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Multi-vendor and multi-buyer collaborative planning, forecasting and replenishment model

Jan-Min Chen
Department of Applied Intelligent Mechanical and Electrical Engineering
Yu Da University of Science and Technology
No. 168, Hsueh-fu Rd.
Tanwen Village, Chaochiao Township
Miaoli County 36143
Taiwan, R.O.C.

Abstract

Inventory management strategy always plays a very important part in supply chain management. The mathematical models proposed by most researchers consider that the production process to be dependable and assume that inventory would not be defective. Therefore, this paper presents a two-tier integrated production-inventory supply chain model that incorporates a defect rate while accounting for an unreliable production process. If defective items are identified, they will be returned to the upstream vendors for reprocessing, after which the repaired products will be shipped back to the buyers. We designed a linear regression equation which combines the concept of decision support system (DSS) to mitigate the uncertainty in ambiguous situations, and then based on the data of the past years to predict more precisely estimate buyer demand. The main goals of this paper are to minimize the expected joint total cost and to identify the best solution considering the presence of defective items, as illustrated by our proposed model. The predicted results based on numerical examples are provided to decision makers or managers for reference to help them make the right decisions and avoid corporate losses.

Subject Classification: *03C13, 62F07.*

Keywords: *Supply chain, Integer-multiplier policies, Defective items, Inventory, DSS System.*

1. Introduction

Nowadays, an enterprise must prioritize sustainable development while maintaining its competitiveness in today's fast-changing business environment. The relationships between companies are crucial in establishing a vertically integrated supply chain that connects all levels to optimize overall profitability

E-mail: **ydjames@ydu.edu.tw**

or reduce costs. Once the alliance is formed, effective management becomes essential, with inventory management strategy being a key factor. Many experts have been researching the inventory management for a lot of years, since inventory costs make up a significant portion of the supply chain expenses. Someone developing economic order quantity (EOQ) model was Harris [1], EOQ was originally formulated based on the ordering and holding costs of a single company. However, from the perspective of supply chain management (SCM), close collaboration with supply chain partners is essential, Harris's study didn't correspond to the real-world scenario. In order to satisfy SCM's conditions, Goyal [2] established the foundation for a joint inventory system involving a single vendor and a single buyer. Banerjee [3] proposed the Joint Economic Lot Size (JELS) model, which considers a delivery system in which the buyer obtains the goods following each production cycle instead of receiving them in batches. Extending Banerjee's [3] model, Goyal [4] incorporated the assumption that each production lot exceeds the buyer's order quantity. Lu [5] developed a unified inventory model for one vendor and multiple buyers, assuming equal shipment quantities for each delivery. Goyal [6] enhanced Lu's [5] model, demonstrating that varying delivery strategies could lead to improved results. Hill [7] built upon the models of Goyal [6] and Lu [5] by introducing the assumption that the shipment lot size during production can range from 1 up to the P/D ratio. Ha and Kim [8] suggested that each production run's lot size matches the buyer's order quantity, with shipments divided into N deliveries. Building upon the work of Goyal [6] and Hill [7], Goyal and Nebebe [9] introduced an improved model to optimize the order quantity more effectively. Pan and Yang [10] introduced a framework that integrates flexible lead times while maintaining consistent shipment sizes. Additionally, Kelle, Al-khateeb, and Miller [11] propose a model in which each production run results in a lot size that is a multiple m of the shipment quantity. This lot is then distributed by the vendor over n separate deliveries, where n may not equal m. Huang and Yang [12] developed a decision support system aimed at addressing the challenges associated with integrated inventory models, particularly focusing on reducing inventory costs in production management. Pan and Yang [13] applied fuzzy theory, assuming that the demand and production rates were variable, and derived the optimal order quantity to propose an integrated inventory model. Yang and Lo [14] investigated a multi-buyer, single-vendor system with adjustable lead times. Subrata [15] designed a two-echelon closed-loop supply chain, incorporating variations distinct from conventional supply chains. More recently, Jha and Shanker [16] incorporated service level restrictions into an inventory model while also accounting for variable lead times.

