Journal of Statistics & Management Systems
ISSN 0972-0510 (Print), ISSN 2169-0014 (Online)
Vol. 28 (2025), No. 6, pp. 1119–1131
DOI : 10.47974/JSMS-1448

Optimizing green inventory management in the textile industry : Addressing linear demand, deterioration and credit periods

M. Balasubramanian *
Department of Mathematics
Vels Institute of Science Technology and Advanced Studies
Pallavaram
Chennai 117
Tamil Nadu
India

and

Department of Mathematics
SRM Arts and Science College
Kattankulathur 603203
Tamil Nadu
India

R. Kamali †
Department of Mathematics
School of Basic Sciences
Vels Institute of Science Technology and Advanced Studies
Pallavaram
Chennai 600117
Tamil Nadu
India

Abstract

This paper presents the linear demand patterns prevalent in the textile industry, providing insights into their implications on inventory dynamics. Leveraging this understanding, a green inventory model is formulated that balances the ecological impact of production and distribution processes with the traditional goals of cost-effectiveness and service level optimization. The model takes into account various elements, including the

* *E-mail:* srmbalu@gmail.com (Corresponding Author)
† *E-mail:* kamali.sbs@velsuniv.ac.in

acquisition of manufacturing procedures, raw materials, transportation techniques, as well as the disposal process at end of its life cycle, to adopt a comprehensive and environmentally conscious approach to sustainability. The findings showcase the model's capacity to diminish its environmental footprint while still upholding operational efficiency and fulfilling customer requirements. Sensitivity analyses were conducted for evaluating model's resilience across different parameters and scenarios. For both phases, the model seeks to identify the ideal credit duration. To clarify the use of the model, numerical examples are provided.

Subject Classification: *90B06.*

Keywords: *Linear demand, Deteriorating items, Partial shortages backlogged, Delay payments, Credit period, Green production items, Inventory and supply chain.*

1. Introduction

In real-world applications, a robust green inventory model specifically tailored for the textile industry is essential, given its unique characteristics and sustainability challenges. This model integrates eco-friendly practices into the supply chain, addressing linear demand patterns typical in the textile sector. It emphasizes minimizing environmental impact across supply chain—from raw material sourcing and production to transportation as well as end-of-life disposal. Sarkar, Sana, and Chaudhuri [21] proposed an inventory model considering trade credit policies as well as variable deterioration for fixed lifetime items, while Maiti [11] utilized a fuzzy genetic algorithm with a dynamic population to handle credit-linked promotional demand under uncertain planning horizons. Managing risk and interruptions in production-inventory systems was the main focus of Paul, Sarker, and Essam [18]. Dey, Roy, and Saha [4] investigated how procurement and stocking methods affect the design of green products in two-period supply chains. Muniappan, Uthayakumar, and Ganesh [13] provided an EOQ model for deteriorating items that takes inflation, deferred payments, and the time value of money into consideration. Hoque [8] presented a realistic optimization approach for single-supplier, multi-buyer systems. Furthermore, in their inventory model, Sarkar, S. Saren, and L. E. Cárdenas-Barrón [20] included price discounts, trade credit, and finite replenishment. Ouyang et al. [16] addressed capacity constraints and order-size-dependent credit, while Srivathsan and Kamath [22] evaluated how inventory information sharing affects two-tier supply chains. Gharaei, Pasandideh, and Khamseh [6] analyzed models handling defective items and backorders, and Muniappan, Uthayakumar, and Ganesh, [14] developed coordination strategies including rework and backordering for perishable goods. Other notable contributions include Kirubhashankar, Muniappan, and Ismail, [10] as well as Ismail et al. [9], who investigated deteriorating inventory models with price breaks. Alvarez and Lippi [1] examined the role of payment flexibility (cash vs. credit), and many studies assumed monotonic demand trends, whereas ramp-type (stabilizing) demand better reflects real product life cycles. Yao and Minner

[23] reviewed multi-supplier inventory strategies, and Birim and Sofyalioglu, [2] highlighted the role of incentives in vendor-managed inventory systems. Birim and Sofyalioglu [5] incorporated inflation and credit into vendor-retailer models, and Muniappan et al. [15] introduced a fixed holding cost EOQ model with price breaks. Sana [19] addressed quality issues in a three-level supply chain. Other relevant works include Das et al. [3] on partial trade credit, Min, Zhou, and Zhao [12] on stock-dependent demand, and Pal, Sana, and Chaudhuri, [17] on three-stage trade credit in multi-tier systems. Hemamalini, Ravithammal, and Muniappan [7] considered screening, disposal costs, and controllable lead times, while Zhang, Yan, and Tang [24] optimized pricing and lot-sizing for profitability and service level goals. Collectively, these studies provide a rich foundation for advancing green inventory practices. This research contributes by proposing a comprehensive model for managing deteriorating items under quadratic time-varying demand as well as permissible payment delays. It demonstrates how integrating sustainability into inventory decisions not only reduces ecological impact but also enhances cost efficiency and customer satisfaction, supporting the evolution of green supply chains.

