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Optimizing transportation network through IFN ranking : A comprehensive approach

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Abstract

The model of an Intuitionistic Uncertain Numeral is important used for computing an uncertain number, and ordering IFNs poses a significant contest in transport. This paper objects to present a method for resolving intuitionistic uncertain transport difficulties using IFNs and applying it to triangular intuitionistic fuzzy numbers. This research introduces an innovative approach to optimizing transportation networks by leveraging Intuitionistic Fuzzy Numbers (IFNs). Real-world case studies validate the superiority of the proposed IFN-based system over traditional methods, emphasizing its ability to address uncertainties in transportation decision-making effectively. This study explores a novel position function intended for intuitionistic three-sided fuzzy numbers and utilizes it trendy a convex

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combination of the orthocentre of centroids of three triangles by dividing the trapezoidal shape and finding the centroid of the centroid of three triangles. The selected transportation problems of balanced types aim to obtain optimal solutions through the use of the orthocentre of centroids ranking methods in IFN, employing existing initial basic feasible solution, optimum solution, and comparison with existing methods. Numerical examples are provided. The study's outcomes provide a valuable tool for policymakers, researchers, and practitioners, enhancing the accuracy of decision models and fostering a deeper understanding of trade-offs in transportation network optimization.

Subject Classification: *90C08, 03E72, 90B10.*

Keywords: *Intuitionistic fuzzy numbers, Transportation problems, Ranking functions, Convex combinations, Orthocenter, Centroid.*

1. Introduction

In mathematics, transportation theory is concerned with the study of the transportation and allocation of resources. There is a need to transfer products from different sources to different destinations at minimum cost. The transference problematic exists a superior type of rectilinear programming problem which deals with optimal allocation of goods from origin to terminal, considering factors such as capacities, demands, and transportation costs, with main objective being to minimalize the transference charge while substantial source and demand constraints. The general transportation problem was originally developed by Hitchcock **[**4**]**, where parameters such as transportation cost, resource availability and demand exist in an uncertain manner. Uncertainty exists in most real-life situations, making it challenging to precisely determine transportation cost. Theory of fuzzy sets was introduced by Zadeh **[**1**]**, various authors have suggested several methods for ordering fuzzy number and is well known field for handling uncertainty. In such cases, the concept of fuzzy transportation problem naturally arises. Fuzzy transportation models offer valuable insights for decision-making in logistics and networks supply.

Numerous researchers have employed fuzzy set theory [5,6,7,8]. The generalized fuzzy number has been utilized by some of them. Intuitionistic fuzzy set proposed by Atanassov [8] such examples are by way of follows. Their sum will also be in the interval [0,1]. as well. Soft set provides a way for decision makers to make decision on transport cost by specifying acceptance and non-acceptance for the transport cost but the degree of satisfaction, degree of dissatisfaction helps decision makers in contradicting the degree of satisfaction with respect to distance provision for quantity and will also helps in taking decision whether to accept and not to accept the amount of transportation cost in any transportation problem. Thus, it would be more accurate to utilize intuitionistic fuzzy number rather than fuzzy number in order to caregiving uncertain data. Intuitionistic fuzzy sets were also investigated by many researchers [8,11,12]. Aggarwal and Gupta [9], Antony et al. [12]; on the other

hand, they developed their methods to address intuitionistic fuzzy transportation problems of type-2 .

Ranking Fuzzy Numbers is a key aspect of the study of the intuitionistic fuzzy set theory. For that, it is necessary to compare with the other intuitionistic fuzzy number to rank them by using a ranking function. Here are various methods used to rank intuitionistic fuzzy numbers such as membership ranking, non-membership ranking, and composite and distance-based ranking. by Babu et al. [2] The authors introduce a new way to order generalized fuzzy numbers based on centroids of centroids in 2012. Their method offers a systematic and effective approach to the comparison of Fuzzy numbers, which is beneficial to the complexity of Fuzzy logic systems.by Chhibber et al. In [10, 11], Authors solve the transportation problem of type-1 and type-2 using North west corner rule by using incentre of centroids.Gupta and Kumari [3] introduced a new solution for dealing with fuzzy transportation issues by considering transportation cost values as triangular intuitionistic fuzzy numbers using a ranking function in their Research paper .Many researchers have implemented the arithmetic operations and the rank function of the IFNs [13].

