

Application of a three-parameter Lindley distribution in quality control

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Abstract

Statistical quality control in industries enables businesses to improve product quality, reduce defects, optimize processes, increase customer satisfaction, and gain a competitive edge in the market. It provides a systematic and data-driven approach to quality management, enabling industries to consistently deliver high-quality products and services while driving continuous improvement across all operations. This paper examines the applications of the Harris extended modified Lindley distribution, a generalization of the well-known Lindley distribution. We demonstrate the practical utility of this distribution in the field of quality control. Using the lifetime distribution, an acceptance sampling plan is devised that can

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determine whether a large batch of products submitted for inspection should be accepted or rejected. The operational characteristic functions that were generated allow for a greater understanding of the performance of the sampling plan. Computed tables show sample sizes and parameters in the article. To demonstrate the efficacy of the proposed methodology, two distinct datasets pertaining to the ordered product failure times of a software development project and ball bearing failure times are utilized.

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1. Introduction

Statistical quality control (SQC) and statistical distributions are interrelated in multiple facets of quality management. Statistical distributions are utilized to analyze data, construct control charts, evaluate process capability, develop sampling plans, and facilitate statistical process control. Effective quality control practices use statistical distributions to make data-driven decisions and improve quality performance. Acceptance sampling plans (ASP) constitute a fundamental component of SQC. They provide a cost-effective structured method for assessing the quality of numbers or batches, managing risks, optimizing inspection efforts, integrating with statistical process control methodologies, and promoting the continuous development of quality management practices. The ASP is a method of inspection that is employed to determine whether a particular batch of items should be accepted or rejected based on pre-determined criteria. The amount of time that must be spent testing the whole lot is one of the most significant obstacles that must be overcome when using acceptance sampling. Further, instead of solely focusing on the number of failures for a certain acceptance criterion, it is essential to determine the minimum sample size necessary to ensure a specified mean life when the life test concludes at a predetermined duration. The goal is to maximize the probability of rejecting a bad lot with a mean life length below the set threshold while simultaneously accepting a lot when it can be proven with high confidence that its mean life fulfills the required criterion. This lowers the possibility of admitting lots with insufficient durability while ensuring that items matching the appropriate quality requirements are accepted.

Contemporary statistical literature has extensively investigated various ASPs that are based on truncated life tests. Researchers have

studied a variety of distributions and developed sampling plans that are tailored to each distribution.

Recent statistical research has studied a variety of acceptance sampling designs based on truncated life tests. These designs include, among others: [22] were the first to develop acceptance sampling using the inverse Rayleigh distribution, laying the groundwork for future study in this field. [21] investigated acceptance sampling designs for the percentiles of the log-logistic distribution in detail. For the generalized log-logistic distribution, [8] devised the two-fold acceptance sampling strategy based on truncated life tests. [3] investigated the exponentiated Fréchet distribution, [5] examined the three-parameter kappa distribution, and [6] investigated the generalized inverted exponential distribution. ASPs for the generalized inverse Weibull distribution were studied in [7]. [2] presented a new family of distributions with applications to the ASPs in risk and quality control, and [1] created a group ASP based on percentiles for the Weibull Fréchet model. Collectively, these research efforts shed insight on the wide variety of acceptance sampling procedures and designs that have been created for use with different distributions, therefore facilitating efficient quality control and decision-making.

On the other hand, recent years have been marked by the development of the Lindley distribution and its variants. The Lindley distribution can be considered as a strong alternative to the exponential distribution for analyzing various kinds of lifetime data. Specifically, the exponential and Lindley distributions depend on a single-parameter, but offer distinct advantages. Unlike the exponential distribution, characterized by a constant hazard rate and a mean residual life function, the Lindley distribution features an increasing hazard rate and a decreasing mean residual life function, enhancing its applicability in specific modeling scenarios. Recent studies on the Lindley distribution include those in [11], [12], [9], [20], and [25].