2. Research Contribution and Focus of Green Inventory Linear Demand Analysis

2.1 *Manufacturing Planning*

Use of mathematical models for production with linear demand helps in planning manufacturing processes efficiently. It enables companies to align their production capacities with the linear demand patterns, ensuring optimal resource utilization.

2.2 *Inventory Management*

Implementing mathematical models assists in determining the appropriate inventory levels to meet linear demand. This ensures that stock levels are neither excessive, leading to overstock situations, nor insufficient, causing stock outs.

2.3 *Supply Chain Optimization*

Mathematical models facilitate the optimization of the entire supply chain by considering linear demand patterns. This involves streamlining procurement, production, and distribution processes to meet customer demand while minimizing costs.

2.4 *Resource Allocation*

The models can be employed to allocate resources efficiently across different stages of the production process. This includes human resources, machinery, and raw materials, ensuring a balanced and cost-effective production environment.

2.5 *Production Scheduling*

Scheduling production activities based on linear demand models helps in maintaining a steady workflow. This minimizes production bottlenecks and ensures that goods are produced in accordance with the linear demand forecast.

2.6 *Cost Management*

Mathematical models assist in cost management by providing insights into the optimal production quantities and timing. This aids in controlling production costs and maximizing profitability.

2.7 *Capacity Planning*

The models contribute to effective capacity planning by aligning production capabilities with linear demand projections. This ensures that the production capacity is neither underutilized nor strained beyond its limits.

2.8 *Risk Assessment*

Employing mathematical models allows for the evaluation of risks associated with production and demand fluctuations. Companies can make informed decisions by considering various scenarios and their potential impact on production outcomes.

2.9 *Continuous Improvement*

Regularly updating and refining the mathematical model based on actual demand data contributes to continuous improvement. This adaptive approach helps companies stay responsive to changing market conditions and improve overall operational efficiency.

2.10 *Sustainability Planning*

Considering environmental concerns, mathematical models can also incorporate sustainability factors into production planning. It consists of reducing waste and the impact on the environment by optimizing production processes and energy consumption.

In summary, the application of mathematical models for production with linear demand extends across various aspects of manufacturing as well as supply chain management, offering companies a systematic and data-driven approach to optimize their operations.

2.11 *Decision Parameters and Consideration*

We use the notation which are the ordering cost (r) , level of green inventory I(t) at t, deterioration rate θ, the inventory purchase cost P, h denote the holding cost, s denote the shortage cost for stock out period, α denote opportunity cost, due to end sale, I_e denote the interest earned and I_r denote the interest charges with $I_r \geq I_e$.

Decision Variables: M, T, T_1, $TC(T_1, T)$, $TC_1(T_1, T)$, for $T1 \geq M$ and $TC_2(T_1, T)$ for $T1 < M$

3. Mathematical Inventory Model Formulation

The mathematical model's formulation for production with linear demand extends across various aspects of manufacturing as well as supply chain management, offering companies a systematic approach to optimize their production level. Based on this we formulate differential equation as follows.

$$\frac{dI_1(t)}{dt} + \theta I_1(T_1) = P - a - bt; [0, T_1).$$

$$I_1(t)e^{\theta t} = \int (P - a - bt)\, e^{\theta t} dt + c_1$$

$$I_1(T_1) = 0 \text{ \& Assume } \frac{\theta(P-a)+b}{\theta^3} = x$$

i.e., “ $I_1(t) = \frac{\theta(P-a)+b}{\theta^2}\left[1 - e^{\theta(T_1-t)}\right] + \frac{b}{\theta}\left[T_1 e^{\theta(T_1-t)} - t\right]$” (1)

When $[T_1, T)$

$$\frac{dI_2(t)}{dt} + \theta I_2(T_1) = -\frac{a}{1+\delta\,(T-t)}$$

“ $I_2(t) = -\frac{a}{\delta}\left[log\left(1 + \delta(T - T_1)\right) - log\left(1 + \delta(T - t)\right)\right]$ (2)