Most researchers have assumed the process is highly reliable, with no consideration for inventory decline or defects. Ghare and Schader [17] were the first to introduce the concept of a defective rate, treating it as a fixed constant to better reflect reality. Covert and Phillip [18] expanded on Ghare and Schader's [17] work by assuming that the defective rate follows a Weibull distribution. Chen [19] built upon Covert and Phillip's [18] research, considering timedependent demand rates and allowing for backorders. Later, Wee and Law [20] assumed a demand-dependent pricing model, where the defective rate follows a Weibull distribution. Aiming to address production variability and lead time fluctuations, Yang and Lin [21] proposed a multi-level integrated inventory model and applied Particle Swarm Optimization (PSO) to optimize shipment frequency and quantity. Yang, Lo, and Lu [22] studied the replenishment and production cycles of a single-vendor, multi-buyer system under an imperfect working environment. Yang et al. [23] develop a model for production and inventory in a single-supplier, single-retailer setting, incorporating the impacts of defective items and inspection inaccuracies. Furthermore, quantity thresholds are provided to absorb transport costs and to promote economies of scale in transport. The aim is to obtain the best-balanced production and replenishment strategy. Chakrabarty, Roy, and Chaudhuri [24] developed a more comprehensive version of the EPL model for defective products by integrating additional real-world factors such as inventory shortages, inflationary effects, and monetary time considerations. Their model also accommodated various demand behaviors, particularly quadratic and exponential demand reductions in response to price changes. Ruidas et al. [25] examined an Economic Order Quantity (EOQ) model that accounts for the presence of defective items in real-world production settings. They formulated the mathematical model and employed Particle Swarm Optimization (PSO) to obtain the optimal solution.

In recent years, companies have been using data in supply chain management to help their decision making and planning in response to changes in inventory that caused by business uncertainty and competition. Morton, McKenney, and Scott [26] has developed the concept of Decision Support Systems (DSS) to assist in commercial or organizational decision-making activities. Consequently, companies are adopting advanced decision support tools more frequently to reduce the impact of market uncertainty. Teerasoponpong and Sopadang [27] introduced a decision support system aimed at enhancing procurement and inventory operations, facilitating data-driven decisions amid market uncertainty. The system also took into account factors such as material demand, price, and supply lead time, with the goal of identifying the most suitable suppliers and optimal order quantities.

According to Sprague and Carson [28], a comprehensive framework for a DSS should consist of the following elements, as depicted in Figure 1:

- **Model Base:** Incorporates the decision evaluation model and the data processing model, or treats these as separate, standalone sub-patterns.
- **Data Base:** Save the necessary data for the decision support system.
- **Software System:** The following three subsystems constitute the software system.
    1. Data Base Management Subsystem (DBMSs)
    2. Model Base Management Subsystem (MBMSs)
    3. Dialog Generation and Management Subsystem (DGMSs)

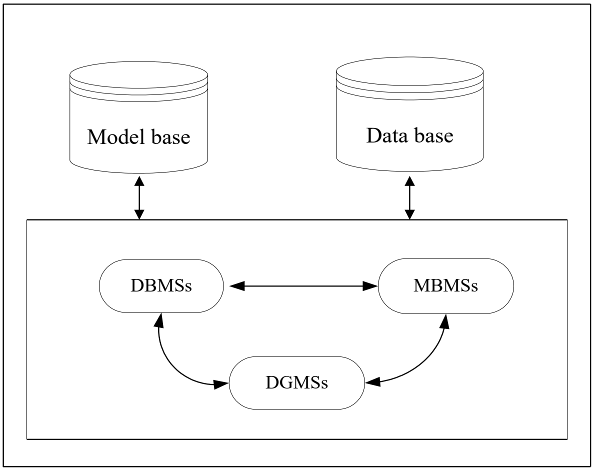

Figure 1
Framework for the DSS

After summarizing the introduction and literature in this section, this research seeks to develop a comprehensive inventory model and establish a DSS procedure. Minimize the expected joint total cost is our main goal. Therefore, a multi-vendor, multi-buyer inventory model is developed to reflect real-world SCM conditions, taking into account product defect rates and repair costs. We assume that each production lot is split into equal shipments and distributed sequentially to downstream buyers, with shipments dispatched immediately after production instead of waiting for the entire lot to be finished. The primary goal is to determine the best shipment volume and frequency under the conditions of an unreliable production process and uncertain lead times, with replenishment and production cycles contributing to total supply chain expenses. Using the concepts to identify the best solution and assist the supervisor in decision-making.