2. Background Information [8]

Intuitionistic fuzzy set $\tilde{A}$ in universal set X.

$$\tilde{A} = \{(x, m_a(x), n_a(x)) : x \epsilon X\}$$

where $m_a(x)$: X → [0, 1] and $n_a(x)$: X → [0, 1] $0 \le m_a(x) + n_a(x) \le 1$

$$\tilde{A} = (a, b, d; \check{a}, \check{b}, \check{d}).$$

$$\mu_A(x) = \begin{cases} \frac{x-a}{b-a}, & a \le x < b \\ \frac{a-x}{d-b}, & b \le x \le d \\ 0, & otherwise \end{cases}$$

$$\vartheta_A(x) = \begin{cases} \frac{\check{a}-x}{\check{b}-\check{a}}, & \check{a} \le x < \check{b} \\ \frac{x-\check{b}}{\check{d}-\check{b}}, & \check{b} \le x \le \check{d} \\ 0, otherwise \end{cases} \quad ; \quad \text{where } \acute{a} \le a \le \check{b} \le b \le d \le \check{d}.$$

"Triangular-IFN" $A = \left(a, \acute{a}, b, \acute{b}, d, \acute{d}\,; m_a, n_a\right)$ with $\acute{a} < a < \acute{b} < b < d < \acute{d}$ is modified "Trapezoidal-IFN"(Fig.1) . Their Membership and non-Membership functions are

$$\mu_A(x) = \begin{cases} \frac{(x-a_1)}{(b_2-a_1)} m_a, & a_1 \le x \le b_2 \\ m_a, & x = b_2 \\ \frac{(d_3-x)}{(d_3-b_2)} m_a, & b_2 \le x \le d_3 \end{cases}$$

and

$$\vartheta_A(x) = \begin{cases} \dfrac{(b'_2 - x) + (x - a'_1)n_a}{(b'_2 - a'_1)}, & a'_1 \le x \le b'_2 \\ n_a, & x = b'_2 \\ \dfrac{(x - b'_2) + (d'_3 - x)n_a}{(d'_3 - b'_2)}, & b'_2 \le x \le d'_3 \end{cases}$$

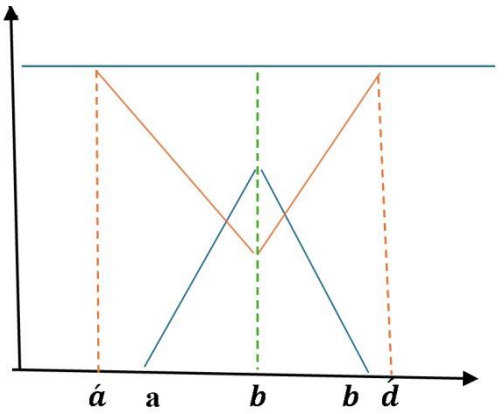


Figure 1
Generalized-"Triangular-IFN"

Arithmetic Operations

$A^I = (a_1, b_1, d_1; \acute{a_1}, \acute{b_1}, \acute{d_1})$ and $B^I = (a_2, b_2, d_2; \acute{a_2}, \acute{b_2}, \acute{d_2})$

(i) Addition: $A^I \oplus B^I = (a_1 + a_2, b_1 + \acute{b_2}, d_1 + d_2; a_1' + a_2', \acute{b_1} + \acute{b_2}, d_3' + d_3')$

(ii) Subtraction: $A^I - B^I = (a_1 - d_2, b_1 - b_2, d_1 - a_2; a_1' - d_2', \acute{b_1} - \acute{b_2}, \acute{d_1} - \acute{a_2})$

(iii) Multiplication: $A^I \otimes B^I = (a_1a_2, b_1b_2, d_1d_2; a_1' a_2', \acute{b_1}\acute{b_2}, d_1' d_2')$

LPP

A general LPP

Max (or) Min = $\sum c_i x_i$ (objective function)

Subject to conditions (constraints):

$$\sum \sum a_{ij} x_j$$

and non-negative restrictions $x_j \ge 0$, Variables $x_1, x_2, \ldots x_n$ are decision variables, the coefficients in the objective function $c_1, c_2, \ldots c_n$ are called as costs. $b_1, b_2, \ldots b_m$ are the availabilities and the coefficients a_{ij}'s are called resources.

