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Analysis of Sudoku square designs for imprecise data

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Abstract

The classic Sudoku square design is applied for testing the null hypothesis when all observations in the data are precise and certain. In some cases, it is difficult for researchers to obtain exact observations due to circumstances beyond their control. Therefore, the observations may be imprecise and noted in intervals. We must therefore search for a suitable generalization of Sudoku square designs in uncertain environments. This paper aims to introduce the four various statistical models of the Sudoku square design under neutrosophic statistics, which provides a solution to the problem of imprecise and uncertain data. The test statistic of the proposed design per model is derived and applied for testing treatment effects. Based on the data analysis, and comparative study, it is concluded that the

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proposed neutrosophic Sudoku square design is informative, efficient, and adequate for use in the presence of uncertainty.

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1. Introduction

Recent years have seen the development of many experimental designs that have been widely used in a variety of statistical applications. Experimental designs have typically utilized one-way or two-way blocking designs (e.g., lattice square, Latin square, random complete block, etc.). However, certain situations might arise where removing sources of variability is more effectively achieved through designs like Sudoku square designs, which are a specific subset of Latin square designs. The Latin square design was introduced by Fisher [1] in the 20s of last century, and researchers expanded this design to include Sudoku squares, which letters and numbers appear once only in each column (row) or a sub-block.

This design is based upon a modern Sudoku puzzle that was created by the American architect Howard Garns in 1979 and published by Dell magazine at the time under the title Number Place. The game became popular in Japan after it was published by Nikolai magazine under the name Sudoku. In 2005, the game became an international.

Hui- Hui-Dong and Ru-Gen [2] first introduced Sudoku Square, which was adapted into a new design and applied to field experiments. In another paper, Sudoku designs of order $k = b^2$ are presented by Subramani and Ponnuswamy [3]. Dauran, et al. [4] have contructed the and analyzed balanced-incomplete Sudoku squares. Related articles about the Sudoku square design can be seen in [5-9]. On the other hand, researchers have focused on introducing applications using Sudoku framework. Hashmi, et al. [10] assigned a number to any chemical structure by analyzing Sudoku graphs. For further reading we can refer to [11] and [12].

The Neutrosophic Statistics (NS) is a direct and natural generalization of fuzzy framework and it provides a reliable measure of imprecise and indeterminacy. Therefore, the NS can extend many different classical statistical approaches, which are suitable to deal with the analysis under indeterminate environments. In fact, Smarandache [13] is the first work introducing the neutrosophic logic as a more efficient method than the fuzzy counterpart. Again, Smarandache [14], [15, 16] and Aslam [17] discussed various applications. More ever, a comprehensive examination of the differences between fuzzy, NS and classical methods are well studied by Aslam [18]. In addition, Aslam [19] has proposed and defined a neutrosophic ANOVA, and recently, AlAita and Aslam [20] established the neutrosophic ANCOVA for each of split-plot, randomized complete block and complete randomized designs. Besides, post hoc multiple comparison tests under neutrosophic statistics have been suggested by Aslam and Albassam [21]. The Z-test was introduced by Aslam [22] under neutrosophic statistics and applied to the Covid-19 dataset. According to Aslam and Al-Marshadi [23], “the Watson-Williams test was introduced to analyze circular/angle data that are uncertain, imprecise, and indeterminate in a neutrosophic approach”. Alhabib and Salama [24] proposed the Neutrosophic time series by studying classical time series in the context of Neutrosophic logic. Various statistical tests have been explored within the framework of neutrosophic statistics in [25-31]. For single-valued neutrosophic sets, Huang [32] proposed a new distance measure. Also, Chutia, et al. [33] have studied the problem of ordering single-valued neutrosophic number sets on the basis of flexible collection of parameters. On the other hand, Nafei, et al. [34], based on fuzzy neutrosophic sets, have studied the subject of optimizing score function, specifically, in multi-attributed group decision-making settings. Besides and by considering the single-valued bipolar neutrosophic environments, Garai and Garg [34] discussed possible multi-attribute decision-making collection by observing one single value in order to treat water resource management issues. On the other hand, Pamucar, et al. [35] proposed and designed a hybrid approach based on fuzzy neutrosophic logic for decision-making in resilient supplier selection topics. Related articles and books about the neutrosophic theories and applications can be viewed in Aslam [17] [36-45].

All previous studies for analysis of Sudoku square design have mainly focused on dealing with determined and certain datasets. There are some disadvantages to these methods as they provide a single, crisp result, which may be incorrect or underestimated at times. There are, however, many situations in which the data are neutrosophic under certain circumstances, and it is necessary to apply neutrosophic statistics, since the old classical methods are no longer appropriate. There is therefore a need for more neutrosophic statistical techniques.