Hence

$$I(T) = \begin{cases} \frac{\theta(P-a)+b}{\theta^2}\left[1 - e^{\theta(T_1-t)}\right] + \frac{b}{\theta}\left[T_1 e^{\theta(T_1-t)} - t\right] & ; 0 \le t < T_1 \\ -\frac{\alpha}{\beta}[log(1 + \delta(T - T_1)) - log(1 + \delta(T - t))] & ; T_1 \le t < T \end{cases} \quad (3)$$

Inventory holding cost and various decision cost calculated as follows”

“HC = $h\int_0^{T_1} I_1(t)dt = h\left\{\frac{\theta(P-a)+b}{\theta^3}\left[1 - e^{\theta T_1}\right] + \frac{T_1}{\theta^2}\left(\theta(P-a) + b\left(e^{\theta T_1} - \frac{\theta T_1}{2}\right)\right)\right\}$” (4)

DC = $p\theta \int_0^{T_1} I_1(t)dt = p\theta\left\{x\left[1 - e^{\theta T_1}\right] + \frac{T_1}{\theta^2}\left(\theta(P-a) + b\left(e^{\theta T_1} - \frac{\theta T_1}{2}\right)\right)\right\}$ (5)

“SC = $s\int_{T_1}^{T} I_2(t)dt = \frac{sa}{\delta^2}[\delta(T - T_1) - \log(1 + \delta(T - T_1))]$ (6)

OC = $\alpha\int_{T_1}^{T} I_2(t)dt = \frac{a\alpha}{\delta}\{\delta(T - T_1) - \log(1 + \delta(T - T_1))\}$" (7)

“Stage 1: $\boldsymbol{M \le T_1}$”

Allowing customers to delay payment for their purchases could be an effective strategy for businesses to attract and retain customers. Providing a permissible delay payment option gives customers the flexibility to manage their cash flow while ensuring timely order fulfillment. However, businesses need to carefully balance the financial implications of delay payment with the costs associated with inventory holding, ordering, and potential stock outs. Therefore, we estimated the credit period M, permissible delay in settling the accounts in two cases.

$$IE_1 = pI_e \int_0^{T_1}(T_1 - t)(a + bt)dt = \frac{pI_e T_1^2}{6}[3a + bT_1] \quad (8)$$

$$I_p = pI_r \int_M^{T_1} I(t)dt$$

= “$pI_r\left\{\frac{1-e^{\theta(T_1-M)}}{\theta^3}[\theta(P-a) + b(1 - \theta T_1)] + \frac{\theta(P-a)+b}{\theta^2}[T_1 - M] - \frac{b}{2\theta}(T_1^2 - M^2)\right\}$" (9)

$$\mathrm{PC} = \int_0^{T_1}(a+bt)c_d dt = c_d\left[aT_1 + \frac{bT_1^2}{2}\right] \tag{10}$$

Total "average cost for this stage is

$$\mathrm{TC}_1 = \frac{r + HC + DC + SC + OC + I_p + PC - IE_1}{T} \tag{11}$$

$= \frac{1}{T}\Big\{r + h\Big\{x\big[1-e^{\theta T_1}\big] + \frac{T_1}{\theta^2}\Big(\theta(P-a) + b\Big(e^{\theta T_1} - \frac{\theta T_1}{2}\Big)\Big)\Big\} + P\theta\Big\{\frac{\theta(P-a)+b}{\theta^3}\big[1 - e^{\theta T_1}\big] + \frac{T_1}{\theta^2}\Big(\theta(P-a) + b\Big(e^{\theta T_1} - \frac{\theta T_1}{2}\Big)\Big)\Big\} + s\Big\{\frac{a}{\delta^2}\big[\delta(T-T_1) - \log\big(1 + \delta(T-T_1)\big)\big]\Big\} + \frac{a\alpha}{\delta}\big\{\delta(T-T_1) - \log\big(1+\delta(T-T_1)\big)\big\} + PI_r\Big\{\frac{1-e^{\theta(T_1-M)}}{\theta^3}\big[\theta(P-a) + b(1-\theta T_1)\big] + \frac{\theta(P-a)+b}{\theta^2}[T_1 - M] - \frac{b}{2\theta}(T_1^2 - M^2)\Big\} + c_d\Big[aT_1 + \frac{bT_1^2}{2}\Big] - \frac{pI_e T_1^2}{6}[3a + bT_1]\Big\}$"