Following an exhaustive search of the available literature, we discovered that the Harris extended modified Lindley (HEML) distribution introduced by [25] has not been studied in the context of ASPs. To address this issue, we suggest a novel ASP for the three-parameter HEML distribution that relies on truncated life tests. It is pertinent to acknowledge that the proposed acceptance sampling scheme has the potential to be employed in various other iterations of the modified Lindley distribution, in order to assess its efficacy and suitability in diverse contexts. The objective of this study is to address the existing void in the academic

literature and furnish a significant addition to the acceptance sampling methodologies for the HEML distribution.

The remaining sections of the article are summarized here. The HEML distribution is briefly discussed in Section 2. Plans for acceptability sampling for truncated life tests are developed in Section 3 using the HEML distribution. In Section 4, illustrated tables of the proposed acceptance sampling and their justifications are provided, along with actual datasets used to demonstrate the effectiveness of the proposed sample strategy. In the final Section 5, a summary of the conclusions and recommendations is provided.

2. HEML distribution

According to [25], the HEML distribution is considered by extending the Harris family defined with the modified Lindley distribution. It thus depends on three parameters, denoted as α , δ and θ . Following the HEML distribution, the probability density function (pdf) of a random variable X is given as follows:

$$f_{HEML}(x; \alpha, \delta, \theta) = \frac{\theta \frac{1}{\alpha} \frac{\delta}{1+\delta} e^{-2\delta x} ((1+\delta)e^{\delta x} + 2\delta x - 1)}{\left[1 - \bar{\theta} \left(\left(1 + \frac{\delta x}{1+\delta} e^{-\delta x} \right) e^{-\delta x} \right)^\alpha \right]^{1 + \frac{1}{\alpha}}}, \quad x > 0, \quad (1)$$

where $\alpha, \delta > 0$, $\bar{\theta} = 1 - \theta$, respectively, and $f_{HEML}(x; \alpha, \delta, \theta) = 0$ for $x \leq 0$. The pdf exhibits a unimodal graph with varying peaks tilted to the right. The survival rate function of the HEML distribution is expressed as follows:

$$\bar{F}_{HEML}(x; \alpha, \delta, \theta) = \frac{\theta \frac{1}{\alpha} \left(1 + \frac{\delta x}{1+\delta} e^{-\delta x} \right) e^{-\delta x}}{\left[1 - \bar{\theta} \left(\left(1 + \frac{\delta x}{1+\delta} e^{-\delta x} \right) e^{-\delta x} \right)^\alpha \right]^{1 + \frac{1}{\alpha}}}, \quad x > 0,$$

and $\bar{F}_{HEML}(x; \alpha, \delta, \theta) = 1$ for $x \leq 0$. For different parametric values, the hazard rate function of the HEML distribution is found to take on several forms, including an increasing (or decreasing) shape and remaining constant. The distribution models hydrological data better than the Harris-extended Lindley distribution established by [10] and other distributions. We refer the reader to [25] in order to examine other HEML distribution-specific characteristics. The subsequent section delves into the sampling plan and elucidates how the HEML distribution can be employed to formulate it.

3. Sampling Plan

The ASP in statistical quality control entails the evaluation of a sample of products from a lot, leading to the acceptance or rejection of the lot depending on product quality. This discussion focuses on the criteria for choosing to accept or decline a lot based on its reliability, contingent upon the product's lifetime adhering to the HEML distribution.

3.1 Method

The main challenges of an ASP include the volume of products requiring evaluation and the threshold of defects permissible prior to acceptance. The quantity of defectives identified upon the conclusion of the test at a specified duration is recorded. The lot is deemed acceptable if the probability is at least p^* , provided that the number of defective items among n inspected does not exceed the maximum allowable defects (c) at time t .

A lot is deemed rejected if the quantity of defective items exceeds the specified number c prior to time t . This study investigates the minimum number of samples required for decision-making criteria.

Let us consider the HEML distribution as the assumed lifetime distribution of the products, where θ and δ are known parameters and λ is unknown. The mean lifetime is therefore entirely determined by the value of λ .

