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The impact of the Covid-19 and the Russia-Ukraine conflict on crude oil and stock market volatility

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Abstract

The global economy has been profoundly affected by two recent crises: the Covid-19 pandemic and the Russia-Ukraine War. These events have also driven market volatility in both West Texas Intermediate (WTI) crude oil and the S&P 500 Index. The objective of this study is to analyze the market volatility of WTI crude oil and the S&P 500 Index in the context of the two aformentioned crises. Through the implementation of randomness tests and an ARCH effect test, it has been determined that ARMA-GARCH models could be useful for modeling the data in each period. Additionally, the GARCH models have been expanded to asymmetric GJR models, which account for the leverage effect. The Box-Jenkins methodology has been employed to identify the best-fitted model. The analysis reveals that WTI crude oil is more strongly impacted by the two events compared to the S&P 500 Index, suggesting that the latter is relatively less volatile. Furthermore, forecast evaluation is conducted by comparing the predicted values obtained from the best-fitted and second-best-fitted models to assess forecast accuracy. Lastly, risk determination is performed by calculating the value at risk. Based on the calculations, it is apparent that WTI crude oil carries greater risk than the S&P 500 Index during major crises. Overall, market volatility analysis provides valuable

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insights into the statistical behavior of different markets, enabling market participants to evaluate market risk and enhance their risk management strategies.

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1. Introduction

Crude oil is a valuable natural resource extracted from the earth and refined to produce various petroleum products, such as gasoline, diesel, and kerosene. In today's world, crude oil plays a crucial role in numerous sectors, especially those related to energy production. Given its diverse applications, crude oil is a significant commodity traded on international markets, where its prices experience daily fluctuations. As a result, crude oil markets often serve as a benchmark for global economic activity. As of August 2022, crude oil prices stood at around $90 per barrel, suggesting an upward trend compared to the previous five years. Brent and West Texas Intermediate (WTI) are the two most widely traded grades of crude oil. Brent crude oil is sourced from the North Sea and is primarily used in European and African markets, while WTI is sourced from the United States and is used in the North American market. The prices of Brent and WTI crude oil are closely monitored by investors and analysts, as they serve as reflections of the global economy and energy markets. Market volatility analysis plays a crucial role in understanding the fluctuations of crude oil prices. Stock market volatility in finance refers to the rate of price fluctuations within a stock market over time. By examining this volatility, investors and traders can identify trends and make well-informed decisions in crude oil trading. The primary driver of volatility in the crude oil market is uncertainty around supply and demand, which can be affected by various factors such as environmental concerns, economic growth, political tensions, and fluctuations in global exchange rates. In essence, understanding crude oil price volatility is essential, given the vital role that crude oil plays in global economic activity.

In this study, we will use datasets from WTI crude oil for volatility analysis, alongside datasets from the S&P 500 stock index for comparative purposes. The S&P 500 is a stock market index comprising 500 leading publicly traded companies in the United States. As a market capitalization-weighted index, it assigns greater influence to companies with larger market caps, thus affecting the overall index value. It is one of the most frequently referenced American indices due to its broad representation of the largest U.S. corporations, making it a widely recognized indicator of U.S. stock market performance and the broader economy. Key companies in the S&P 500 include Apple, Microsoft, and Amazon. Additionally, various significant events have historically impacted the dynamics of energy market prices. Major occurrences such as the 2008 global financial crisis, the 2018 U.S.-China trade war, the pandemic in 2019, and the war conflict in 2022 have all contributed to fluctuations in crude oil prices.

Wang and Sun [1] investigated the relationship between oil price fluctuations and various determinants using a structural equation model (SEM). Their study revealed that global economic activity is the most influential factor driving crude oil price volatility. Additionally, political tensions and conflicts, particularly among major oil-producing nations or those heavily reliant on crude oil, were found to significantly raise oil prices. Although wars may not directly influence crude oil prices, they can disrupt oil supply, thereby causing price fluctuations. A notable historical event that led to a downturn in the energy market was the global financial crisis of 2008. Originating from the subprime mortgage crisis between 2007 and 2010, the financial turmoil compelled companies to cut production due to a sharp decline in aggregate demand, resulting in soaring unemployment rates and reduced consumer spending. This decline in demand from both consumers and suppliers caused the price of crude oil to plunge dramatically, dropping from $133.88 to $39.09 per barrel within less than a year.

In the research study by Jia et.al. [2], the effects of the pandemic on local and international oil markets were investigated using a structural vector autoregressive model. As the second-largest oil importer in the market, the Daqing oil price in China experienced a significant decline. However, China did not benefit from the price drop due to the simultaneous decrease in oil demand. For Nigeria, the impact of pandemic is considered to be severe, particularly for the Bonny crude oil market, which has been significantly affected by the pandemic impact. Both local and foreign oil prices are not expected to experience substantial long-term increases as a result of these circumstances.

Oluwasegun et al. [3], utilizing the time-varying parameter vector autoregressive (TVP-VAR) technique, found that the interconnectedness between oil and other financial assets intensifies during periods of conflict. During such times, oil acts as a significant transmitter of spill-overs, exerting a strong influence on other financial assets; however, this effect is generally temporary. Additionally, the ongoing pandemic, which continues to impact people worldwide, has made pandemic-related news a crucial factor in predicting crude oil price volatility. Numerous studies have explored the pandemic's effects on global stock markets [4,5].

Another important hypothesis that related to this study is the Efficient Market Hypothesis (EMH) which has been employed as a method to assess market efficiency. However, recent research has indicated that the EMH may not accurately predict market behavior since it assumes a constant level of market efficiency over a specified period. To address this limitation, a new approach called the Adaptive Market Hypothesis (AMH) has been introduced as an evolutionary alternative to the EMH. According to the AMH, market efficiency and inefficiency coexist in a logical and consistent manner. The findings presented by Ghazani and Ebrahimi [6] also support the concept of the AMH, suggesting that the WTI and Brent markets exhibit characteristics that align with this adaptive market efficiency framework. This highlights the importance of considering market dynamics and evolving conditions when analysing and forecasting crude oil markets.

2.0 Methodology

Returns are commonly used in financial data analysis as they can be presented as percentages and tend to exhibit more statistical stationarity compared to price series. The data of daily prices are converted to continuously compounded percentage rates of return by using the formula:

$$r_t = 100 \times (\ln P_t - \ln P_{t-1}) \tag{1}$$

where P_t and P_{t-1} are the daily closing prices at time *t* and *t–1* respectively

In financial markets, the Efficient Market Hypothesis (EMH) serves as a useful benchmark for measuring relative efficiency. When a market is efficient, meaning it incorporates all available information, it becomes unpredictable. The concept of EMH is closely associated with the random walk hypothesis. As stated in Lo and MacKinlay [7], the greater the efficiency of the market, the more random the sequence of price changes generated by the market. In order to examine the random walk hypothesis, we have conducted unit-root test, Ljung-Box correlation test, variance ratio test and ARCH test.