$$= \frac{1}{T}\Big\{r + (h+P\theta)\Big\{x\big[1-e^{\theta T_1}\big] + \frac{T_1}{\theta^2}\Big(\theta(P-a) + b\Big(e^{\theta T_1} - \frac{\theta T_1}{2}\Big)\Big)\Big\} + \frac{a(s+\delta\alpha)}{\delta^2}\big\{\delta(T-T_1) - \log\big(1+\delta(T-T_1)\big)\big\} + PI_r\Big\{\frac{1-e^{\theta(T_1-M)}}{\theta^3}\big[\theta(P-a) + b(1-\theta T_1)\big] + \frac{x}{\theta}[T_1 - M] - \frac{b}{2\theta}(T_1^2 - M^2)\Big\} + c_d\Big[aT_1 + \frac{bT_1^2}{2}\Big] - \frac{pI_e T_1^2}{6}[3a + bT_1]\Big\} \tag{12}$$

$$\frac{\partial TC_1(T_1,T)}{\partial T_1} = 0 \; and \; \frac{\partial TC_1(T_1,T)}{\partial T} = 0 \tag{13}$$

Necessary and Sufficient conditions

$$\left[\frac{\partial^2 TC_1(T_1,T)}{\partial T_1^2}\right]_{at\ (T_1^*,T^*)} > 0\,, \left[\frac{\partial^2 TC_1(T_1,T)}{\partial T^2}\right]_{at\ (T_1^*,T^*)} > 0 \; and$$

$$\left[\left(\frac{\partial^2 TC_1(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TC_1(T_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC_1(T_1,T)}{\partial T_1 \partial T}\right)^2\right] > 0$$

To solve $\frac{\partial TC_1(T_1,T)}{\partial T_1} = 0 \; and \; \frac{\partial TC_1(T_1,T)}{\partial T} = 0$ gives T^* and T_1^* " $\frac{\partial TC_1(T_1,T)}{\partial T_1} = 0$

$$\frac{1}{T}\Big\{(h+P\theta)\Big\{(\theta(P-a)+b)\Big(\frac{-e^{\theta T_1}}{\theta^2}\Big) + \frac{T_1}{\theta}\Big[b\Big(e^{\theta T_1} - \frac{1}{2}\Big)\Big] + \frac{1}{\theta^2}\Big[\Big(\theta(P-a) + b\Big(e^{\theta T_1} - \frac{\theta T_1}{2}\Big)\Big)\Big]\Big\} - \frac{a(s+\delta\alpha)(T-T_1)}{1+\delta(T-T_1)} + PI_r\Big\{-b\Big(\frac{1-e^{\theta(T_1-M)}}{\theta^2}\Big) - \frac{e^{\theta(T_1-M)}}{\theta^2}\big[\theta(P-a) + b(1-\theta T_1)\big] + \frac{\theta(P-a)+b}{\theta^2} - \frac{bT_1}{\theta}\Big\} + c_d[a + bT_1] - \frac{pI_e T_1}{3}[3a + bT_1] - \frac{pI_e bT_1^2}{6}\Big\} = 0 \tag{14}$$

$$\frac{\partial TC_1(T_1,T)}{\partial T} = 0$$

$\Rightarrow \frac{1}{T}\Big\{\frac{a(s+\delta\propto)(T-T_1)}{1+\delta(T-T_1)}\Big\} - \frac{1}{T^2}\Big\{r + (h+P\theta)\Big\{x\big[1-e^{\theta T_1}\big] + \frac{T_1}{\theta^2}\Big(\theta(P-a) + b\Big(e^{\theta T_1} - \frac{\theta T_1}{2}\Big)\Big)\Big\} + \frac{a(s+\delta\alpha)}{\delta^2}\big\{\delta(T-T_1) - \log\big(1+\delta(T-T_1)\big)\big\} +$

$PI_r \left\{\frac{1-e^{\theta(T_1-M)}}{\theta^3}[\theta(P-a)+b(1-\theta T_1)] + \frac{\theta(P-a)+b}{\theta^2}[T_1-M] - \frac{b}{2\theta}(T_1^2 - M^2)\right\} + c_d\left[aT_1 + \frac{bT_1^2}{2}\right] - \frac{pI_e T_1^2}{6}[3a+bT_1]\right\}$ " = 0 (15)

Stage 2: $T_1 < M$

$$\text{IE}_2 = \text{“}pI_e\left\{\int_0^{T_1}(T_1 - t)(a+bt)dt + (M-T_1)\int_0^{T_1}(a+bt)dt\right\}$$

$$= \frac{pI_e T_1^2}{6}[3a+bT_1] + \frac{pI_e T_1(M-T_1)}{2}[2a+bT_1] \quad (16)$$