Fuzzy Transportation Problem:

Fuzzy transportation problem(FTP) is an extension of transportation modeling approaches by incorporate fuzzy logic to address uncertainty in input

data. In the FTP, parameters such as supply, demand, and transportation cost are imprecise.

Consider 'm' companies or source producing a product and let there be 'n' destinations or places of requirement, requiring this product.

The objective function is

$$\text{Min } Z = \sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}\, X_{ij}$$

Subject to Constraints

$$\sum_{i=1}^{m} X_{ij} = a_i$$

$$\sum_{j=1}^{n} X_{ij} = b_j$$

$$X_{ij} \geq 0; \;\; \& \;\; a_i \geq 0, \;\; b_j \geq 0$$

This problem can be represented in a table 1.

Table 1

Transportation Problem

Destinations (j) Sources(i)	D_1	D_2	. . .	D_n	Availability (a_i)
S_1	$C_{11}X_{11}$	$C_{12}X_{12}$	. . .	$C_{1n}X_{1n}$	a_1
S_2	$C_{21}X_{21}$	$C_{22}X_{22}$	. . .	$C_{2n}X_{2n}$	a_2
. .	. .	. .	. .	. .	. .
S_m	$C_{m1}.X_{m1}$	$C_{m2}X_{m2}$	. . .	$C_{mn}X_{mn}$	a_m
Requirement(b_j)	b_1	b_2	. . .	b_n	$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

3. Methodology

This paper deals with Intuitionistic triangular fuzzy numbers using a ranking function. This ranking function is utilized to find Convex Combination point and is also employed in solving transportation problem to determine the minimum optimum cost when costs are represented by Intuitionistic triangular fuzzy numbers.

3.1 *Methods of finding initial basic feasible Solution (IBFS)and VAM*

Step-1: Take the difference of first lowest cost and second lowest cost for all the rows and columns. These differences are called as penalties.

Step-2: Select highest penalty (any row and column).

Step-3: Repeat the above steps until all the allocations have been made.

3.2 *MODI method*

Step-1: Obtain initial BFS

Assign variables u_i's to rows and v_j's to columns such that $u_i + v_j = c_j$ for basic cells.

Step-2: Solve the equations in step-1 to find u_i's vj's ($u_i = v_j = 0$ initially).

Step-3: For non-basic cells find $d_{ij} = c_{ij} - u_i + v_j$

Step-4: If $d_{ij} \leq 0$, then basic feasible Solution is optimal and if at least one $d_{ij} = 0$, then alternate optimal Solution exists.

Step-5: If at least one$d_{ij} > 0$, then the basic feasible Solution is not optimal. Now select largest positive d_{ij} and to maintain the total number of basic cells (m + n - 1), one of the existing basic variables has to be removed.

Step-6: Start from the cell of entering basic variable and move horizontally and vertically, take turns only at a basic cell and return back to the starting cell to get a closed loop. This loop will have corners as basic cells.

Step-7: Let θ be the maximum units that can be allocated at the entering variable cell. To maintain row and column totals, subtract θ and add θ from the row edges and column edges respectively. Find the minimum value of θ which is minimum of allocations made at the corners of the loop from which have been θ subtracted. This will be a new basic feasible solution.

Step-8: Test the new basic feasible Solution obtained in step-7 using step-2, 3, and 4. If it is not optimal, then repeat steps 5, 6, 7 until optimality is achieved.

4. New Ranking Methods using mean's of Orthocentre of centroids FOR MEMBERSHIP FUNCTION AND NON-MEMBERSHIP FUNCTION

For membership function (MF)