2.1 *ARMA-GARCH models*

The autoregressive process of order p, labeled as AR(p), employs past values from the original series to predict future values. In contrast, the moving average process denoted as MA(q), with an order of q, entails a linear combination of the current white noise term and the q most recent preceding white noise terms. The ARMA(p,q) is expressed as

$$r_t = \phi_0 + \sum_{i=1}^{p} \phi_i r_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t \tag{2}$$

where ϕ_0 is a constant term, ϕ_i are coefficients of AR, θ_j are coefficients of MA and ε_t is the error term. A generalised autoregressive conditional heteroskedastic (GARCH) model which combines both moving average component and autoregressive component can be expressed as:

$$\begin{aligned} \varepsilon_t &= \omega_t \sigma_t \\ \sigma_t^2 &= \alpha_0 + \sum_{i=1}^{s} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2 \end{aligned} \tag{3}$$

such that ω is a positive constant, α_i and β_j are the coefficients for ARCH and GARCH terms.

For GARCH models, one of the disadvantages is that they assume the effects of positive and negative shocks on volatility are equal. In reality, financial asset prices tend to react differently to positive and negative shocks. The Glosten-Jagannathan-Runkle (GJR) model would be more useful in modeling volatility clustering as it accounts for asymmetric responses in volatility by including the leverage effect. The GJR model is written as:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{s} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{r} \beta_j \sigma_{t-j}^2 + \sum_{i=1}^{s} \gamma_i I_t (\varepsilon_{t-1} < 0) \varepsilon_{t-i}^2 \tag{4}$$

where I_t is the dummy variable with $I_t = \begin{cases} 1 & \text{if } \varepsilon_t < 0 \\ 0 & \text{otherwise} \end{cases}$.

2.2 *Model Selections and Forecast Evaluations*

The model selection process relies on two information criteria: the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). The model with the lowest AIC and BIC values is deemed the most suitable. Once the optimal model is identified, accuracy measures are applied to assess its forecasting performance. Forecast error, which represents the difference between the observed and predicted values, is evaluated using two widely adopted metrics: mean absolute error (MAE) and root mean squared error (RMSE). For forecast evaluation, the final 100 observations of each period will be reserved, with the selected model used to predict these 100 one-step-ahead values. Following this, the MAE and RMSE metrics will be computed to compare the performance of both the best and second-best models for each period.

2.3 *Risk Determination*

Value at Risk (VaR) is a statistical measure used to estimate the potential loss of an investment, given a specified confidence level, over a defined time horizon.It provides investors with a single number that summarizes the total risk, enabling them to assess and compare the risks associated with different investments. VaR can be interpreted as the level of confidence, $X\%$ that the loss amount will not exceed the amount of VaR in the next N days. For example, a 10-day 95% VaR of $1 million means that there is a 95% confidence level that the investment will not lose more than $1 million in the next 10 days. In practice, risk analysts typically calculate VaR with N using the following formula:

$$N\text{-}day\ VaR = 1\text{-}day\ VaR \times \sqrt{N} \tag{5}$$

There are various methods available to calculate VaR, including historical simulation, Deltanormal, Monte-Carlo simulation, and the ARMA-GARCH method. In this report, the ARMA-GARCH method is employed to calculate VaR for WTI and S&P 500 returns during two specific crises. Suppose that the residuals are normally distributed. In this case, the 1-day VaR at X % confidence level can be calculated as follows:

$$X\%\ 1\text{-}day\ VaR = Capital \times [\hat{r}_t(1) \times z\hat{\sigma}_t(1)] \tag{6}$$

where z is the z-score for Xth percentile, $\hat{r}_t(1)$ and $\hat{\sigma}_t(1)$ are the one-step-ahead predicted return and predicted standard deviation obtained from the forecast result by using the respective fitted model. Under student-t distribution, the standardised distribution,

$$t_v^*(p) = \frac{t_v(p)}{\sqrt{\dfrac{v}{v-2}}} \tag{7}$$

$t_v(p)$ is the critical value of t distribution with probability p. Then the VaR is calculated by using the formula:

$$X\%\ 1\text{-}day\ \ VaR = Capital \times [\hat{r}_t(1) \times t_v^*(p)\hat{\sigma}_t(1)] \tag{8}$$

3.0 Empirical study

3.1 *Data Collection*

A key objective of this study is to analyze the impact of major crises on different markets, with a focus on two recent events: the pandemic and the war. The analysis will evaluate how these crises have affected West Texas Intermediate (WTI) crude oil prices and the S&P 500 stock index. The daily price data for WTI is sourced from the U.S. Energy Information Administration, while data for the S&P 500 is obtained from Yahoo Finance. Each dataset is segmented into four distinct periods, as outlined in Table 1.

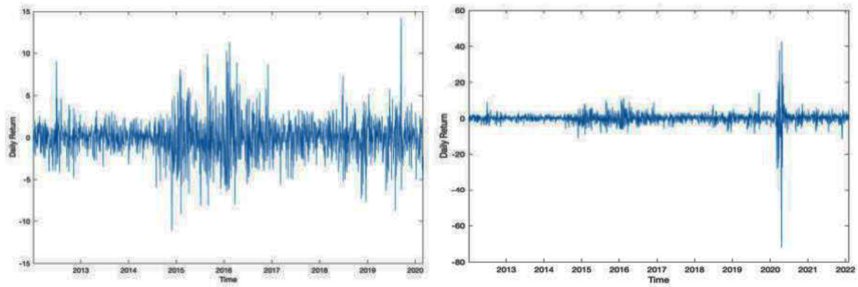
Table 1

Time Frame of Each Event

WTI Crude Oil		
Events	**Periods**	**Daily Prices**
Before Pandemic	Jan 2012- Jan 2020	2030
During Pandemic	Jan 2012- Jan 2022	2531
Before war	Jan 2012- Jan 2022	2531
During war	Jan 2012- Nov 2022	2739
S&P 500 Stock Index		
Events	**Periods**	**Daily Prices**
Before Pandemic	Jan 2012- Jan 2020	2033
During Pandemic	Jan 2012- Jan 2022	2537
Before war	Jan 2012- Jan 2022	2537
During war	Jan 2012- Nov 2022	2746

3.2.1 WTI Crude Oil: Before and During Pandemic

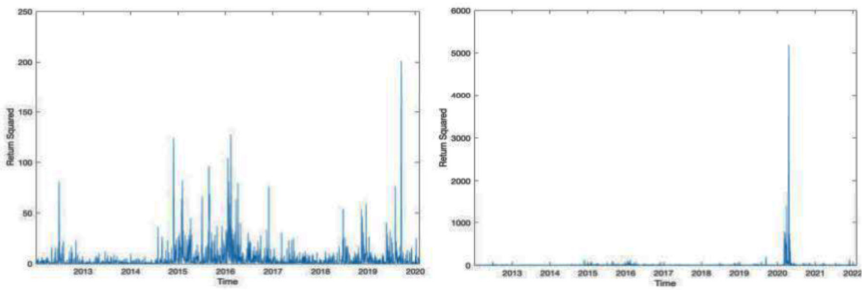
From Figure 1, the variance of the return series appears to be smaller in 2013 and 2014, followed by a greater variance from 2015 to 2017. Concerning the returns of WTI crude oil amid the pandemic, there is evident volatility clustering, particularly around March 2020, coinciding with the unprecedented occurrence of WTI crude oil prices dropping to negative values for the first time in history.