Total average cost is

$$\text{TC}_2 = \frac{r + HC + DC + SC + OC + PC - IE_2}{T} \quad (17)$$

$= \left\{r + h\left\{x\left[1-e^{\theta T_1}\right] + \frac{T_1}{\theta^2}\left(\theta(P-a) + b\left(e^{\theta T_1} - \frac{\theta T_1}{2}\right)\right)\right\} + P\theta\left\{\frac{\theta(P-a)+b}{\theta^3}\left[1 - e^{\theta T_1}\right] + \frac{T_1}{\theta^2}\left(\theta(P-a) + b\left(e^{\theta T_1} - \frac{\theta T_1}{2}\right)\right)\right\} + s\left\{\frac{a}{\delta^2}\left[\delta(T-T_1) - \log\left(1 + \delta(T-T_1)\right)\right]\right\} + \frac{a\alpha}{\delta}\left\{\delta(T-T_1) - \log\left(1+\delta(T-T_1)\right)\right\} + c_d\left[aT_1 + \frac{bT_1^2}{2}\right] - \frac{PI_e T_1^2}{6}[3a+bT_1] - \frac{pI_e T_1(M-T_1)}{2}[2a+bT_1]\right\}$

$= \frac{1}{T}\left\{r + (h+P\theta)\left\{x\left[1-e^{\theta T_1}\right] + \frac{T_1}{\theta^2}\left(\theta(P-a) + b\left(e^{\theta T_1} - \frac{\theta T_1}{2}\right)\right)\right\} + \frac{a(s+\delta\alpha)}{\delta^2}\left\{\delta(T-T_1) - \log\left(1+\delta(T-T_1)\right)\right\} + c_d\left[aT_1 + \frac{bT_1^2}{2}\right] - \frac{pI_e T_1^2}{6}[3a + bT_1] - \frac{pI_e T_1(M-T_1)}{2}[2a+bT_1]\right\}$" (18)

To solve,

$$\frac{\partial TC_2(T_1,T)}{\partial T_1} = 0 \; and \; \frac{\partial TC_2(T_1,T)}{\partial T} = 0 \quad (19)$$

Sufficient conditions are

$$\left[\frac{\partial^2 TC_2(T_1,T)}{\partial T_1^2}\right]_{at\ (T_1^*,T^*)} > 0, \left[\frac{\partial^2 TC_2(T_1,T)}{\partial T^2}\right]_{at\ (T_1^*,T^*)} > 0 \; and$$

$$\left[\left(\frac{\partial^2 TC_2(T_1,T)}{\partial T_1^2}\right)\left(\frac{\partial^2 TC_2(T_1,T)}{\partial T^2}\right) - \left(\frac{\partial^2 TC_2(T_1,T)}{\partial T_1 \partial T}\right)^2\right] > 0$$

$\frac{\partial TC_2(T_1,T)}{\partial T_1} = 0 \; and \; \frac{\partial TC_2(T_1,T)}{\partial T} = 0$ implies T^*, T_1^*

$$\frac{\partial TC_2(T_1,T)}{\partial T_1} = 0$$

$\Rightarrow \frac{1}{T}\left\{(h+P\theta)\left\{(\theta(P-a)+b)\left(\frac{-e^{\theta T_1}}{\theta^2}\right) + \frac{T_1}{\theta}\left[b\left(e^{\theta T_1} - \frac{1}{2}\right)\right] + \frac{1}{\theta^2}\left[\left(\theta(P - a) + b\left(e^{\theta T_1} - \frac{\theta T_1}{2}\right)\right)\right]\right\} - \frac{a(s+\delta\alpha)(T-T_1)}{1+\delta(T-T_1)} + c_d[a+bT_1] - pI_e(a+bT_1)(M - T_1)\right\} = 0$ (20)

$$\frac{\partial TC_2(T_1,T)}{\partial T} = 0$$

$$\frac{1}{T}\left\{\frac{a(s+\delta\alpha)(T-T_1)}{1+\delta(T-T_1)}\right\} - \frac{1}{T^2}\left\{r + (h+P\theta)\left\{\frac{\theta(P-a)+b}{\theta^3}\left[1-e^{\theta T_1}\right] + \frac{T_1}{\theta^2}\left(\theta(P-a) + b\left(e^{\theta T_1} - \frac{\theta T_1}{2}\right)\right)\right\} + \frac{a(s+\delta\alpha)}{\delta^2}\left\{\delta(T-T_1) - \log\left(1+\delta(T-T_1)\right)\right\} + c_d\left[aT_1 + \frac{bT_1^2}{2}\right] - \frac{pI_e T_1^2}{6}\left[3a+bT_1\right] - \frac{pI_e T_1(M-T_1)}{2}\left[2a+bT_1\right]\right\} \quad (21)$$

Hence, we obtain the various decision variables for both cases optimal values of T_1^*, T^* as well as $TC_1(T_1, T)$ and the optimal values of T^* and T_1^* and $TC_2(T_1, T)$ by using program.