It is pertinent to note that the cumulative density function (cdf) of the HEML distribution is given by:

$$F_{HEML}(t; \alpha, \delta, \theta, \lambda) = 1 - \frac{\theta^{\frac{1}{\alpha}} \left(1 + \delta + \delta \left(\frac{t}{\lambda}\right) e^{-\delta \left(\frac{t}{\lambda}\right)}\right) e^{-\delta \left(\frac{t}{\lambda}\right)}}{\left[(1 + \delta)^{\alpha} - \theta \left\{\left(1 + \delta + \delta \left(\frac{t}{\lambda}\right) e^{-\delta \left(\frac{t}{\lambda}\right)}\right) e^{-\delta \left(\frac{t}{\lambda}\right)}\right\}^{\alpha}\right]^{\frac{1}{\alpha}}}, \quad t > 0,$$

where α, θ, δ are shape parameters, and $\lambda > 0$ is a scale parameter. If α, θ, δ are known, the average lifetime depends only on λ .

Assume that the required minimum average lifetime is λ_0 . The following equivalency is therefore valid.

$$F_{HEML}(t; \alpha, \delta, \theta, \lambda) \leq F_{HEML}(t; \alpha, \delta, \theta, \lambda_0) \Leftrightarrow \lambda \geq \lambda_0.$$

Assuming that the lifetime of sample products follows the HEML distribution, we seek to develop an ASP within this framework. The ASP possesses the following attributes:

- The quantity of items n present on the test;

- The number of items to be accepted denoted as c ;
- The highest possible time period of the test denoted as t ;
- The relationship between the highest possible test time period and the specified mean lifespan is expressed as t / λ_0 .

The sampling plan should reject the lot if the true average life λ is less than λ_0 for the benefit of the consumers. Therefore, the consumer's risk should not exceed $1 - p^*$, where p^* is a lower bound on the probability that a lot will be rejected by the ASP. The triplet $(n, c, t / \lambda_0)$ for a particular p^* is referred to as the ASP. To ascertain the acceptance probability for adequately large lots, we can utilize a binomial distribution. For known values of c and t / λ_0 , the primary objective is to determine the minimum sample size n , so

$$L(p_0) = \sum_{i=0}^c \binom{n}{i} p_0^i (1 - p_0)^{n-i} \leq 1 - p^*, \quad (2)$$

where $p_0 = F_{HEML}(t; \alpha, \delta, \theta, \lambda_0)$ is the probability of failure prior to time t . The smallest possible values of n that satisfy Equation (2) are achieved for $\alpha = 2.5$, $\delta = 1.3$, $\theta = 2$, and $p^* = 0.75, 0.95$. Additionally, the values of t / λ_0 are 0.68, 0.84, 0.99, 1.1, 1.3, 1.42, 1.62, 1.81, 2, and 2.5. The findings are displayed in Table 1. The transformation of Equation (2) to Equation (3) occurs when the binomial probability can be approximated by the Poisson probability with parameter $\eta = np_0$ if $p_0 = F_{HEML}(t; \alpha, \delta, \theta, \lambda_0)$ is extremely small and n is big.

$$L_1(p_0) = \sum_{i=0}^c \frac{\eta^i}{i!} e^{-\eta} \leq 1 - p^*. \quad (3)$$

Table 2 presents the minimal numbers for n that satisfy Equation (3) for identical set of values across varying values of p^* .

The operating characteristic (OC) function of the sampling strategy $(n, c, t / \lambda_0)$ represents the likelihood of accepting the entire batch $L(p_1)$.

Here, we have

$$L(p_1) = \sum_{i=0}^c \binom{n}{i} p_1^i (1 - p_1)^{n-i},$$

where $p_1 = F_{HEML}(t; \alpha, \delta, \theta, \lambda)$. The mean lifetime of the product is positively correlated with λ , resulting in a decrease in the failure probability p_1 , which indicates that the OC function is increasing with respect to λ .