(a) Before Pandemic (b) During Pandemic

Figure 1

Daily return series of WTI Crude Oil



(a) Before Pandemic (b) During Pandemic

Figure 2

Volatility plots of WTI Crude Oil

It can also be observed from volatility plots in Figure 2(a) that there is volatility occurring around the period of 2015 to 2017. Additionally, from Figure 2(b), it is evident that a single peak volatility occurred in 2020, which was attributed to the sharp decrease in oil prices. These findings indicate that the pandemic had the most significant impact on WTI crude oil in the last 10 years (from 2012 to 2022).

As shown in Table 2, both the average daily return of WTI crude oil before and during the pandemic are negatives. The noticeable increase in the value of variance indicates higher volatility resulting from the coronavirus outbreak. Additionally, the skewness value has changed from positive to negative. With negative skewness, investors would expect frequent small gains and a few large losses. In comparison to a normal distribution with a kurtosis value of 3, both periods of returns exhibit heavier tails. However, the kurtosis value of WTI returns during the pandemic is significantly greater than 3 compared to that before the pandemic. The large kurtosis value suggests a higher likelihood of experiencing extreme return observations more frequently during the pandemic.

Table 2

Descriptive statistics of WTI returns

Parameters	Before Pandemic	During Pandemic
Mean	–0.0341	–0.0057
Median	0.0399	0.0824
Variance	4.4671	10.8495
Standard deviation	2.1136	3.2939
Skewness	0.2024	3.1426
Kurtosis	6.9059	121.5529

By observing histograms with normal fitting from Figure 3, the peaks of the histograms are higher compared to those of a normal distribution, and they also exhibit a heavy tail. In contrast, the histograms before and during the pandemic show a better fit with a student-t distribution. This

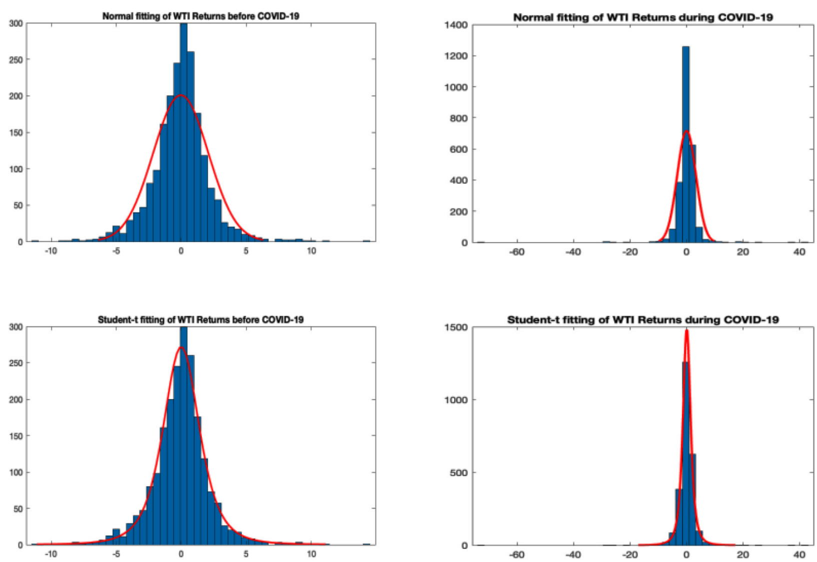


Figure 3

Histograms of WTI Crude Oil

indicates that a student-t fitting is more appropriate for the return series in both periods.

From the normal Q-Q plots as shown in Figure 4, it can be observed that both tails deviate from the straight line, indicating the presence of fat tails in the distribution. While there are some points at the tails of the WTI return series with student-t fitting that also deviate from the straight line, they exhibit a better fit with the student-t distribution compared to the normal distribution.

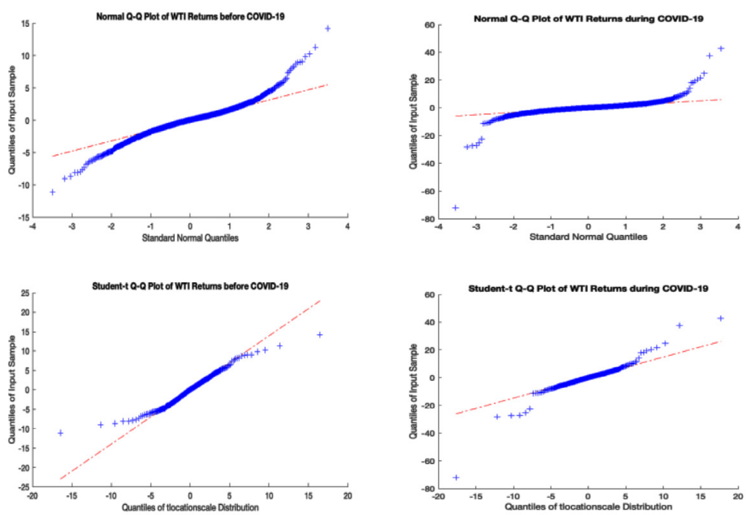


Figure 4

Q-Q plots of WTI Crude Oil (Covid-19)

The parameters of the normal distribution and student-t distribution for WTI returns before and after the pandemic are estimated using maximum likelihood estimation and summarized in Table 3.

Table 3

Parameters estimation for WTI Crude Oil

Before the COVID-19 pandemic			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	−0.0341	2.1130	–
Student-t	−0.0001	1.4459	3.4482
During the COVID-19 pandemic			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	−0.0057	3.2932	–
Student-t	0.0651	1.4379	2.5378

3.2.2 S&P500: Before and During Pandemic

Similar observations have been noted for the S&P 500, mirroring those observed for WTI crude oil. As presented in Table 4, the S&P 500 Index demonstrates positive average returns both before and during the pandemic period. The variance increases from 0.6524 to 1.0718, indicating a rise in the variability of stock index returns during the pandemic. Additionally, there is a notable increase in the kurtosis value. Since the kurtosis values exceed that of a normal distribution (which has a kurtosis value of 3), the return series of the S&P 500 index before and during the pandemic outbreak seem to follow a student-t distribution.

Furthermore, the kurtosis value of S&P 500 returns is close to that of WTI for the period before pandemic, but the latter is significantly greater than that of the S&P 500 index after the pandemic outbreak (121.5529 > 24.6187). This observation suggests that WTI crude oil carries a higher level of risk compared to the S&P 500 Index.

