.0000

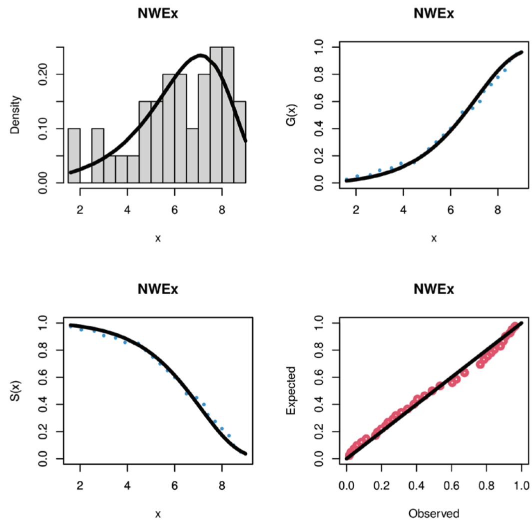

Figure 5

Plots of the fitted functions for the NWEx distribution based on the first dataset.

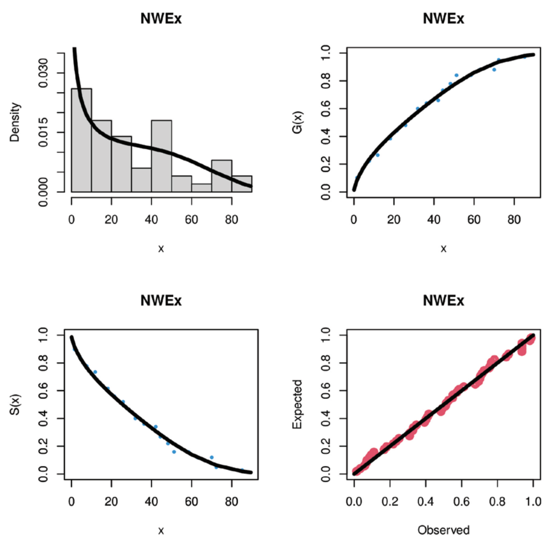

Figure 6

Plots of the fitted functions for the NWEx distribution based on the second dataset.

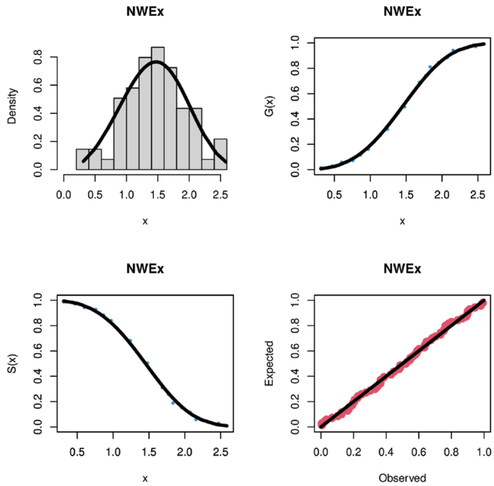

Figure 7

Plots of the fitted functions for the NWEx distribution based on the third dataset.

8. Conclusions

This study presents a flexible family of probability distributions known as the new Weibull-H (NW-H) family. This family includes special sub-models that offer a range of failure rate behaviors, including both monotonic and non-monotonic rates. The sub-models also feature a variety of density shapes such as unimodal, left-skewed, bimodal, right-skewed, J-shaped, and reversed-J-shaped. The densities within the NW-H family can be characterized by reversed-J shapes, right skewness, symmetry, left skewness, or patterns of decreasing-increasing-decreasing. A valuable mixture representation of the NW-H family is developed using the exponentiated-G family. One particular model within this family, the NWEx, is examined in detail. The study applies eleven classical and Bayesian estimation methods to estimate the parameters of the NWEx model and evaluates their performance through simulation. The analysis reveals that Bayesian estimation, based on partial and overall ranks, is the most effective approach for parameter estimation under various loss functions. Additionally, the NWEx distribution is applied to three real-world datasets, demonstrating its superior fit compared to several competing exponential models.

Data Availability

"The data are fully available in the article and the mentioned references."

Author contributions

"All authors contributed to this work equally. All authors reviewed the manuscript."

Competing interests

"The authors declare no competing interests."

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