Upon exhaustive review of the published studies, no research has been found on Sudoku square design for neutrosophic data. In this paper, we define new neutrosophic designs as generalizations of previously defined ones under CS, providing the capability to handle both precise and vague forms of data. A numerical example will be used to evaluate the efficiency of proposed designs under NS. Based on comparative analyses, it has been demonstrated that the proposed designs can be analyzed in cases in which observations are in neutrosophic form, imprecise, indeterminate, or uncertain interval-valued.

2. Preliminaries

This section provides some basic concepts about neutrosophic data and NS that will be useful as we prepare this paper.

Neutrosophic data encompass uncertain and ambiguous observations, undefined arguments, and interval values influenced by uncertainty. Consequently, the information obtained from experiments or populations can be represented using interval-valued neutrosophic numbers (INN).

The neutrosophic normally distributed random variable, namely, $X_N \in [X_L, X_U]$ has a neutrosophic mean $\mu_N \in [\mu_L, \mu_U]$ and a neutrosophic standard deviation $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$, and it also can be written in neutrosophic form as $X_N = X_L + X_U I_N$, where X_L presents determinate part, $X_U I_N$ presents indeterminate part, and $I_N \in [I_L, I_U]$ refers to the associated measure of indeterminacy or uncertainty.

Now, let N be the total population size of observations that include indeterminacy, and take n as the sample size of randomly drawn observations. In consequence, one can define both neutrosophic sample mean and variance as

$$\bar{x}_N \in \left[\frac{\sum_{i=1}^{n} x_{Li}}{n}, \frac{\sum_{i=1}^{n} x_{Ui}}{n}\right];\ \bar{x}_N \in [\bar{x}_L, \bar{x}_U\ ] \text{ and}$$
$$s_N^2 \in \left[\frac{\sum_{i=1}^{n}(x_{Li}-\bar{x}_L)^2}{n-1}, \frac{\sum_{i=1}^{n}(x_{Ui}-\bar{x}_U)^2}{n-1}\right];\ s_N^2 \in [s_L^2, s_U^2],$$

respectively. According to [18, 19], a brief description of the difference between fuzzy, neutrosophic, and classical statistics can be given as follows.

Classical statistics	**Fuzzy statistics**	**Neutrosophic statistics**
Classical statistics require that all observations or parameters be precisely determined to be analyzed. Therefore, the Sudoku square design can be analyzed using classical statistics when all observations have been determined.	Fuzzy statistics are used when some or all of the observations are fuzzy and uncertain, and they do not account for the measure of indeterminacy. Consequently, fuzzy statistics can analyze Sudoku square design when some or all observations are fuzzy and uncertain.	Neutrosophic statistics are an extension of fuzzy statistics with determined and undetermined parts. These statistics have the measure of indeterminacy and can be used with observations that are in intervals and not fuzzy. That is, Sudoku square design under neutrosophic statistics is an extension of Sudoku square design under classical and fuzzy statistics.

3. Analysis of Neutrosophic Sudoku Square Design

The classic Sudoku square design literature explains how to construct the design, discusses four different types, and provides several examples. For more information see Subramani and Ponnuswamy [3].

In this paper, we are going to present a Sudoku square design under NS, where the statistical model will be defined and NANOVA tables will be created for each type of Sudoku square designs. In what follows, we list four different kinds of neutrosophic Sudoku squares.

3.1. *Neutrosophic Sudoku square design - Type I (NSSD-I)*

In this model, neutrosophic effects of rows, columns, and treatments are the same as in neutrosophic Latin square designs. Additionally, there are neutrosophic effects caused by row block, column block, and squares. Therefore, the statistical NSSD-I model is

$$y_{Nij(klpq)} = \mu_N + \alpha_{Ni} + \beta_{Nj} + \tau_{Nk} + r_{Nl} + c_{Np} + s_{Nq} + \varepsilon_{Nij(klpq)},$$

$$\begin{cases} i = 1,2,\ldots,b \\ j = 1,2,\ldots,b \\ k = 1,2,\ldots,b^2 \\ l = 1,2,\ldots,b^2 \\ p = 1,2,\ldots,b^2 \\ q = 1,2,\ldots,b^2 \end{cases} \quad (1)$$

In addition, we can write the neutrosophic form of $y_{Nij(klpq)}$ as

$$y_{Nij(klpq)} = y_{Lij(klpq)} + y_{Uij(klpq)} I_N;\ I_N \in [I_L, I_U], \quad (2)$$

where μ_N is the neutrosophic common mean, α_{Ni} is the i^{th} neutrosophic row block effect, β_{Nj} is the j^{th} neutrosophic column block effect, τ_{Nk} is the k^{th} neutrosophic treatment effect, r_{Nl} is the l^{th} neutrosophic row effect, c_{Np} is the p^{th} neutrosophic column effect, s_{Nq} is the q^{th} neutrosophic square effect, and $\varepsilon_{Nij(klpq)}$ is a neutrosophic random error with zero mean and neutrosophic variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$.