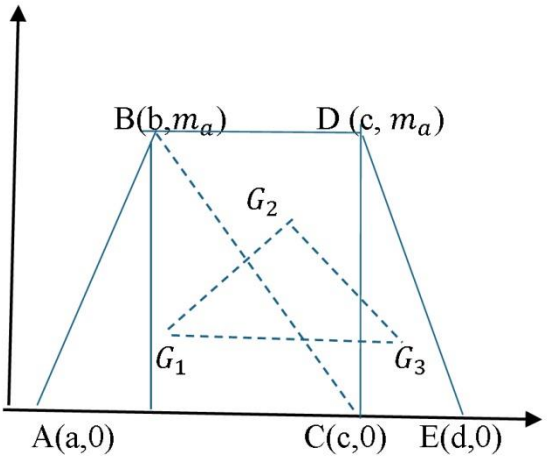


Figure 2
GTFN for Membership function

A^I= **(a, b, c, d; w) (Fig.2).**

$$G_1 = \left(\frac{(a+b+c)}{3}, \frac{m_a}{3}\right)$$

$$G_2 = \left(\frac{(2b+c)}{3}, \frac{2m_a}{3}\right)$$

$$G_3 = \left(\frac{(2c+d)}{3}, \frac{m_a}{3}\right).$$

Orthocentre $O_{A^I}(x,y)=\left(\frac{c+b}{2}, \frac{(b+2a-3c)(3b-2d-c)+2m_a^2}{6m_a}\right)$

$$G(x,y) = \left(\frac{(a+2b+5c+d)}{9}, \frac{4m_a}{9}\right).$$

$$O_{A^I}(x,y)=\left(b, \frac{(b-a)(d-b)+m_a^2}{3m_a}\right)$$

For non-membership (Non-MF)

Since the point of position for centroids of non-membership of the trapezoidal.

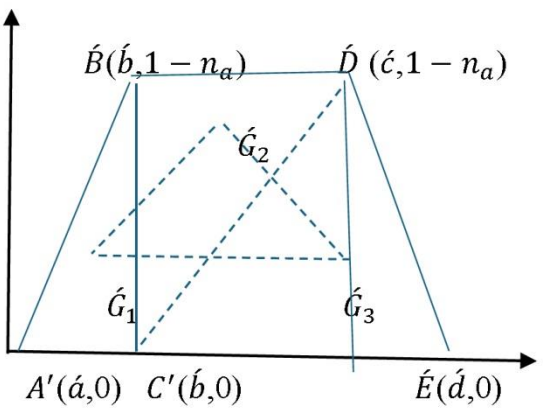


Figure 3
GTFN for non-membership

$A^{\acute{I}}$=($\acute{a},\acute{b},\acute{c},\acute{d}$; 1-n_a) (Fig.3).

$$\acute{G}_1 = \left(\frac{(\acute{a}+2\acute{b})}{3}, \frac{1-n_a}{3}\right)$$

$$\acute{G}_2 = \left(\frac{(2\acute{b}+\acute{c})}{3}, \frac{2(1-n_a)}{3}\right)$$

$$\acute{G}_3 = \left(\frac{(\acute{b}+\acute{c}+\acute{d})}{3}, \frac{1-n_a}{3}\right).$$

Orthocentre $\acute{O}_{A^I}(x,y)$ $=\left(\acute{b}, \frac{(\acute{b}-\acute{a})(\acute{d}-\acute{b})+(1-n_a)^2}{3(1-n_a)}\right)$

$$G' = (x,y) = \left(\frac{\acute{a}+2\acute{b}+5\acute{c}+\acute{d}}{9}, \frac{4(1-n_a)}{9}\right)$$

The mean' s of orthocentre of points OO' of "Triangular-IFN" using MF and non-MF is represented by

$$O = \left(\underline{X}_{A^I}, \underline{Y}_{A^I}\right) = \left(\frac{b+\acute{b}}{2}, \frac{1}{6}\left(\frac{(b-a)(d-b)+m_a^2}{m_a} + \frac{(\acute{b}-\acute{a})(\acute{d}-\acute{b})+(1-n_a)^2}{(1-n_a)}\right)\right)$$

$$\left(\underline{X}_{A^I}, \underline{Y}_{A^I}\right) = \left(\frac{(\mathrm{a}+\acute{a})+7(\mathrm{b}+\acute{b})+(\mathrm{d}+\acute{d})}{18}, \frac{4(1+m_a-n_a)}{18}\right)$$

Ranking index for orthocentre is defined by

$$R(O^I) = \underline{X}_{A^I} \times \underline{Y}_{A^I}$$

$$R(A^I) = \left(\frac{(\boldsymbol{b}+\acute{\boldsymbol{b}})}{\mathbf{12}} * \left(\frac{(\boldsymbol{b}-\boldsymbol{a})(\boldsymbol{d}-\boldsymbol{b})+\boldsymbol{m}_{\boldsymbol{a}}^{\mathbf{2}}}{\boldsymbol{m}_{\boldsymbol{a}}} + \frac{(\acute{\boldsymbol{b}}-\acute{\boldsymbol{a}})(\acute{\boldsymbol{d}}-\acute{\boldsymbol{b}})+(\mathbf{1}-\boldsymbol{n}_{\boldsymbol{a}})^{\mathbf{2}}}{(\mathbf{1}-\boldsymbol{n}_{\boldsymbol{a}})}\right)\right)$$