For specified values of p^* and t/λ_0 , the selection of c and n is determined by the OC values. Given that

$$p = F_{HEML}\left(\frac{t}{\lambda_0}/\frac{\lambda}{\lambda_0};\alpha,\delta,\theta,\lambda\right).$$

The OC values for different sampling strategies are calculated and displayed in Table 3. Additionally, the OC curve is graphically represented in Figure 1 for $p^* = 0.75$ and for the sampling plan $(n,c,k = t/\lambda_0)$.

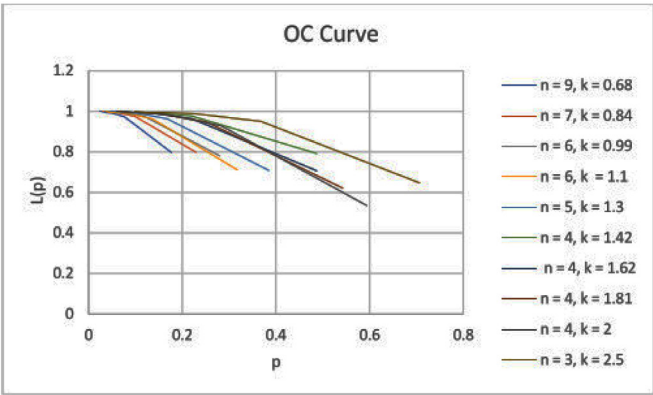


Figure 1
OC curve of the ASP $(n,c,k = t/\lambda_0)$

Table 1
Minimum sample size required for a given confidence level p^* , ratio t/λ_0 , acceptance number c , $\alpha = 2.5$, $\delta = 1.3$, $\theta = 2$ for the binomial approximation of the HEML distribution

p^*	c	t/λ_0									
		0.68	0.84	0.99	1.1	1.3	1.42	1.62	1.81	2	2.5
0.75	0	3	2	2	2	2	1	1	1	1	1
	1	6	5	4	4	3	3	3	3	2	2
	2	9	7	6	6	5	4	4	4	4	3
	3	12	9	8	7	6	6	5	5	5	4
	4	15	10	10	9	8	7	7	6	6	5
	5	18	14	12	10	9	9	8	8	7	7
	6	20	16	14	12	11	10	9	9	8	8

Contd...

	7	23	18	15	14	12	11	11	10	10	9
	8	26	20	17	16	14	13	12	11	11	10
	9	28	23	19	17	15	14	13	12	12	11
	10	31	25	21	19	17	16	14	14	13	12
0.95	0	6	5	4	4	3	3	2	2	2	2
	1	10	8	7	6	5	4	4	4	3	3
	2	14	11	9	8	7	6	5	5	5	4
	3	17	13	11	10	8	7	7	6	6	5
	4	21	16	13	12	10	9	8	8	7	6
	5	24	18	15	14	12	11	10	9	8	8
	6	27	21	17	16	13	12	11	10	10	9
	7	30	23	19	17	15	14	13	12	11	10
	8	33	26	21	19	16	15	14	13	12	11
	9	36	28	23	21	18	17	15	14	13	12
	10	39	30	25	23	20	18	17	15	15	13

Table 2

Minimum sample size required for a given confidence level The parameters include p^* , the ratio t/λ_0 , the acceptance number c , $\alpha = 2.5$, $\delta = 1.3$, and $\theta = 2$, which pertain to the Poisson approximation of the HEML distribution.

p^*	c	t/λ_0									
		0.68	0.84	0.99	1.1	1.3	1.42	1.62	1.81	2	2.5
0.75	0	4	3	3	3	2	2	2	2	2	2
	1	7	6	5	5	4	4	4	4	4	3
	2	10	8	7	7	6	6	5	5	5	5
	3	13	11	9	9	8	7	7	6	6	6
	4	16	13	11	10	9	9	8	8	8	7
	5	19	15	13	12	11	10	10	9	9	8
	6	22	18	15	14	12	12	11	10	10	10
	7	25	20	17	16	14	13	12	12	11	11
	8	27	22	19	17	15	15	14	13	13	12
	9	30	24	21	19	17	16	15	14	14	13
	10	33	26	23	21	19	18	16	16	15	14
	0	8	7	6	5	5	54	4	4	3	4
	1	12	10	9	8	7	7	6	6	6	6

Contd...