4. Numerical Examples

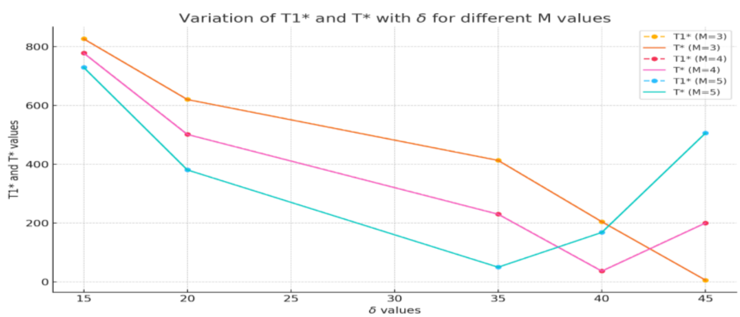
We Obtain the Effect of Changes Various Parameters as Follows

Example 4.1: Various parameter: r =200, a=1000, b =0.3, p =20, P =2000, h =1.2, s =30, $C_d = 5$, $\alpha = 15$, I_e=0.13, $I_r = 0.15$, $\theta = 0.08$.

Table 1

The variation of T_1* and T* with different δ values

		δ	
M	**3**	**4**	**5**
15 TC (T_1, T)	4.4963 X 10^{28}	9.2133 X 10^{26}	1.8418 X 10^{25}
T_1^*	827.1952	778.6209	729.7397
T^*	826.6952	778.2875	729.4897
20 TC (T_1, T)	3.0291 X 10^{21}	2.2013 X 10^{17}	1.3824 X 10^{13}
T_1^*	620.8900	501.8787	381.1116
T^*	620.3900	501.5454	380.8616
35 TC (T_1, T)	1.8893 X 10^{14}	7.8069 X 10^{7}	7.1454 X 10^{4}
T_1^*	413.7335	230.4927	50.2504
T^*	413.2335	230.1594	49.9543
40 TC (T_1, T)	1.0331 X 10^{7}	5.3775 X 10^{4}	7.5777 X 10^{5}
T_1^*	205.0625	36.8832	168.5181
T^*	204.5625	36.4306	168.2681
45 TC (T_1, T)	1.3602 X 10^{4}	7.2276 X 10^{6}	3.1132 X 10^{17}
T_1^*	5.9916	200.5001	506.2087
T^*	6.2550	200.1668	505.9587



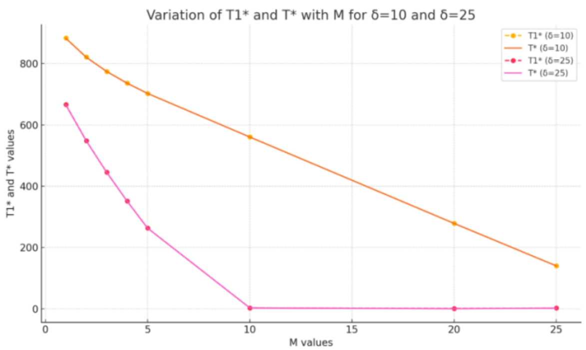
The diagram above visualizes the variation of T_1* and T* with different δ values for the three different M values (3, 4, and 5) refer -Table 1

Example 4.2: The various decision parameter vales are

r =200, a=1000, b =0.3, p =20, P =2000, h =1.2, s =30, $C_d = 5$, $\alpha = 5$, I_e=0.13, I_r = 0.15, $\theta = 0.08$.

Table 2
The variation of T_1* and T* for different M values when δ=10 and δ=25

	M	
δ	10	25
1 TC (T_1, T) T_1^* T^*	4.0237 X 10^{30} 883.3441 882.3441	1.1256 X 10^{23} 666.0408 665.0408
2 TC (T_1, T) T_1^* T^*	2.7562 X 10^{28} 821.0799 820.5799	8.9539 X 10^{18} 548.1489 547.6489
3 TC (T_1, T) T_1^* T^*	6.5556 X 10^{26} 774.3687 774.0354	2.3585 X 10^{15} 445.2460 444.9126
4 TC (T_1, T) T_1^* T^*	3.0652 X 10^{25} 736.1033 735.8533	1.2527 X 10^{12} 351.1609 350.9109
5 TC (T_1, T) T_1^* T^*	2.1144 X 10^{24} 702.6979 702.4979	1.0747 X 10^{9} 263.1678 262.9678
10 TC (T_1, T) T_1^* T^*	2.3901 X 10^{19} 560.4200 560.3200	8.7991 X 10^{3} 2.7822 2.8177
20 TC (T_1, T) T_1^* T^*	3.6397 X 10^{9} 278.3660 278.3160	1.8342 X 10^{35} 1.0174 X 10^{3} 0.0174 X 10^{3}
25 TC (T_1, T) T_1^* T^*	2.4325 X 10^{5} 139.7781 139.7381	1.0987 X 10^{73} 2.1046 X 10^{3} 2.1046 X 10^{3}