In Section 2, the parameters and assumptions are introduced first, leading to the formulation of the integrated inventory model. Following this, Section 3 presents the solution to the model, leading to the optimal results. Numerical examples and a sensitivity analysis are discussed in Sections 4 and 5, and the conclusions are drawn in Section 6.

2. Model

In this paper, we present a mathematical model based on the literature compiled in Section 1, modified and presented. First, we specify the notations of our model and key assumptions. Second, we described the annual total cost of vendor and buyer, then combined them into a joint total cost. Last, we discussed the minimum cost at a distinct T's value in next section. This model examines a production system that manufactures a single product type while accounting for defective items, with the objective of maximizing profit

2.1 *Variables notation*

The following notations are introduced for the formulation of a two-echelon integrated inventory model.

T = The basic ordering interval for the buyer.

K_V = An integer factor is applied to modify the cycle time at the vendor level.

$D_{V,g}$ = The rate of demand for vendor g.

$P_{V,g}$ = The rate of production for vendor g.

n_v = The quantity of vendors.

h_s = The vendor's yearly inventory cost per material unit.

h_v = The per-unit yearly inventory cost for finished products held by the vendor.

S_v = The cost incurred for each production setup by the vendor.

Q_r = The unit of manufacturing cost for vendor.

$D_{b,j}$ = The buyer j 's demand rate. $D_{b,j} = \hat{E}_{b,j} = \hat{\alpha}_{b,j} + \hat{\beta}_{b,j}Q_n$

n_b = Buyer's number.

h_b = Annual per-unit holding cost of finished products at the buyer's side.

S_b = The cost of ordering for the vendor.

R = Recycling-related expenses incurred by the vendor.

L = The repair-related expenditure incurred by the vendor.

d = The cost per unit for screening items by the buyer.

X = The rate at which the buyer conducts screening.

B = The cost incurred by the buyer for purchasing each unit.

K = The percentage of defective products that are non-repairable.

Y = The proportion of defective products.

σ = Buyer's discount rate (1-mKY, m=5).

$\hat{\alpha}_{b,j}$ = The gradient of the population regression function.

$\hat{\beta}_{b,j}$ = The y-intercept of the population regression function.

$\hat{E}_{b,j}$ = Forecasted demand.

$TC_{v,g}$ = The total cost of vendor g.

$TC_{b,j}$ = The total cost of buyer j.

$JTC_{(K_v,T)}$ = The combined total cost of the entire supply chain.

2.2 *Assumptions*

There are 8 assumptions we assumed as below:

(1) This model is built on a multi-vendor, multi-buyer framework for a single item.

(2) Defective rate is related directly to quantity discount.

(3) No shortages are allowed.

(4) Since shortages are not permitted, the production rate of non-defective products must exceed the buyer's demand.

(5) This system will repair the returned defective production, but those won't be completely repaired.

(6) The production rate is finite.

(7) Owing to pulling production $\sum_{j=1}^{n_b}(D_{b,j}) = \sum_{g=1}^{n_v}(D_{v,g})$. Demands are equal at all echelons.

(8) The supply chain's cycle time is consistent across all stages where each step is an integer multiple of the downstream stage's cycle time. The vendor's cycle time is $T_v = K_v * T$

2.3 *Formulation of the model*

In this part, the total costs incurred by the supplier and buyer are consolidated into one integrated inventory model. As demonstrated in Fig. 2, When the cycle begins, the inventory level starts at its initial value and gradually increases until it reaches its maximum point. Subsequently, the product is consumed at a specific rate until the cycle ends, while the vendor's cycle length remains unchanged as previously stated.