0.95	2	16	13	11	10	9	9	8	8	8	7
	3	20	16	13	13	11	11	10	10	9	9
	4	24	19	16	15	13	13	12	11	11	10
	5	27	22	18	17	15	14	14	13	12	12
	6	30	24	21	19	17	16	15	14	14	13
	7	33	27	23	21	19	18	17	16	15	15
	8	36	30	25	23	20	20	18	17	17	16
	9	40	32	28	25	22	21	20	19	18	17
	10	43	34	30	27	24	23	21	20	20	18

Table 3
The OC values for the ASP of the HEML distribution

p^*	n	c	t/λ_0	λ/λ_0					
				2	4	6	8	10	12
0.75	9	2	0.68	0.796519271	0.97308992	0.992410384	0.996921915	0.998468623	0.999132494
	7	2	0.84	0.797800571	0.975126477	0.993273133	0.99733945	0.998698053	0.999271028
	6	2	0.99	0.780485951	0.973704269	0.993081167	0.997312734	0.998701278	0.99927937
	6	2	1.1	0.711615369	0.962949885	0.990204518	0.996203817	0.998170779	0.998987754
	5	2	1.3	0.708771947	0.963935537	0.990804145	0.996521448	0.998351667	0.999098891
	4	2	1.42	0.792074785	0.977581414	0.994577969	0.998005863	0.999071481	0.999498453
	4	2	1.62	0.70696685	0.964664593	0.991396026	0.996852523	0.998543777	0.999217864
	4	2	1.81	0.621089059	0.948525057	0.987305307	0.995367476	0.9978668	0.998859538
	4	2	2.00	0.535374133	0.928338505	0.981998308	0.993430867	0.996985501	0.998394628
	3	2	2.50	0.646905532	0.95018371	0.988266087	0.995919793	0.998192596	0.999062624
0.95	14	2	0.68	0.536925726	0.912466914	0.972544566	0.988300617	0.994012004	0.996545099
	11	2	0.84	0.520381202	0.912974914	0.993273133	0.99733945	0.998698053	0.999271028
	9	2	0.99	0.517206609	0.916166146	0.975465074	0.990000651	0.995034657	0.997196779
	8	2	1.1	0.50802395	0.915919905	0.975871857	0.990298259	0.995227617	0.997324186
	7	2	1.3	0.453780637	0.902839209	0.972511008	0.989109676	0.994705367	0.997058227
	6	2	1.42	0.494679666	0.916669363	0.977288261	0.991196083	0.995780062	0.997678636
	5	2	1.62	0.524652185	0.926012746	0.980631176	0.992686832	0.996557322	0.998130804
	5	2	1.81	0.420564106	0.951720169	0.987596843	0.995316083	0.997787068	0.998793688
	5	2	2.00	0.328095157	0.858234106	0.960774587	0.985067467	0.992988052	0.996212039
	4	2	2.50	0.646905532	0.95018371	0.988266087	0.995919793	0.998192596	0.999062624

4. Illustration

In this scenario, the lifetime distribution is assumed to follow the HEML distribution with the given parameters: $\alpha = 2.5$, $\delta = 1.3$, $\theta = 2$. The experimenter's objective is to furnish evidence substantiating the assertion that the unknown average lifespan of the item is a minimum of 1000 hours. In the reliability test, we evaluate the risk of consumers as $1 - p^* = 0.25$. The experiment aims to complete the testing phase at $t = 680$ hours. According to Table 1, an acceptance number of $c = 2$ corresponds to a required sample size of $n = 9$. The established sampling plan is defined by $n = 9$, $c = 2$, and a ratio of $t / \lambda_0 = 0.68$. In a duration of 680 hours, if a maximum of 2 failures is observed among 9 samples, the experimenter can confidently claim, with a confidence limit of 0.75, that the mean lifespan of the item is a minimum of 1000 hours. The Poisson approximation to the binomial distribution indicates that the suitable sample size is $n = 10$, with parameters $c = 2$ and a ratio of $t / \lambda_0 = 0.68$, while accounting for a consumer risk of 0.25 under the HEML distribution. The corresponding OC values can be obtained from Table 4.