Table 4

Desciptive statistics of S&P 500 Index returns

Parameters	**Before COVID-19**	**During COVID-19**
Mean	0.0456	0.0498
Median	0.0547	0.0671
Variance	0.6524	1.0718
Standard deviation	0.8077	1.0353
Skewness	−0.4523	−0.9844
Kurtosis	6.2542	24.6187

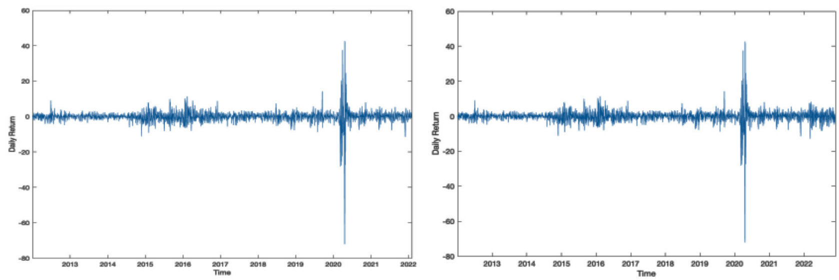
Table 5

Parameters estimation for S&P 500 Index

Before the COVID-19 pandemic			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	0.0456	0.8075	–
Student-t	0.0755	0.5389	3.1739
During the COVID-19 pandemic			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	0.0498	1.0351	–
Student-t	0.0873	0.5635	2.5986

3.2.3 WTI Crude Oil

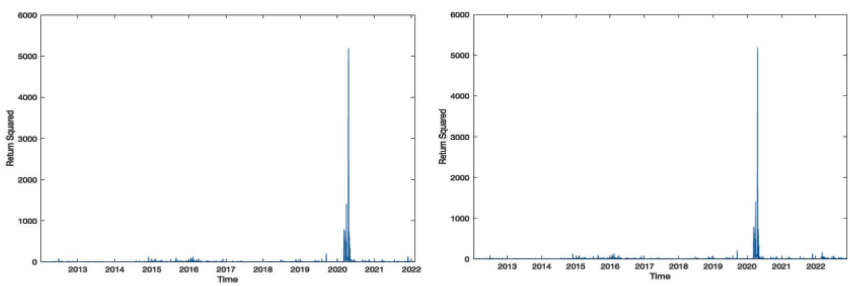
Figure 5 and Figure 6 illustrated the return series and volatility for WTI. Upon closer examination, the pattern of the return series for WTI crude oil during the war appears similar to that before the conflict. This observation might imply that the pandemic had a more pronounced effect on the volatility of WTI crude oil compared to the war. However, upon further scrutiny, fluctuations are still evident around March 2022, coinciding with the period when the price of WTI crude oil surged to new highs due to uncertainties in oil supply stemming from Russia's invasion of Ukraine.



(a) Before war (b) During war

Figure 5

Daily return series of WTI Crude Oil before and during War



(a) Before War (b) During War

Figure 6

Volatility plots of WTI Crude Oil

Examining the descriptive statistics in Table 6, the average daily return of WTI crude oil is negative for both periods before and after the Russian invasion of Ukraine. There is minimal difference in the variance and standard deviation before and after the invasion, indicating that the outbreak of the war did not result in a significant increase in volatility. The existing volatility caused by the pandemic appears to have overshadowed any additional volatility stemming from the war.

Furthermore, the kurtosis value of WTI returns before and during the invasion was much greater than 3, indicating heavier tails than a normal distribution. The elevated kurtosis value before the war may be attributed to the lingering impact of the pandemic.

Table 6

Desciptive statistics of WTI returns

Parameters	Before Russia-Ukraine War	During Russia-Ukraine War
Mean	−0.0057	−0.0106
Median	0.0824	0.0817
Variance	10.8495	10.8017
Standard deviation	3.2939	3.2866
Skewness	−3.1426	−2.9509
Kurtosis	121.5529	113.5585

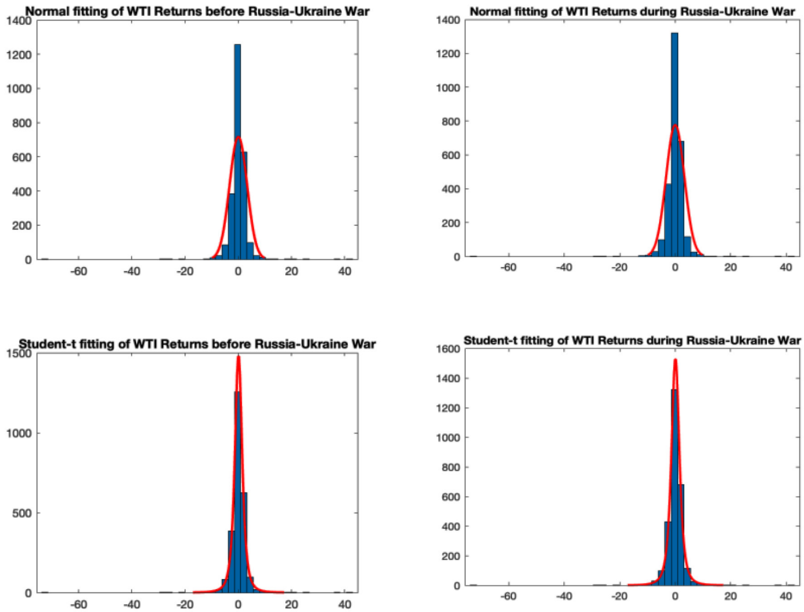


Figure 7

Histograms of WTI Crude Oil (Russia-Ukraine War)

From the histograms in Figure 7, it can be seen that the WTI return series fit better to student-t distribution than normal distribution. The the Q-Q plots in Figure 8(b) further confirm that the distribution of the WTI returns during the war fits better with a student-t distribution.

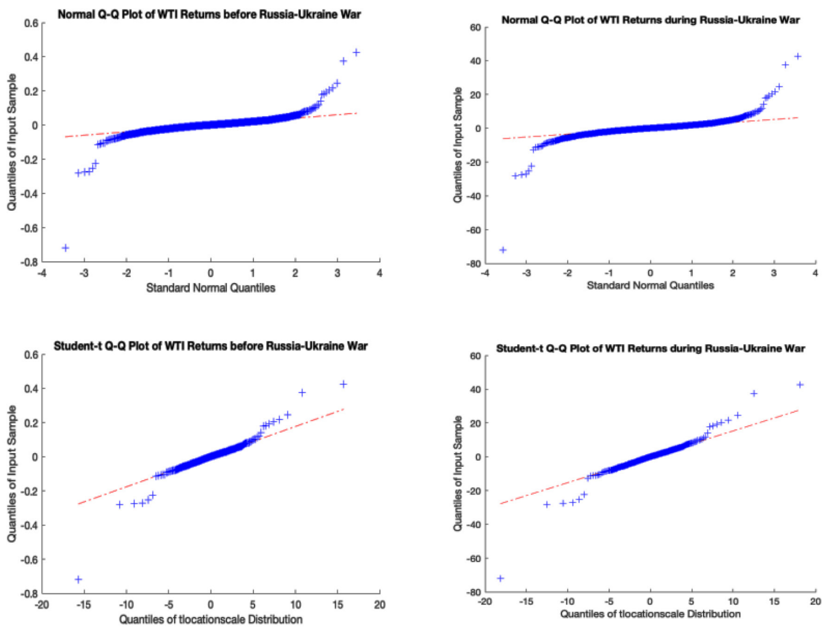


Figure 8

Q-Q plots with Normal and Student-t Fitting of WTI Crude Oil (War)

Table 7 shows the parameters estimation of normal and student-t distributions for WTI returns before and after the war.