The null and alternative hypotheses for NSSD-I are

$$H_{N0}: \alpha_{Ni} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \alpha_{Ni} \neq 0, i = 1.2.,,,.b,$$
$$H_{N0}: \beta_{Nj} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \beta_{Nj} \neq 0, j = 1.2.,,,.b,$$
$$H_{N0}: \tau_{Nk} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \tau_{Nk} \neq 0, k = 1.2.,,,.b^2,$$
$$H_{N0}: r_{Nl} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ r_{Nl} \neq 0, l = 1.2.,,,.b^2,$$
$$H_{N0}: c_{Np} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ c_{Np} \neq 0, p = 1.2.,,,.b^2,$$
$$H_{N0}: s_{Nq} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ s_{Nq} \neq 0, q = 1.2.,,,.b^2.$$

Neutrosophic sums of squares for NSSD-I are calculated as following:

$$CF_N = \frac{\left(\sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b^2}\sum_{p=1}^{b^2}\sum_{q=1}^{b^2} y_{Nij(klpq)}\right)^2}{b^4},$$

$$SS_{NT} = \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b^2}\sum_{p=1}^{b^2}\sum_{q=1}^{b^2} y_{Nij(klpq)}^2 - CF_N;\ SS_{NT} \in [SS_{LT}, SS_{UT}],$$

$$SS_{NRB} = \frac{\sum_{i=1}^{b} y_{Ni.(....)}^2}{b^3} - CF_N;\ SS_{NRB} \in [SS_{LRB}, SS_{URB}],$$

$$SS_{NCB} = \frac{\sum_{j=1}^{b} y_{N.j(....)}^2}{b^3} - CF_N;\ SS_{NCB} \in [SS_{LCB}, SS_{UCB}],$$

$$SS_{NTr} = \frac{\sum_{k=1}^{b^2} y_{N..(k...)}^2}{b^2} - CF_N;\ SS_{NTr} \in [SS_{LTr}, SS_{UTr}],$$

$$SS_{NR} = \frac{\sum_{l=1}^{b^2} y_{N..(.l..)}^2}{b^2} - CF_N;\ SS_{NR} \in [SS_{LR}, SS_{UR}],$$

$$SS_{NC} = \frac{\sum_{p=1}^{b^2} y_{N..(..p.)}^2}{b^2} - CF_N;\ SS_{NC} \in [SS_{LC}, SS_{UC}],$$

$$SS_{NS} = \frac{\sum_{q=1}^{b^2} y_{N..(...q)}^2}{b^2} - CF_N;\ SS_{NS} \in [SS_{LS}, SS_{US}],$$

$$SS_{NE} = SS_{NT} - SS_{NRB} - SS_{NCB} - SS_{NTr} - SS_{NR} - SS_{NC} - SS_{NS};$$
$$SS_{NE} \in [SS_{LE}, SS_{UE}].$$

Table 1 presents the NANOVA table for the NSSD-I.

Table 1
NANOVA Table for the NSSD-I

Source of Variation (S. O. V)	NSS	Ndf	NMS	F_N
Row Blocks	SS_{NRB}	$b-1$	$MS_{NRB} = \frac{SS_{NRB}}{b-1}$	$\frac{MS_{NRB}}{MS_{NE}}$
Column Blocks	SS_{NCB}	$b-1$	$MS_{NCB} = \frac{SS_{NCB}}{b-1}$	$\frac{MS_{NCB}}{MS_{NE}}$
Treatments	SS_{NTr}	b^2-1	$MS_{NTr} = \frac{SS_{NTr}}{b^2-1}$	$\frac{MS_{NTr}}{MS_{NE}}$
Rows	SS_{NR}	b^2-1	$MS_{NR} = \frac{SS_{NR}}{b^2-1}$	$\frac{MS_{NR}}{MS_{NE}}$
Columns	SS_{NC}	b^2-1	$MS_{NC} = \frac{SS_{NC}}{b^2-1}$	$\frac{MS_{NC}}{MS_{NE}}$
Squares	SS_{NS}	b^2-1	$MS_{NS} = \frac{SS_{NS}}{b^2-1}$	$\frac{MS_{NS}}{MS_{NE}}$
Error	SS_{NE}	$(b-1)[(b+1)(b^2-3)-2]$	$MS_{NE} = \frac{SS_{NE}}{(b-1)[(b+1)(b^2-3)-2]}$	
Total	SS_{NT}	b^4-1		

3.2. *Neutrosophic Sudoku square design - Type II (NSSD-II)*

Here, the neutrosophic row effects (or neutrosophic column effects) are nested in neutrosophic row block effects (or neutrosophic column block effects).