Ranking index for centroids is given by [2]

$$R(A^I) = \frac{1}{81}[(a_1 + \acute{a}) + 7(\mathrm{b} + \acute{b}) + (\mathrm{d} + \acute{d})] \times [(1 + m_a - n_a)]$$

Theorem: *If A is GITFN, then the convex combination of mean's of orthocentre and centroids point of centroids satisfy the following equations.*

1. $(\beta C_x + (1-\beta)O_x)_{-A} = -(\beta C_x + (1\text{-}\beta)O_x)_A$
2. $(\alpha C_y + (1-\alpha)O_y)_{-A} = (\alpha C_y + (1-\alpha)O_y)_A$

Proof: Where $C = \left(\underline{X}_{A^I}, \underline{Y}_{A^I}\right) = \left(\frac{(a+\acute{a})+7(\mathrm{b}+\acute{b})+(\mathrm{d}+\acute{d})}{18}, \frac{4(1+m_a-n_a)}{18}\right)$

$$O = \left(\underline{X}_{A^I}, \underline{Y}_{A^I}\right) = \left(\frac{b+\acute{b}}{2}, \frac{1}{6}\left(\frac{(b-a)(d-b)+m_a^2}{m_a} + \frac{(\acute{b}-\acute{a})(\acute{d}-\acute{b})+(1-n_a)^2}{(1-n_a)}\right)\right)$$

$$-A^I = (-a_1, -b_1, -d_1; -\acute{a}_1, -\acute{b}_1, -\acute{d}_1 : m_a, n_a)$$

then, $(\beta C_x + (1-\beta)O_x)_{-A} = -(\beta C_x + (1\text{-}\beta)O_x)_A$

$$= \left[\beta\left(\frac{(-a-\acute{a})+7(-\mathrm{b}-\acute{b})+(-\mathrm{d}-\acute{d})}{18}\right) + (1-\beta)\left(\frac{-b-\acute{b}}{2}\right)\right]$$

$$= \left[\beta\left(\frac{-(a+\acute{a})-7(\mathrm{b}+\acute{b})-(\mathrm{d}+\acute{d})}{18}\right) - (1-\beta)\left(\frac{b+\acute{b}}{2}\right)\right]$$

$$= -\left[\beta\left(\frac{(a+\acute{a})+7(\mathrm{b}+\acute{b})+(\mathrm{d}+\acute{d})}{18}\right) + (1-\beta)\left(\frac{b+\acute{b}}{2}\right)\right]$$

$$= -(\beta C_x + (1\text{-}\beta)O_x)_A$$

Similarly, we can solve for $(\alpha C_y + (1-\alpha)O_y)_{-A} = (\alpha C_y + (1-\alpha)O_y)_A$

- Convex Combination of mean' s of orthocentre of centroids of centroids and the centroid of the centroids of triangular intuitionistic fuzzy number

$$=\mathrm{S} = \beta(O_x + C_x) + (1-\beta)(C_y + O_y)$$

$$\mathrm{S} = \alpha\left(\frac{16(b+\acute{b})+(a+\acute{a})+(d+\acute{d})}{18}\right) + (1\text{-}\alpha)$$

$$\left[\frac{4(1+m_a-n_a)}{18}+\frac{1}{6}\left(\frac{2(b-a)*(d-b)+{m_a}^2}{m_a}+\frac{2(\acute{b}-\acute{a})*(\acute{d}-\acute{b})+(1-n_a)^2}{1-n_a}\right)\right]$$

where $0\leq \beta \leq 1$, the orthocentre of centroids of TIFN

$$O=\left(\underline{X}_{A^I},\underline{Y}_{A^I}\right)=\left(\frac{b+\acute{b}}{2},\frac{1}{6}\left(\frac{2(b-a)(d-b)+m_a^2}{m_a}+\frac{2(\acute{b}-\acute{a})(\acute{d}-\acute{b})+(1-n_a)^2}{(1-n_a)}\right)\right)$$

and the centroid of the centroids of TIFN

$$C=\left(\underline{X}_{A^I},\underline{Y}_{A^I}\right)=\left(\frac{(a+\acute{a})+7(\mathrm{b}+\acute{b})+(\mathrm{d}+\acute{d})}{18},\frac{4(1+m_a-n_a)}{18}\right)$$