Table 7

Parameters estimation of normal and student-t distributions for WTI returns (War)

Before Russia-Ukraine War			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	−0.0057	3.2932	–
Student-t	0.0651	1.4379	2.5378
During Russia-Ukraine War			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	−0.0106	3.2860	–
Student-t	0.0648	1.5126	2.6095

3.2.3 S&P500

By observing the return series of S&P 500 Index before and during the war, there are clear fluctuations around February to March 2022 when

Russia invaded Ukraine. However, the fluctuations caused by the war are smaller than those caused by the pandemic.

According to Table 8, the average return of WTI crude oil shows a slight decrease, and there is only a slight increase in the variance compared to the period before the war. The negative skewness observed in the oil prices before and during the war suggests that the prices tend to have larger and more significant downward movements when negative shocks occur.

Additionally, both the period before and during the war exhibit larger kurtosis values compared to that of a normal distribution, indicating a heavier tail. However, the decrease in kurtosis during the war implies a reduction in the occurrence of extreme events, suggesting a relatively more stable price behavior compared to the pre-war period.

It's noteworthy that the kurtosis value of the stock index during the war is significantly smaller than that of WTI crude oil (19.7973 < 113.5585). This implies that the WTI crude oil market carries a higher level of risk and is more susceptible to extreme price movements compared to the S&P 500 Index during the war period.

Table 8

Desciptive statistics of S&P 500 Index returns

Parameters	Before Russia-Ukraine War	During Russia-Ukraine War
Mean	0.0498	0.0412
Median	0.0671	0.0594
Variance	1.0718	1.1793
Standard deviation	1.0353	1.0860
Skewness	−0.9844	−0.8301
Kurtosis	24.6187	19.7861

Table 9

Parameters estimation of normal and student-t distributions for S&P 500 returns

Before Russia-Ukraine War			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	0.0498	1.0351	–
Student-t	0.0873	0.5635	2.5986
During Russia-Ukraine War			
Distribution	**Mean**	**Standard deviation**	**Degrees of freedom**
Normal	0.0412	1.0858	–
Student-t	0.0798	0.6035	2.5805

Table 9 reported the estimated parameters of normal and student-t distributions. As a short summary of this chapter, it can be observed that WTI crude oil displays negative returns for all four periods (before and during the pandemic, and before and during the war). On the other hand, the S&P 500 Index demonstrates positive returns during these same periods. Moreover, all four periods of each dataset exhibit heavier tails than a normal distribution, indicating a better fit with a student-t distribution. This suggests that extreme events or outliers are more likely to occur in these periods. Based on the previously discussed variance and kurtosis values, it can be concluded that the pandemic had a greater impact on both WTI crude oil and the S&P 500 Index compared to the war. The higher kurtosis value during both the pandemic and the war indicates that WTI crude oil carries a higher level of risk than the S&P 500 Index.

3.3 *Randomness Test*

Table 10

Results of the ADF test for WTI and S&P 500 daily prices and returns

WTI Crude Oil		
Series	**p-value**	**Test statistics**
Prices	0.3003	−0.9599
Returns	0.003	−58.9843
S&P500 Stock Index		
Series	**p-value**	**Test statistics**
Prices	0.9531	1.3193
Returns	0.003	−59.9883

Table 11

Serial Correlation test of Ljung-Box Q-test for WTI and S&P 500 returns

WTI Crude Oil			
Lags	**5**	**10**	**15**
p-value	0	0	0
Q-statistics	104.8550	167.2308	220.5338
S&P500 Stock Index			
Lags	**5**	**10**	**15**
p-value	0	0	0
Q-statistics	87.0173	269.9321	317.6716

The entire dataset, spanning from January 2012 to November 2022, is employed to execute three randomness tests on the return series of WTI crude oil and the S&P 500 stock index. Based on the results presented in Table 10 and 11, since all the p-values are extremely small, the null hypothesis is rejected. There is sufficient evidence to conclude that there is serial correlation present in both the WTI and S&P500 return series at lags 5, 10 and 15. Consequently, the return series of WTI and S&P 500 index do not follow a random walk. ARMA(p, q) model would be appropriate to model these return series.

Referring to Table 12, the p-values are all less than $\alpha = 0.05$. The null hypothesis is rejected, and thus there is enough evidence to conclude that the WTI return series is not a random walk.

$H_0 : \varepsilon_t$ is independent and identically distributed. $H_1 : \varepsilon_t^2$ is correlated.

Table 12

Results of the Variance Ratio Test for WTI returns

Lags	2	4	8
p-value	0.0000	0.0000	0.0011
Test statistics	−6.2247	−5.7224	−3.2738
Ratio	0.8810	0.7953	0.8149

As for S&P 500 Index in Table 13, all the p-values are less than $\alpha = 0.05$. Hence, the null hypothesis is rejected, indicating the return series S&P 500 Index is not a random walk.

$H_0 : \varepsilon_t$ is independent and identically distributed. $H_1 : \varepsilon_t^2$ is correlated.

Table 13

Variance Ratio Test for S&P500 returns

Lags	2	4	8
p-value	0.0000	0.0004	0.0014
Test statistics	−7.2433	−3.5623	−3.1979
Ratio	0.8618	0.8728	0.8194

In summary, the return series for both the indices exhibit non-independent and non-identically distributed characteristics, indicating a heteroskedastic effect in the variance of the returns. Overall, considering the outcomes of the three randomness tests, it can be concluded that the return series of both WTI crude oil and the S&P 500 index do not conform to a random walk process. Hence, the application of modeling is deemed valuable for additional analysis and making future predictions.

3.4 *Estimations, model selection and model diagnose*

Before fitting the model, the Ljung-Box Q-test is conducted on each period (before and during pandemic, before and during war) for the WTI crude oil and S&P 500 returns. The results of the tests show that the return series in each period are serially correlated starting from lag 1. Along with the results of the ARCH test in the previous section, ARMA(p, q)-GARCH(r, s) models will be used to fit the models for each period. The

Box-Jenkins methodology will be applied for model selection and evaluation. The Box-Jenkins methodology involves several steps, including identification, parameter estimation, model selection, and model diagnostics.

3.4.1 The Covid-19 Period

3.4.1.1 WTI Crude Oil Series before pandemic

The selection of the optimal model relies on the AIC and BIC values. The model displaying the lowest AIC and BIC values is deemed the best. Table 14 showcases four distinct models along with their corresponding AIC and BIC values. Among these models, the ARMA(2,2)-GJR(1,1) model featuring a student-t distribution demonstrates the lowest AIC and BIC values, establishing it as the best model among the available options.