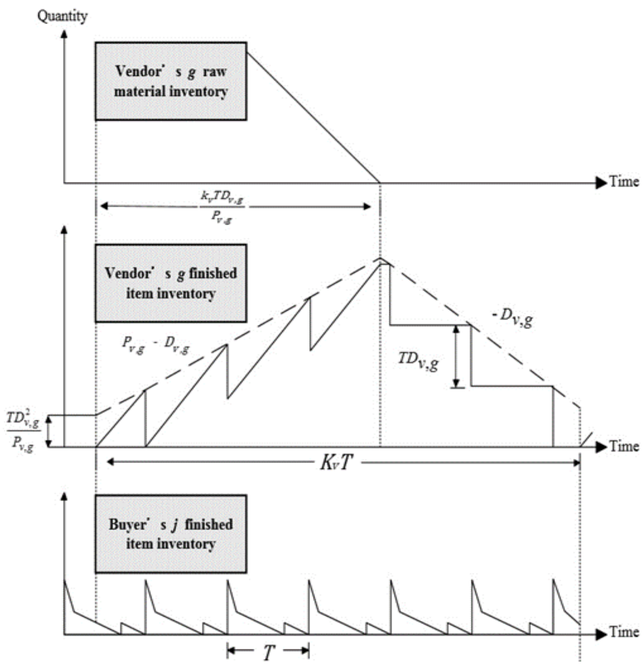

Figure 2
The inventory levels of the vendor and the buyer

A. The vendor's annual total cost

During each production run, the cost of vendor consists of manufacturing cost, setup cost, holding cost, repairing cost, recycling cost. The following is elements of the vendor's total annual cost.

(i) Manufacturing cost $= Q_r P_{v,j}$

(ii) Set up cost $= \frac{S_v}{K_v T}$

(iii) Holding cost $= \frac{h_s T D_{v,g}^2}{2P_{v,g}} + \frac{h_v T}{2}\left[(2-K_v)\frac{D_{v,g}^2}{P_{v,g}} + (K_v-1)D_{v,g}\right]$

(iv) Repairing cost $= LYD_{v,g}$

(v) Recycling cost $= RKYP_{v,g}$

B. The buyer's annual total cost

During each production run, the vendor incurs costs associated with ordering, holding, purchasing, and screening. The following outlines the primary components of the buyer's total annual cost.

(i) Ordering cost $= \frac{S_b}{T}$

(ii) Holding cost is given by:

$$= \frac{hb}{T}\left[\frac{(D_{b,j}T)^2 Y}{2(X)(1-KY)^2} + \frac{D_{b,j}\left(T(1-Y)\right)^2}{2(1-KY)} + \frac{D_{b,j}\left((1-K)YT\right)^2}{2(1-KY)^2}\right]$$

$$= \frac{h_b D_{b,j} T}{2(1-KY)}\left[\frac{D_{b,j}Y}{(X)(1-KY)} + (1-Y)^2 + \frac{(Y-KY)^2}{(1-KY)}\right]$$