The diagram above illustrates the variation of T_1* and T* for different M values when δ=10 and δ=25, based on Table 2

In the both cases, to obtain the optimal inventory control decisions, a combination of numerical optimization techniques and simulation-based methods is utilized. The Numerical analysis to determine order quantity, optimal replenishment time, as well as permissible delay period that minimize total cost while satisfying customer demand requirements. The case study considers a deteriorating product with time-dependent linear demand as well as allows customers to delay payment within a specified period. The results show that the model leads to significant cost savings, improved customer service levels, and reduced environmental impact by promoting sustainable inventory management practices.

5. Results and Conclusion

This paper conducts a case study using real-world data from the textile industry to validate effectiveness of proposed green inventory model. The results illustrate model's capacity to minimize environmental impact, uphold operational efficiency, and meet customer demand. Sensitivity analyses are undertaken to evaluate the model's robustness across different parameters and scenarios. These research findings contribute to the expanding body of literature on sustainable supply chain management, presenting a tailored solution for textile industry with linear demand patterns. The suggested green inventory model emerges as a valuable tool for industry practitioners and policymakers, offering a means to enhance sustainability practices in textile supply chains. This model promotes a more environmentally responsible approach to inventory management in the textile sector. Notably, the model indicates that setting a limit on the delay in payments cycle can be beneficial to the manufacturer. Conversely, if the delay in payments cycle exceeds a certain threshold, the advantage shifts to the buyer. An example using numbers is given to demonstrate the optimum credit period and overall system benefits. Future developments in this paper will consider factors such as constant demand rates, variable holding costs, and other relevant aspects.

Conflicts of Interest: The author(s) disclose no conflicts of interest with this paper.

References

[1] F. Alvarez and F. Lippi, "Cash burns: An inventory model with a cash-credit choice," *Journal of Monetary Economics*, vol. 90, pp. 99–112 (2017), doi: 10.1016/j.jmoneco.2017.07.001.

[2] S. Bir Birim and C. Sofyalioglu, "Evaluating vendor managed inventory systems: How incentives can benefit supply chain partners," *Journal of Business Economics and Management*, vol. 18, no. 1, pp. 163–179 (2017), doi: 10.3846/16111699.2016.1266695.

[3] S. Das, M. A. A. Khan, E. E. Mahmoud, A. H. Abdel-Aty, K. M. Abualnaja, and A. A. A. Shaikh, "A production inventory model with partial trade credit policy and reliability," *Alexandria Engineering Journal*, vol. 60, no. 1, pp. 1325–1338 (2021), doi: 10.1016/j.aej.2020.10.054.

[4] K. Dey, S. Roy, and S. Saha, "The impact of strategic inventory and procurement strategies on green product design in a two-period supply chain," *International Journal of Production Research*, vol. 57, no. 7, pp. 1915–1948 (2019), doi: 10.1080/00207543.2018.1511071.

[5] M. Esmaeili and M. Nasrabadi, "An inventory model for single-vendor multi-retailer supply chain under inflationary conditions and trade credit," *Journal of Industrial and Production Engineering*, vol. 38, no. 2, pp. 75–88 (2021), doi: 10.1080/21681015.2020.1845248.

[6] A. Gharaei, S. H. R. Pasandideh, and A. A. Khamseh, "Inventory model in a four-echelon integrated supply chain: Modeling and optimization," *Journal of Modelling in Management* (2017), doi: 10.1108/JM2-07-2016-0065.

[7] S. Hemamalini, M. Ravithammal, and P. Muniappan, "EOQ inventory model for buyer-vendor with screening, disposed cost and controllable lead time," in AIP Conference Proceedings, vol. 2095, no. 1, p. 030010, AIP Publishing LLC (Apr. 2019), doi: 10.1063/1.5097521.

[8] M. A. Hoque, "An optimal solution technique to the single-vendor multi-buyer integrated inventory supply chain by incorporating some realistic factors," *European Journal of Operational Research,* vol. 215, no. 1, pp. 80–88 (2011), doi: 10.1016/j.ejor.2011.05.036.