Table 14

AIC and BIC results (WTI retruns before Pandemic)

Models	Distribution	AIC	BIC
ARMA(2,2)-GARCH(1,1)	Normal	8334.6	8368.3
ARMA(2,2)-GARCH(1,1)	Student-t	8197.1	8236.4
ARMA(2,2)-GJR(1,1)	Normal	8296.7	8336.0
ARMA(2,2)-GJR(1,1)	Student-t	8179.4	8224.3

Table 15 shows that the p-value of each coefficient is less than significance level of 0.05, except for the constant value in the mean equation and the coefficient of the ARCH term (α_1). The GARCH term is statistically significant, while the ARCH term is not, indicating that the volatility before pandemic is highly persistent. This suggests that changes in volatility have a long-lasting effect, and volatility clusters over time. Additionally, the leverage effect is significant, indicating that negative shocks in prices have a greater impact on volatility than positive shocks.

The selected model, ARMA(2,2)-GJR(1,1) with student-t distribution is expressed as:

$$r_t = 0.0090304 - 0.89681 r_{t-2} + 0.90711 \varepsilon_{t-2} + \varepsilon_t$$

$$\sigma_t^2 = 0.020084 + 0.95271 \sigma_{t-1}^2 + 0.01019 \varepsilon_{t-1}^2 + 0.063979 I_{t-1} \varepsilon_{t-1}^2$$

where $I_t = \begin{cases} 1 & \text{if } \varepsilon_t < 0 \\ 0 & \text{otherwise} \end{cases}$

Table 15

Parameters estimation of the selected best model for WTI returns before pandemic

ARMA(2,2)-GJR(1,1) with Student-t Distribution (before COVID-19)			
Parameters	**Coefficient**	**Standard Error**	**P-value**
Mean Equation			
ϕ_0	0.0090304	0.066131	0.89138
ϕ_2	-0.89681	0.058225	0.000
θ_2	0.90711	0.055615	0.000
Variance Equation			
ω	0.020084	0.0088069	0.022579
β_1	0.95271	0.0080026	0
α_1	0.01019	0.0083553	0.22262
γ_1	0.063979	0.01377	0.000
ν	6.0069	0.74799	0.000

After choosing the most suitable model, a diagnostic test was conducted on the residuals of the model to further assess its appropriateness. An appropriate model should have residual series that are not serially correlated. From Table 16, since p-values that are greater than 0.05 at lags 5, 10 and 15, there is insufficient evidence to conclude that the residuals are serially correlated. This suggests that the residual series could resemble white noise. Additionally, the result of the ARCH test on the residuals indicates the absence of ARCH effects. Therefore, the chosen model appears to be appropriate.

Table 16

Diagnostic test on the residuals of the selected best model for WTI before pandemic

Serial Correlation Test			
Lags	**5**	**10**	**15**
P-value	0.4233	0.7013	0.8914
Test statistics	4.9399	7.2536	8.7254
ARCH Test			
Lags	**5**	**10**	**15**
P-value	0.8716	0.8940	0.9216
Test statistics	1.8339	4.9568	8.0548

3.4.1.2 WTI Crude Oil series during Pandemic

Potential models were identified using a trial and error approach, and their corresponding AIC and BIC values are presented in Table 17.

ARMA(4, 4)-GJR(1, 1) model with student-t distribution has the lowest AIC and BIC values among the four potential models, making it the preferred choice as the best model for the WTI returns during the pandemic.

Table 17

AIC and BIC values of the potential models (WTI returns during Pandemic)

Models	Distribution	AIC	BIC
ARMA(4, 4)-GARCH(1,1)	Normal	10886	10921
ARMA(4, 4)-GARCH(1,1)	Student-t	10645	10686
ARMA(4, 4)-GJR(1,1)	Normal	10834	10875
ARMA(4, 4)-GJR(1,1)	Student-t	10628	10675

Based on the parameters estimated, the model can be expressed as:

$$r_t = 0.013936 - 0.759785 r_{t-4} - 0.74618 \varepsilon_{t-4} + \varepsilon_t$$

$$\sigma_t^2 = 0.11138 + 0.87512 \sigma_{t-1}^2 + 0.052583 \varepsilon_{t-1}^2 + 0.10264 I_{t-1} \varepsilon_{t-1}^2$$

with degrees of freedom, $\nu = 5.4474$.

Unlike the selected model for the period before pandemic, both the GARCH and ARCH terms in the model during pandemic are statistically significant. However, the magnitude of the GARCH term is much larger than that of the ARCH term, suggesting highly persistent volatility and relatively less significant impact of recent shocks on current volatility. The results of the serial correlation test and the ARCH effect test support that the chosen model is appropriate

3.4.1.3 S&P 500 series before Pandemic

After careful experimentation, it is observed that the AR(1) model is suitable. In Table 18, the AR(1)-GJR(1, 1) model with a student-t distribution

Table 18

AIC and BIC values of the potential models (S&P 500 returns before Covid-19)

Models	Distribution	AIC	BIC
AR(1)-GARCH(1, 1)	Normal	4475.1	4503.2
AR(1)-GARCH(1, 1)	Student-t	4341.2	4374.9
AR(1)-GJR(1, 1)	Normal	4364.8	4398.5
AR(1)-GJR(1, 1)	Student-t	4247.3	4286.7

exhibits the lowest AIC and BIC values. Consequently, it is selected as the most suitable model for the S&P 500 Index.

By estimating the coefficients, all the coefficients are statistically significant except for the ARCH term. Since the coefficient of the ARCH term is extremely small (2×10^{-12}), it is not included in the fitted model. ARMA(1,0)-GJR(1, 1) model with student-t is expressed as:

$$r_t = 0.063731 - 0.04675 r_{t-1} + \varepsilon_t$$
$$\sigma_t^2 = 0.033892 + 0.78516\sigma_{t-1}^2 + 0.34441 I_{t-1}\varepsilon_{t-1}^2$$

with degrees of freedom, $\nu = 5.7206$.

The diagnostic test suggests that the residuals are not serially correlated and the ARCH effect is not present. Thus, the choice of model appears to be appropriate.

3.4.1.4 S&P 500 series during Pandemic

The possible models with each AIC and BIC values are displayed in Table 19.

Table 19

AIC and BIC results (S&P 500 returns during Pandemic)

Models	Distribution	AIC	BIC
AR(1)-GARCH(1, 1)	Normal	6017.4	6046.6
AR(1)-GARCH(1, 1)	Student-t	5838.2	5873.3
AR(1)-GJR(1, 1)	Normal	5938.8	5973.8
AR(1)-GJR(1, 1)	Student-t	5742.6	5783.5

AR(1)-GJR(1,1) model with student-t is chosen as the best model. All the estimated coefficients are stastically significant except for the ARCH term. The fitted model is:

$$r_t = 0.070757 - 0.047256 r_{t-1} + \varepsilon_t$$
$$\sigma_t^2 = 0.034545 + 0.79278\sigma_{t-1}^2 + 0.00234870\varepsilon_{t-1}^2 + 0.34452 I_{t-1}\varepsilon_{t-1}^2$$

with degrees of freedom, $\nu = 5.4038$.