Table 4
The OC values for the ASP

$\frac{\lambda}{\lambda_0}$	2	4	6	8	10	12
$L(p)$	0.796519271	0.97308992	0.992410384	0.996921915	0.998468623	0.999132494

Application 1

The dataset made available consists of data on product failure rates that was taken from a software development project that was first published by [26]. Numerous researchers, including [24], [19], [18], have thoroughly examined this dataset in the context of acceptance sampling. Individual observations are indicated by the notation x_i , where i is a number between 1 and 10, and it represents an ordered sample of size 10. The values of the observations in the sample specifically are as follows: 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, 5218, 5823.

Based on Table 1 for $p^* = 0.95$, the ratio t / λ_0 is 0.68 for an average lifespan of 1000 hours over a testing period of 680 hours, with n and c values of 10 and 1, respectively. Thus, the sampling strategy for the sample data presented above is $n = 10$, $c = 1$, $t / \lambda_0 = 0.68$. We will only accept the

lot if there are one or fewer failures prior to 680 hours. The adoption of the ASP is predicated on the assumption that the system's lifetime adheres to the HEML distribution. The Q-Q plot and goodness-of-fit statistics indicate a high degree of agreement, with parameters $\theta = 27.467560$, $\alpha = 0.121727$, and $\delta = 0.002909$. A single failure occurs 519 hours before termination ($t = 680$ hours) in the specified sample. Therefore, we approve the proposal. The termination time t is shorter than the sampling strategies suggested by [19], [24], [14], [13], [22], [15], [23], [4], and [16]. In light of this, using the current sampling plan enables the experiment to be conducted at a significantly reduced cost and in much less time.

Application 2

The second dataset is derived from endurance testing of deep groove ball bearings as reported in [17]. The dataset is as follows: 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, and 93.12.

When a testing period of 56 hours and a specified average life of 66 hours are taken into consideration, the ratio t/λ_0 would be equal to 0.84. The ASP is calculated based on the premise that the dataset conforms to the HEML distribution. The Quantile-Quantile plot and adequacy measures indicate a strong concordance ($\theta = 1.016 \times 10^8$, $\alpha = 6.812$, $\delta = 5.585 \times 10^{-2}$). Taking into account, $t/\lambda_0 = 0.84$, the desired level of confidence $p^* = 0.75$, and $n = 10$, we are able to determine that $c = 4$ from Table 1. Consequently, the sampling methodology for the specified sample dataset would be $n = 10$, $c = 4$, $t/\lambda_0 = 0.68$. The lot under consideration will only be accepted if there are four or fewer failures noted before 64 hours. It can be seen from the sample data that four of the ten failures—at the precise times of 51.84, 51.96, 54.12, and 55.56—failed before the 56-hour mark. Consequently, the product is regarded as acceptable in accordance with the specified sampling strategy.

5. Conclusion

When dealing with items that follow the HEML distribution, a reliability test plan is created. Our study provides tables that use binomial and Poisson approximations to determine the minimum sample size necessary based on specified parameters such as the ratio, confidence level, and acceptance number. The evaluation of OC values is incorporated into the plan. We make use of two datasets pertaining to the failure rates of ordered goods and ball bearings to demonstrate the efficacy of the plan.

The producer's risk is substantially decreased by the suggested plan. In addition, to improve the efficiency of making final decisions on a lot, it is recommended to use the suggested method, which can reduce both the time of the trial and the cost.

Data Availability

The data used in this study are referenced into the text and bibliography.

Conflicts of Interest

The authors wish to affirm that there are no conflicts of interest to disclose.

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Author's contributions

Every author contributed equally and substantially to all sections of this article. All authors reviewed and endorsed the final version of the manuscript.

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