Example 1: Let A=(2,4,5;1,4,6) and B=(3,4,6;1,4,8)

$$O=\left(\underline{X}_{A^I},\underline{Y}_{A^I}\right)=\left(\frac{b+\acute{b}}{2},\frac{1}{6}\left(\frac{2(b-a)(d-b)+m_a^2}{m_a}+\frac{2(\acute{b}-\acute{a})(\acute{d}-\acute{b})+(1-n_a)^2}{(1-n_a)}\right)\right)$$

$$\mathrm{C}=\left(\underline{X}_{A^I},\underline{Y}_{A^I}\right)=\left(\frac{(\mathrm{a}+\acute{a})+7(\mathrm{b}+\acute{b})+(\mathrm{d}+\acute{d})}{18},\frac{4(1+m_a-n_a)}{18}\right).$$

The centroid point orthocentre of two intuitionistic fuzzy numbers of A and B using above formula which is presented in Table 2.

Table 2
Rank of two intuitionistic fuzzy numbers of A and B

	O_x	O_y	C_x	C_y	$CCP(\alpha=0.5)$
A(2,4,5;1,4,6)	4	3	7.777	0.888	7.8325
B(3,4,6;1,4,8)	4	5	8.222	0.888	9.055
Rank					A<B

5. Numerical Example

Illustrative Example: Solving Transportation problem (Table3) using VAM using TORA software and apply MODI method for optimum Solution by using proposed ranking method.

Table 3
Balanced model with number of supplies is equal to demands

Supply → Demand↓	D_1	D_2	D_3	D_4	**Supply** s_i
S_1	(2,4,5; 1,4,6)	(2,5,7;1,5,8)	(4,6,8;3,6,9)	(4,7,8;3,7,9)	11
S_2	(4,6,8; 3,6,9)	(3,7,12;2,7,13)	(10,15,20; 8,15,22)	(11,12,13; 10,12,14)	11
S_3	(3,4,6; 1,4,8)	(8,10,13;5,10,16)	(2,3,5;1,3,6)	(6,10,14; 5,10,15)	11
S_4	(2,4,6; 1,4,7)	(3,9,10;2,9,12)	(3,6,10;2,6,12)	(3,4,5;2,4,8)	12
Demand d_j	16	10	8	11	**45**

We have solved the transportation problem by using VAM method for IBFS and apply MODI method for optimum Solution, where the costs are in intuitionistic fuzzy number.

Step1, using the mean' s of orthocentre of centroids ranking function and presented in Table 4.

Table 4
Ranking triangular intuitionistic fuzzy numbers

Supply → Demand↓	D_1	D_2	D_3	D_4	**Supply s_i**
S_1	(2,4,5;1,4,6) R= 6.667	(2,5,7;1,5,8) R=16.667	(4,6,8;3,6,9) R=15	(4,7,8;3,7,9) R=15.167	11
S_2	(4,6,8;3,6,9) R=15	(3,7,12;2,7,13) R=60.667	(10,15,20; 8,15,22) R=190	(11,12,13;1 0,12,14) R=14	11
S_3	(3,4,6;1,4,8) R=10.667	(8,10,13; 5,10,16) R=63.333	(2,3,5; 1,3,6) R=5	(6,10,14; 5,10,15) R=71.667	11
S_4	(2,4,6;1,4,7) R=18.66	(3,9,10;2,9,12) R=43.500	(3,6,10; 2,6,12) R=38	(3,4,5;2,4,8) R=7.333	12
Demand d_j	16	10	8	11	**45**=45

Step2: Use the methods VAM to find Original basic feasible solution through ranks and check in TORA (Iteration 1 is presented in Fig.4)

Step-3: Take the difference of first lowest cost and second lowest cost for all the rows and columns. These differences are called as penalties.

Select highest penalty (any row and column). Check lowest cost in that row or column and allocate the maximum possible units and remove that row and column in Table 5.