From the results of the diagnostic test, the residuals of the best model are not serially correlated and do not show evidence of ARCH effects. After obtaining the best models for each period before and during pandemic, we can use them to analyse the impact of the event on the WTI and S&P 500 returns.

3.4.1.5 Discussions

Table 20

Comparison of estimated coefficients for best models before and during pandemic

WTI Crude Oil		
Estimates	**Before COVID-19**	**During COVID-19**
ω	0.020084	0.11138
β_1	0.95271	0.87512
S&P 500 Index		
Estimates	**Before COVID-19**	**During COVID-19**
ω	0.033892	0.034545
β_1	0.78516	0.79278

For the WTI returns, the estimated coefficient of the constant term in the variance equation has increased significantly from 0.020084 to 0.11138 due to the impact of the pandemic outbreak. This suggests that the long-run average variance of the return series has increased, indicating higher risk in the WTI crude oil market during the pandemic. Additionally, the coefficient of the GARCH term has decreased by 8.1441% (from 0.95271 to 0.87512) as shown in Table 20, indicating a reduction in the persistence of volatility due to the impact of the pandemic pandemic. In other words, previous periods of high volatility may have less influence on the current levels of volatility. In contrast, the coefficient of the constant term in the variance equation for S&P 500 returns only decreases slightly with the impact of the pandemic. Unlike the GARCH term estimated coefficient for WTI returns, which decreases in value during pandemic, the estimated coefficient of the GARCH term for S&P 500 returns has increased slightly, indicating a small increase in volatility persistence. It is worth noting that the leverage terms are statistically significant before and during pandemic for both WTI and S&P 500 returns. The positive leverage for both return series in each period indicates that bad news is more likely to have an impact on WTI and S&P 500 returns than good news.

3.4.2 The Russia-Ukraine War Period

3.4.2.1 WTI Crude Oil Before Russia-Ukraine War

The period of WTI crude oil return series before war is the same as the period during pandemic. Therefore, the same fitted model is used for the WTI returns before the war.

3.4.2.2 WTI Crude Oil During Russia-Ukraine War

The order of ARMA model is selected using the trial and error method. Four possible models are considered, and their respective AIC and BIC values are summarized in Table 21.

Table 21

AIC and BIC Results (WTI returns during War)

Models	Distribution	AIC	BIC
ARMA(4, 4)-GARCH(1,1)	Normal	11951	11987
ARMA(4, 4)-GARCH(1,1)	Student-t	11713	11755
ARMA(4, 4)-GJR(1,1)	Normal	11916	11958
ARMA(4, 4)-GJR(1,1)	Student-t	11702	11749

The chosen best model: ARMA(4,4)-GJR(1,1) with student-t distribution, is express as:

$$r_t = 0.010281 + 0.815835 r_{t-4} - 0.79693\varepsilon_{t-4} + \varepsilon_t$$

$$\sigma_t^2 = 0.10907 + 0.87238\sigma_{t-1}^2 + 0.06912\varepsilon_{t-1}^2 + 0.084419 I_{t-1}\varepsilon_{t-1}^2$$

with degrees of freedom, $\nu = 5.5346$.

After diagnostic test, the residual series of the model has neither serial correlation nor ARCH effect, suggesting that the choice of model is appropriate.

3.4.2.3 S&P500 Before Russia-Ukraine War

Given that the time periods during the pandemic and before the war are identical, the same model—AR(1)-GJR(1, 1) with a student-t distribution—is applied for the S&P 500 returns before the war.

3.4.2.4 S&P500 During Russia-Ukraine War

Table 22 summarizes the four possible models with their respective AIC and BIC values. AR(1)-GJR(1,1)-student t model with is chosen as the

Table 22

AIC and BIC Results (S&P 500 returns during Russia-Ukraine War)

Models	Distribution	AIC	BIC
AR(1)-GARCH(1,1)	Normal	6825.6	6861.1
AR(1)-GARCH(1,1)	Student-t	6649.8	6691.2
AR(1)-GJR(1,1)	Normal	6749.0	6790.4
AR(1)-GJR(1,1)	Student-t	6555.8	6603.2

best model for the S&P 500 return series during the war as it has the lowest values of AIC and BIC.

The fitted AR(1)-GJR(1,1)-student t is:

$$r_t = 0.06752 - 0.042426 r_{t-1} + \varepsilon_t$$

$$\sigma_t^2 = 0.028114 + 0.81228\sigma_{t-1}^2 + 0.010846\varepsilon_{t-1}^2 + 0.31564 I_{t-1}\varepsilon_{t-1}^2$$

with degrees of freedom, $\nu = 5.5821$.

Based on the coefficient estimation result, the ARCH term is not statistically significant. Since the GARCH term is statistically significant and its value is much larger than that of the ARCH term, it can be concluded that the volatility is highly persistent.

3.4.2.5 Discussions

Table 23

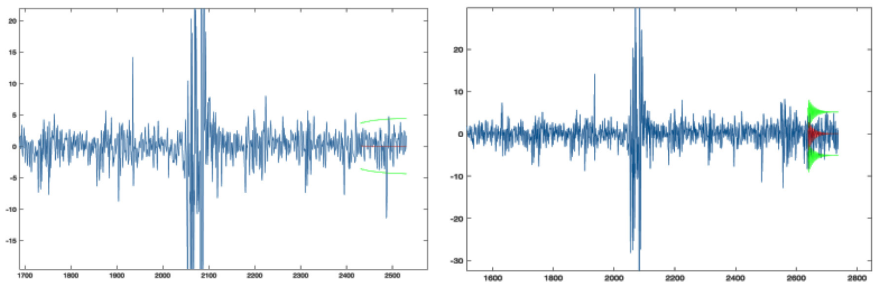
Comparison of estimated coefficients for best models before and during Russia-Ukraine War

WTI Crude Oil		
Estimates	**Before War**	**During War**
ω	0.11138	0.10907
β_1	0.87512	0.87238
S&P500 Index		
Estimates	**Before War**	**During War**
ω	0.034545	0.028114
β_1	0.79278	0.81228

In Table 23, it can be observed that the difference between the constant term in the variance equation for WTI returns before and during the war is 0.00231 (decreases from 0.11138 to 0.10907). On the other hand, there is also a very small decrease in the constant term value of S&P 500 returns (different by 0.006431), indicating that the volatility did not change much before and during the war. Hence, the event of the war only had a minor impact on both WTI crude oil and S&P 500 Index. As for the GARCH terms, the coefficient of the GARCH term for WTI returns decreases slightly from 0.87512 to 0.87238, indicating a decrease in volatility persistence due to the war. Conversely, the coefficient of the GARCH term for S&P 500 returns increases slightly from 0.79278 to 0.81228. For both WTI and S&P 500 returns before and during war, the leverage terms are statistically significant. The positive leverage term at each period suggests that when the value of WTI or S&P 500 Index decreases, the volatility will increase.