(iii) Purchasing cost $= D_{b,j}\beta(1-\sigma)$

(iv) Screening cost $= \frac{dD_{b,j}}{(1-KY)}$

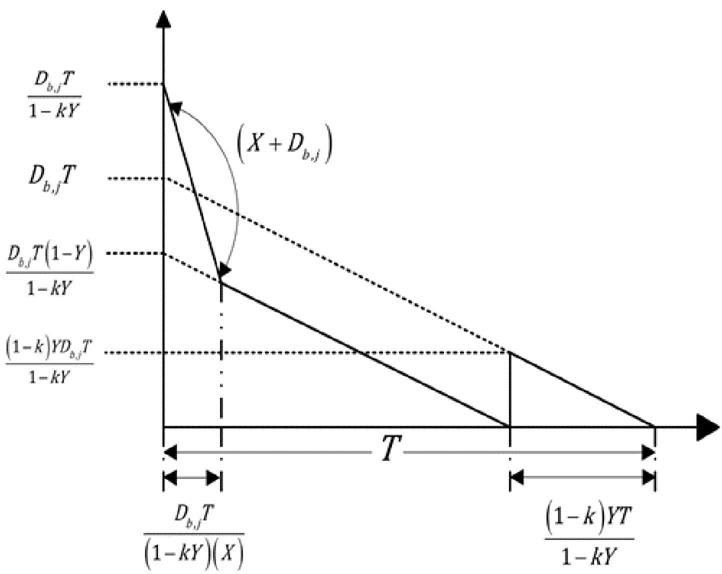

Figure 3
The buyer's inventory level

C. The joint total cost per year

(i) The annual cost incurred by the vendor can be represented as:

$$TC_v = \frac{S_v}{K_v T} + Q_r P_{v,g} + RKYP_{v,g} + LYD_{v,g} \frac{h_s T D_{v,g}^2}{2P_{v,g}} + \frac{h_v T}{2}\left[(2-K_v)\frac{D_{v,g}^2}{P_{v,g}} + (K_v - 1)D_{v,g}\right] \quad (1)$$

(ii) The annual cost of buyer is given by:

$$TC_b = \frac{S_b}{T} + \frac{dD_{b,j}}{(1-KY)} + D_{b,j}\beta(1-\sigma) + \frac{h_b D_{b,j} T}{2(1-KY)} \times \left[\frac{D_{b,j}Y}{(X)(1-KY)} + (1-Y)^2 + \frac{(Y-KY)^2}{(1-KY)}\right] \quad (2)$$

(iii) The annual total cost encompassing the entire supply chain $JTC_{(K_v,T)} = \sum_{g=1}^{n_v} TC_{v,g} + \sum_{j=1}^{n_b} TC_{b,j}$, is given by:

$$\sum_{g=1}^{n_v}\left(\frac{S_v}{K_v T} + Q_r P_{v,j} + RKYP_{v,g} + LYD_{v,g}\frac{h_s T D_{v,g}^2}{2P_{v,g}} + \frac{h_v T}{2}\left[(2-K_v)\frac{D_{v,g}^2}{P_{v,g}} + (K_v-1)D_{v,g}\right]\right) + \sum_{j=1}^{n_b}\left(\frac{S_b}{T} + \frac{dD_{b,j}}{(1-KY)} + D_{b,j}\beta(1-\sigma) + \frac{h_b D_{b,j}T}{2(1-KY)} \times \left[\frac{D_{b,j}Y}{(X)(1-KY)} + (1-Y)^2 + \frac{(Y-KY)^2}{(1-KY)}\right]\right) \quad (3)$$

3. Solution procedure

For given K_v, it is obvious that $JTC_{(K_v,T)}$ is a convex function of T > 0 in T. Consequently, minimum cost at a distinct T's value. We apply the derivative of $JTC_{(K_v,T)}$regarding T to make sure of the minimum cost, let $\partial JTC(K_v,T)/\partial T = 0$, which yields

$$T^* = \sqrt{2}\sqrt{\frac{(S_v n_v + S_b n_b K_v)(YK-1)}{K_v(\pi h_b - 2\theta(1-YK))}} \quad (4)$$

where

$$\pi = \sum_{j=1}^{n_b}\left(D_{b,j}\left(\frac{Y^2(K-1)^2}{(YK-1)} - (Y-1)^2 + \frac{D_{b,j}Y}{(YK-1)(X)}\right)\right)$$

$$\theta = \sum_{g=1}^{n_v}\left(\frac{-D_{v,g}}{2}\left(h_v\left(\frac{D_{v,g}(K_v-2)}{P_{v,g}} - K_v + 1\right) - \frac{D_{v,g}h_s}{P_{v,g}}\right)\right)$$