Step-4 Repeat the step 2 until all the allocations have been made and we got Initial basic feasible solution through ranks table which is used in MODI method in Table 6

Table 5
First allocation of triangular intuitionistic fuzzy numbers

	D_1	D_2	D_3	D_4	**Supply s_i**	**Penalties**
S_1	(2,4,5; 1,4,6) R= 6.667	**10(2,5,7; 1,5,8) R=16.667**	(4,6,8; 3,6,9) R=15	(4,7,8; 3,7,9) R=15.167	11 (1)	8.333
S_2	(4,6,8; 3,6,9) R=15	(3,7,12; 2,7,13) R=60.667	(10,15,20; 8,15,22) R=190	(11,12,13; 10,12,14) R=14	11	1
S_3	(3,4,6; 1,4,8) R=10.667	(8,10,13; 5,10,16) R=63.333	(2,3,5; 1,3,6) R=5	(6,10,14; 5,10,15) R=71.667	11	5.667

S_4	(2,4,6; 1,4,7) R=18.66	(3,9,10; 2,9,12) R=43.500	(3,6,10; 2,6,12) R=38	(3,4,5; 2,4,8) R=7.333	12	11.327
Demand d_j	16	10	8	11	**45**=45	
Penalties	4	26.833	10	6.667		

Table 6
Initial basic feasible solution through ranks.

Supply → Demand↓	D^1	D^2	D^3	D^4	**Supply** s_i
S^1	**1(2,4,5; 1,4,6)**	**10(2,5,7; 1,5,8)**	(4,6,8; 3,6,9)	(4,7,8; 3,7,9)	11
S^2	**11(4,6,8; 3,6,9)**	(3,7,12; 2,7,13)	(10,15,20; 8,15,22)	(11,12,13; 10,12,14)	11
S^3	**3(3,4,6; 1,4,8)**	(8,10,13; 5,10,16)	**8(2,3,5; 1,3,6)**	(6,10,14; 5,10,15)	11
S^4	**1(2,4,6; 1,4,7)**	(3,9,10; 2,9,12)	(3,6,10; 2,6,12)	**11(3,4,5 2,4,8)**	12
Demand d_j	16	10	8	11	**45**=45

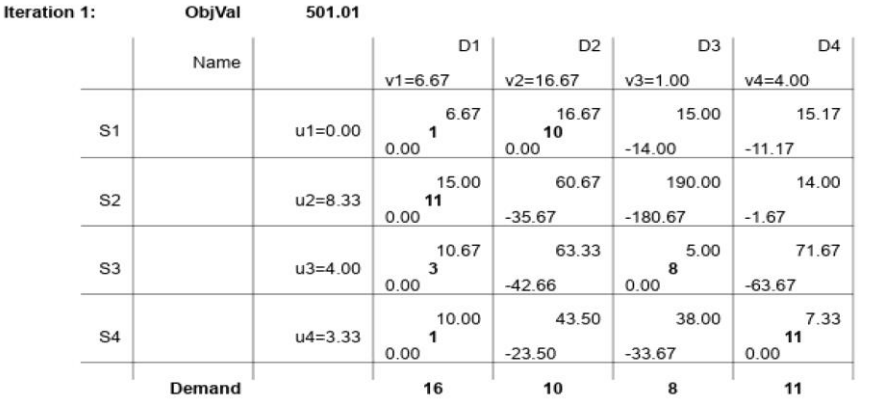


Figure 4
Using TORA initial basic feasible Solution Vogel's approximation method

MODI method

Step-1 Find the original basic feasible Solution VAM for comparison we used to rank them.

Now, check this solution for optimality. Here, the number of occupied cells are exactly m+n-1 (i.e. 3+4-1=6), the IBFS is non-degenerate and also the occupied cells are in independent positions (do not form loop).

Therefore, this IBFS can go for optimality test.

Step-2 Determine the values of dual variables, u_i's to rows and v_j's to columns ($u_i = v_j = 0$ initially) such that $u_i + v_j = c_j$ for basic cells in T**able** 7.

Step-3 Solve the equations in step-2 for non-basic cells.

Step-4 For non-basic cells find $d_{ij} = c_{ij} - u_i + v_j$ in Table **8**.