3.5 *Forecast Evaluations*

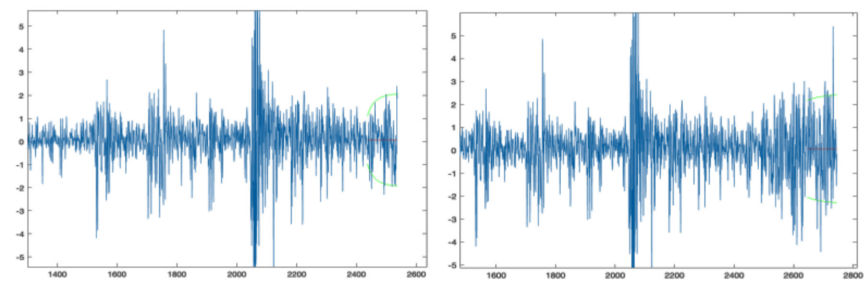
In reality, it is often more insightful to include all significant events and factors that influence volatility. Therefore, the models during the pandemic and during the war are used to forecast 100 values ahead of time. In Figure 9 and Figure 10, for each dataset, the last 100 actual observations are reserved as testing data, while the remaining data are used as the training set for forecasting.



(a) During Pandemic (b) During War

Figure 9

100-one-step-ahead forecast for WTI Crude Oil



(a) During Pandemic (b) During War

Figure 10

100-one-step-ahead forecast for S&P500 Index

For the WTI crude oil return series, it can be observed from Table 24 that the MAE and RMSE values for both the best models during pandemic and the war are slightly higher than those of the second-best model. One

Table 24

Forecast accuracy measures of the best and second-best models

WTI Crude Oil				
Events	**Models**	**Distribution**	**MAE**	**RMSE**
COVID-19	ARMA(4, 4)-GJR(1, 1) (Best)	Student-t	1.5629	2.1659
	ARMA(4, 4)-GARCH(1, 1)	Student-t	1.5578	2.1641
Russia-Ukraine War	ARMA(4, 4)-GJR(1, 1) (Best)	Student-t	2.2778	2.8148
	ARMA(4, 4)-GARCH(1, 1)	Student-t	2.2765	2.8140
S&P500 Index				
Events	**Models**	**Distribution**	**MAE**	**RMSE**
COVID-19	AR(1)-GJR(1,1) (Best)	Student-t	0.7464	0.9595
	AR(1)-GARCH(1,1)	Student-t	0.7463	0.9616
Russia-Ukraine War	AR(1)-GJR(1,1) (Best)	Student-t	1.1883	1.5133
	AR(1)-GARCH(1,1)	Student-t	1.1924	1.5144

possible reason for this could be that the last 100 observations reserved for forecast evaluation do not align well with the best model compared to the second-best model, as the volatility may have changed due to economic behavior. However, the difference in values is not significant. In contrast, for the S&P 500 returns during pandemic and the war, the best models have slightly lower MAE and RMSE values compared to the second-best models, which supports our choice of the best models.

3.6 *Risk Determination*

In the following calculations, we assume that the investment capital is \$1 million. The one-step-ahead return and variance obtained from the fitted ARMA(4, 4)-GJR(1, 1) with student-t distribution for WTI return series during pandemic are 0.01236 and 3.4573. The estimated degrees of freedom, ν from the model is 5.4474. Then, the 5% and 1% quantiles are:

5% quantile: $0.01236 - 1.5744(\sqrt{3.4573}) = -2.9150\%$

1% quatile: $0.01236 - 2.5874(\sqrt{3.4573}) = -4.7986\%$

Therefore, the VaR at the 95% and 99% confidence level are:

$$95\%\text{ 1-day VaR: }\$1{,}000{,}000 \times 2.9150\% = \$29{,}150$$

$$99\%\text{ 1-day VaR: }\$1{,}000{,}000 \times 4.7986\% = \$47{,}986.$$

Under 95% confidence, there will be a loss of not more than \$29, 150 in the next day. On the other hand, there is 99% confidence that the investor will lose not more than \$47, 986 in the next day. By applying the similar procedures, the 95% and 99% VaR for the WTI during the war, and S&P 500 during the pandemic outbreak and during the war are as follows:

VaR of WTI during War:

$$95\%\ \text{1-day VaR: } \$1{,}000{,}000 \times 3.2447\% = \$32{,}447$$

$$99\%\ \text{1-day VaR: } \$1{,}000{,}000 \times 5.9274\% = \$59{,}274.$$

VaR of S&P 500 during pandemic:

$$95\%\ \text{1-day VaR: } \$1{,}000{,}000 \times 0.7635\% = \$7{,}635$$

$$99\%\ \text{1-day VaR: } \$1{,}000{,}000 \times 1.3062\% = \$13{,}062.$$

VaR of S&P 500 during War:

$$95\%\ \text{1-day VaR: } \$1{,}000{,}000 \times 1.6231\% = \$16{,}231$$

$$99\%\ \text{1-day VaR: } \$1{,}000{,}000 \times 2.7013\% = \$27{,}013.$$

Upon computation, the results suggest that WTI crude oil carries a higher level of risk in comparison to the S&P 500 Index.

4.0 Conclusion

Analyzing market volatility is crucial for understanding the statistical behavior of financial markets, particularly considering the substantial macroeconomic impact of oil price volatility on market investors and policymakers. This report delves into the market volatility of WTI crude oil, examining the influence of two recent global crises. The inclusion of volatility analysis on the S&P 500 Index provides a comparative perspective for WTI crude oil. The datasets are segmented into four periods—before and during the pandemic, and before and during the war—to investigate how these events affect market volatility.

Descriptive statistics reveal that the average returns of WTI crude oil during the four periods are negative, contrasting with the positive average returns of the S&P 500 Index for the same periods. Additionally, the kurtosis value for each period exceeds 3, indicating a heavier tail than a normal distribution and suggesting a better fit with a student-t distribution. Analyzing the descriptive statistics and estimated coefficients of the ARMA-GARCH models for each period, the results indicate that both events induce market volatility in WTI crude oil and the S&P 500 Index. However, the pandemic exerts a more substantial impact on market volatility compared to the war.

Furthermore, both events have a greater impact on WTI crude oil than on the S&P 500 Index, highlighting the higher volatility of WTI crude oil. Additionally, a positive leverage effect is observed in each period, signifying that volatility reacts differently to significant price increases or decreases. In the final section, the assessment of market risk through Value at Risk (VaR) calculations demonstrates that WTI crude oil poses a higher potential for losses compared to the S&P 500 Index at the 95% and 99% confidence intervals.

In future work, it is recommended to explore more types of models, such as exponential GARCH (EGARCH), GJR-GARCH, and GARCH-M models to fit the data and then further analyse the market volatility in order to understand the market's statistical behaviour better.

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