[9] A. M. Ismail, P. Muniappan, and C. K. Kirubhashankar, "Optimum buyer-vendor inventory model with coordination and price breaks," in *Journal of Physics: Conference Series*, vol. 1770, no. 1, p. 012104, IOP Publishing (2021), doi: 10.1088/1742-596/1770/1/012104.

[10] C. K. Kirubhashankar, P. Muniappan, and A. M. Ismail, "A production inventory model for deteriorating product with price breaks," in *Journal of Physics: Conference Series*, vol. 1770, no. 1, p. 012102, IOP Publishing (2021), doi: 10.1088/1742-6596/1770/1/012104.

[11] M. K. Maiti, "A fuzzy genetic algorithm with varying population size to solve an inventory model with credit-linked promotional demand in an imprecise planning horizon," *European Journal of Operational Research*, vol. 213, no. 1, pp. 96–106 (2011), doi: 10.1016/j.ejor.2011.02.014.

[12] J. Min, Y. W. Zhou, and J. Zhao, "An inventory model for deteriorating items under stock-dependent demand and two-level trade credit," *Applied Mathematical Modelling*, vol. 34, no. 11, pp. 3273–3285 (2010), doi: 10.1016/j.apm.2010.02.019.

[13] P. Muniappan, R. Uthayakumar, and S. Ganesh, "An EOQ model for deteriorating items with inflation and time value of money considering time-dependent deteriorating rate and delay payments," *Systems Science & Control Engineering*, vol. 3, no. 1, pp. 427–434 (2015), doi: 10.1080/21642583.2015.1073638.

[14] P. Muniappan, R. Uthayakumar, and S. Ganesh, "A production inventory model for vendor–buyer coordination with quantity discount, backordering and rework for fixed lifetime products," *Journal of Industrial and Production Engineering*, vol. 33, no. 6, pp. 355–362 (2016), doi: 10.1080/21681015.2015.1070374.

[15] P. Muniappan, C. K. Kirubhashankar, and A. M. Ismail, "An optimum EOQ model for buyer-vendor with price breaks and fixed holding cost," in *Journal of Physics: Conference Series*, vol. 1770, no. 1, p. 012103, IOP Publishing (2021), doi: 10.1088/1742-6596/1770/1/012104.

[16] L. Y. Ouyang, C. H. Ho, C. H. Su, and C. T. Yang, "An integrated inventory model with capacity constraint and order-size dependent trade credit," *Computers & Industrial Engineering*, vol. 84, pp. 133–143 (2015), doi: 10.1016/j.cie.2014.12.035.

[17] B. Pal, S. S. Sana, and K. Chaudhuri, "Three stage trade credit policy in a three-layer supply chain – a production-inventory model," *International Journal of Systems Science*, vol. 45, no. 9, pp. 1844–1868 (2014), doi: 10.1080/00207721.2012.757383.

[18] S. K. Paul, R. Sarker, and D. Essam, "Managing risk and disruption in production-inventory and supply chain systems: A review," *Journal of Industrial and Management Optimization*, vol. 12, no. 3, pp. 1009–1029 (2016), doi: 10.3934/jimo.2016.12.1009.

[19] S. S. Sana, "A production-inventory model of imperfect quality products in a three-layer supply chain," *Decision Support Systems*, vol. 50, no. 2, pp. 539–547 (2011), doi: 10.1016/j.dss.2010.11.012.

[20] B. Sarkar, S. Saren, and L. E. Cárdenas-Barrón, "An inventory model with trade-credit policy and variable deterioration for fixed lifetime products," *Annals of Operations Research*, vol. 229, no. 1, pp. 677–702 (2015), doi: 10.1007/s10479-014-1745-9.

[21] B. Sarkar, S. S. Sana, and K. Chaudhuri, "An inventory model with finite replenishment rate, trade credit policy and price-discount offer," *Journal of Industrial Engineering* (2013), doi: 10.1155/2013/672504.

[22] S. Srivathsan and M. Kamath, "Performance modeling of a two-echelon supply chain under different levels of upstream inventory information sharing," *Computers & Operations Research*, vol. 77, pp. 210–225 (2017), doi: 10.1016/j.cor.2016.07.011.

[23] M. Yao and S. Minner, "Review of multi-supplier inventory models in supply chain management: An update," *SSRN Electronic Journal* (2017), doi: 10.2139/ssrn.2995134.

[24] J. Zhang, H. Yan, and W. Tang, "Optimal pricing and lot-sizing under a service level constraint in a two-echelon supply chain," *International Journal of Production Economics* (2017), doi: 10.5267/j.ijiec.2010.01.002.

Received November, 2024