Table 7
Representations of ui's and vj's

	D^1	D^2	D^3	D^4	u_i's
S^1	(2,4,5, 1,4,6)	(2,5,7, 1,5,8)			u_1= (0,0,0;0,0,0)
S^2	(4,6,8;3,6,9)				u_2= (−1,2,6; −3,2,8)
S^3	(3,4,6;1,4,8)		(2,3,5;1,3,6)		u_3= (−2,0,4; −5,0,7)
S^4	(2,4,6;1,4,7)			(3,4,5;2,4,8)	u_4= (−3,0,4; −5,0,6)
v_j's	v_1= (2,4,5;1,4,6)	v_2= (2,5,7;1,5,8)	v_3 = (−2,3,7;6,3,11)	v_4= (−1,4,8; −4,4,13)	

Step-5 If $d_{ij} \geq 0$, then basic feasible Solution is optimal and if at least one $d_{ij} = 0$, then alternate optimal Solution exists.

Table 8
For non-basic cells

	D^1	D^2	D^3	D^4	u_i's
S^1			(4,6,8; 3,6,9)	(4,7,8; 3,7,9)	u_1= (0,0,0;0,0,0)
S^2		(3,7,12; 2,7,13)	(10,15,20; 8,15,22)	(11,12,13; 10,12,14)	u_2= (–1,2,6; -3,2,8)
S^3		(8,10,13; 5,10,16)		(6,10,14; 5,10,15)	u_3= (–2,0,4; –5,0,7)
S^4		(3,9,10; 2,9,12)	(3,6,10; 2,6,12)		u_4= (–3,0,4; –5,0,6)
v_j's	v_1= (2,4,5; 1,4,6)	v_2= (2,5,7; 1,5,8)	v_3= (−2,3,7; 6,3,11)	v_4= (–1,4,8; –4,4,13)	

Table 9
Representation for d_{ij}

	D^1	D^2	D^3	D^4	Supply s_i
S^1			(−3,3,10; −8,3,15) R= 88	(−4,3,9; −10,3,13) R= 87	11
S^2		(−10,0,11; 14,0,15)	(−3,10,23; −11,10,31)	(−3,6,15; −11,6,21)	11

		R=0	R= 1020	R= 338	
S^3		(−3,5,13; −10,5,20) R= 242.5		(−6,6,17; −15,6,24) R= 512	11
S^4		(−8,4,11; −12,4,16) R= 185.33	(2,3,5;1,3,6) R= 247		12
Demand d_j	16	10	8	11	**45**=45

Since all the $d_{ij} \geq 0$ (Table 9) then basic feasible Solution is optimal and if at least one $d_{ij} = 0$, then alternate optimal Solution exists if not then continue the step 6,7,8 and 9

Step-6: The optimal solution, obtained in Step 4, is x_{11}=1, x_{12}=10, x_{21}=11, x_{31}=3, x_{33}8, x_{41}=1 and x_{44}=11. The intuitionistic fuzzy intuitionistic fuzzy transportation problem, given in Table10, is

1⊗(2,4,5;1,4,6)⊕10⊗(2,5,7;1,5,8)⊕11⊗(4,6,8;3,6,9)⊕3⊗(3,4,6;1,4,8) ⊕8⊗(2,3,5;1,3,6)⊕1⊗ (2, 4,6;1,4,7) ⊕ 11(3,4,5;2,4,8) = (126,204,282;78,204,352)

Table 10
Optimal solution and intuitionistic fuzzy optimal value

Supply → Demand↓	D^1	D^2	D^3	D^4	**Supply** s_i
S^1	1 (2,4,5; 1,4,6)	10 (2,5,7; 1,5,8)			11
S^2	11 (4,6,8; 3,6,9)				11
S^3	3 (3,4,6; 1,4,8)		8 (2,3,5; 1,3,6)		11
S^4	1 (2,4,6; 1,4,7)			11 (3,4,5; 2,4,8)	12
Demand d_j	16	10	8	11	45

6. Results and Discussion

We have achieved minimum costs by applying the various methods to the considered transportation problems to find the minimum transportation cost using ranking function. We utilized ranking technique to compare TIFN to finding IBFS, MODI method, and compared them with existing method.

If we compare solutions obtained with the IBFS methods and the existing method, we find that the optimum solution are same. (Table 11)

Table 11
Result

Methods	**IBFS (VAM Method)**	**Optimum solution**	**Iteration**
Method 1	(126,204,282;78,204,352)	(126,204,282;78,204,352)	1
Method 2	-	(116,204,302;68,204,372) (126,204,282;78,204,352)	1 2

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