In order to predict the buyer's yearly demand, the $D_{b,j}$ will be substituted with $\hat{E}_{b,j}$,and

$$\hat{E}_{b,j} = \hat{\alpha}_{b,j} + \hat{\beta}_{b,j} Q_n \quad (5)$$

$$\hat{\beta}_{b,j} = \frac{\sum_{t=1}^{n}(Q_{t,j}-\bar{Q}_j)(F_{t,j}-\bar{F}_j)}{\sum_{t=1}^{n}(Q_{t,j}-\bar{Q}_j)^2} \quad \forall j = 1{\sim}n_b \tag{6}$$

$$\hat{\alpha}_{b,j} = \bar{F}_j - \hat{\beta}_{b,j} \quad \forall j = 1{\sim}n_b \tag{7}$$

Algorithm

To find the minimum values of $JTC_{(K_v,T)}$, we proceed with the following steps:

Step 1. Using by Eq. (5) to Define $D_{b,j}$.

Step 2. Evaluate $T^{(Kv)}$o according to Eq. (4).

Step 3. Compute $JTC(K_v, T^{(K_v)})$ using Eq. (3).

Step 4. Let $K = K_v + 1$, repeat steps 1 to 3 to find $JTC(K_v, T^{(K_v)})$.

Step 5. If $JTC(K_v, T^{(K_v)}) \leq JTC(K_v - 1, T^{(K_v-1)})$, returned to

Step 6. Otherwise, the optimal solution is $(K_v{}^*, T^*) = \left(K_v - 1, T^{(K_v-1)}\right)$

4. Numerical example

The analytic solution proposed on procedure is an application of solving the following numerical example. The following is the number of vendors what we set. Buyer is 3, Q_r = \$10/unit, R= \$3/unit, L = \$6/unit, d = \$0.5/unit, X = 6000 units/year, B = \$9/unit, h_s = \$0.9/unit/year, h_v = \$2/unit/year, S_v = \$400/cycle, S_b = \$150/cycle, K = 0.7, Y = 0.003, σ = 1-mkY = 0.937, and the demand of buyer is shown as Table.1 in past 7 year.

Table 1
The buyer's demand data (units/year) in past 7 years.

J	2009	2010	2011	2012	2013	2014	2015
1	1380	1450	950	1780	1900	1840	**1760**
2	860	970	800	1230	1450	1500	**1600**
3	650	630	780	985	1060	1240	**1400**

Calculate Eq. (5), we get the value $\hat{\beta}_{b,j}, \hat{\alpha}_{b,j}$ by using Eq.(6) and Eq.(7) before forecasting buyer's demand. Hence, we obtain $D_{b,1}$ =1727 units per year, $D_{b,2}$ =1649 units per year, $D_{b,3}$ =1588 units per year, $D_{v,g}$ and $P_{v,g}$is shown in Table 2.

Table 2
The data of vendor's $D_{v,g}$ and $P_{v,g}$.

G	$D_{v,g}$ (units/year)	$P_{v,g}$ (units/year)
1	1550	**5500**
2	1640	**4500**
3	1774	**3000**

By Using the algorithm, we obtain the optimal solution as follows:

(i) Cycle time of buyer $T^* = 0.273(\text{year})$, $K_v{}^* = 6$,

(ii) Cycle time of vendor $T_v = 1.636(\text{year})$,

(iii) The joint total cost = $ 201,204.14.

The results show that we use the model to predict the annual demand of three buyers, and then calculate the optimal replenishment cycle time to obtain the joint total cost. With the optimal value of the buyer's replenishment cycle time (T) the vendor can calculate the manufacturing cycle time for the entire supply chain by multiplying it by an integer (Kv), thereby enhancing efficiency and lowering costs.

5. Sensitivity analysis

Building upon the previous section, this section conducts a sensitivity analysis to examine how variations in certain parameters influence the joint total cost. In Table. 3 and Table. 4, we do sensitivity analysis to observe the JTC^*under parameters are changed in −25% and +25% of T, Y and K, the results were shown as below.

Table 3
Sensitivity analysis for *Y* and *k*.

	JTC^*	Y		K	
		-25%	+25%	-25%	+25%
JTC	$201,204	$200,777	$201,630	$200,975	$201,433
Compare with JTC^*		-$426	+$426	-$228	+$229

Table 4
Sensitivity analysis for replenishment cycle T.

	JTC^*	T	
		-25%	+25%
JTC	$201,204.14	$199,807.74	$209,918
Compare with JTC^*		-$1,396.4	+$8,713.86

In Fig 4, a sensitivity analysis of percentage of defective product, defective product which cannot be repaired and the buyer’s basic cycle time is performed. The results show that T has the greatest impact on the joint total cost, in particular, there is a dramatic change at 25%, resulting in massive cost increases. Next, the variation of the parameter Y is slightly sensitive to the cost, and the cost is close to the positive variation. Finally, the change of parameter K is less sensitive to cost, and the cost is also close to positive variation. Although the magnitude of Y and K changes are relatively small, they also have a certain degree of impact on the cost.

According to the above results, an increase in the values of these three variables has an effect on the total supply chain costs. On the vendor side, repairing and recycling costs are impacted by Y and K, whereas changes in the cycle time T substantially affect setup and holding costs. In addition, on the buyer side, Y and K affect holding cost and screening cost, for T has an impact on ordering cost and holding cost. The decision makers can adjust the strategy for the costs that they would like to reduce in the production run

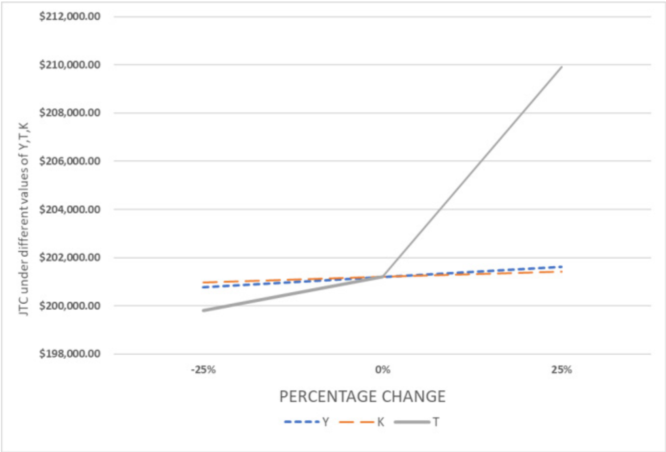

Figure 4

6. Conclusions

Considering the instability of the production process, this paper introduces a two-echelon integrated inventory model. The primary aim is to optimize supply chain efficiency by reducing total costs through strategic and synchronized decision-making. As shown in Table 3, it is evident that the combined total cost increases as K and Y rise, since defective products result in greater time and expenses for procurement and manufacturing. The defective rate and repair rate are crucial in determining the inventory policy. In this model, the focus is placed on the demand from the downstream segment of the supply chain, as production patterns influence the manufacturing process.

In the context of Collaborative Planning, Forecasting, and Replenishment (CPFR), this paper contributes by developing a linear regression model to predict the buyer's demand. Using the DDS approach, the defective rate and repair rate are incorporated into the uncertainty, and the forecasted results help decision-makers or managers make informed choices to minimize losses. The model also assists companies in determining whether production strategies need adjustment, such as improving the yield rate to reduce the occurrence of defects or increasing the repair rate, as well as considering further investment in new equipment and technology upgrades. As seen in Table 4, the joint total cost exhibits an upward trend as the replenishment cycle time increases, demonstrating the significant impact of cycle duration on overall cost.

As previously stated, the model determines the buyer's replenishment cycle time (T), which represents the optimal reference point for the entire supply chain. The vendor can calculate the production cycle time by multiplying it with an integer factor (Kv), thereby improving efficiency and reducing overall costs.

Finally, we plan to refine our future research. First, we aim to incorporate real-world complexities, such as collaborating with industry cases to obtain actual data and provide real numerical examples for future studies. Additionally, more factors can be included in the linear regression function to enhance the forecasting accuracy. Lastly, considerations such as inflation and carbon emissions can be integrated into the supply chain costs to further improve the proposed model.

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