This statistical NSSD-II model is given by

$$y_{Nij(klpq)} = \mu_N + \alpha_{Ni} + \beta_{Nj} + \tau_{Nk} + r(\alpha)_{Nl(i)} + c(\beta)_{Np(j)} + s_{Nq} + \varepsilon_{Nij(klpq)},$$
$$\begin{cases} i = 1,2,\dots,b \\ j = 1,2,\dots,b \\ k = 1,2,\dots,b^2 \\ l = 1,2,\dots,b \\ p = 1,2,\dots,b \\ q = 1,2,\dots,b^2 \end{cases} \tag{3}$$

The neutrosophic form of $y_{Nij(klpq)}$ is the same as in (2), where μ_N is the neutrosophic common mean, α_{Ni} is the i^{th} neutrosophic row block effect, β_{Nj} is the j^{th} neutrosophic column block effect, τ_{Nk} is the k^{th} neutrosophic treatment effect, $r(\alpha)_{Nl(i)}$ is the l^{th} neutrosophic row effect nested in i^{th} neutrosophic row block , $c(\beta)_{Np(j)}$ is the p^{th} neutrosophic column effect nested in j^{th} neutrosophic column block, s_{Nq} is the q^{th} neutrosophic square effect, and $\varepsilon_{Nij(klpq)}$ is a neutrosophic random error with zero mean and neutrosophic variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$.

The null and alternative hypotheses for NSSD-II are

$$H_{N0}: \alpha_{Ni} = 0 \;\; vs \;\; H_{N1}: at\ least\ one\ \alpha_{Ni} \neq 0, i = 1.2.,,,.b,$$

$$H_{N0}: \beta_{Nj} = 0 \;\; vs \;\; H_{N1}: at\ least\ one\ \beta_{Nj} \neq 0, j = 1.2.,,,.b,$$

$$H_{N0}: \tau_{Nk} = 0 \;\; vs \;\; H_{N1}: at\ least\ one\ \tau_{Nk} \neq 0, k = 1.2.,,,.b^2,$$

$$H_{N0}: r(\alpha)_{Nl(i)} = 0 \;\; vs \;\; H_{N1}: at\ least\ one\ r(\alpha)_{Nl(i)} \neq 0, i = 1.2.,,,.b, \\ l = 1.2.,,,.b,$$

$$H_{N0}: c(\beta)_{Np(j)} = 0 \;\; vs \;\; H_{N1}: at\ least\ one\ c(\beta)_{Np(j)} \neq 0, j = 1.2.,,,.b, \\ p = 1.2.,,,.b,$$

$$H_{N0}: s_{Nq} = 0 \;\; vs \;\; H_{N1}: at\ least\ one\ s_{Nq} \neq 0, q = 1.2.,,,.b^2.$$

Neutrosophic sums of squares for NSSD-II are calculated as following:

$$CF_N = \frac{\left(\sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b}\sum_{p=1}^{b}\sum_{q=1}^{b^2} y_{Nij(klpq)}\right)^2}{b^4},$$

$$SS_{NT} = \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b}\sum_{p=1}^{b}\sum_{q=1}^{b^2}\left(y_{Nij(klpq)}\right)^2 - CF_N; \\ SS_{NT} \in [SS_{LT}, SS_{UT}],$$

$$SS_{NRB} = \frac{\sum_{i=1}^{b} y_{Ni.(....)}^2}{b^3} - CF_N; SS_{NRB} \in [SS_{LRB}, SS_{URB}],$$

$$SS_{NCB} = \frac{\sum_{j=1}^{b} y_{N.j(....)}^2}{b^3} - CF_N; \\ SS_{NCB} \in [SS_{LCB}, SS_{UCB}],$$

$$SS_{NTr} = \frac{\sum_{k=1}^{b^2} y_{N..(k...)}^2}{b^2} - CF_N; SS_{NTr} \in [SS_{LTr}, SS_{UTr}],$$

$$SS_{NR} = \frac{\sum_{i=1}^{b}\sum_{l=1}^{b} y_{Ni.(.l..)}^2}{b^2} - \frac{\sum_{i=1}^{b} y_{Ni.(....)}^2}{b^3}; SS_{NR} \in [SS_{LR}, SS_{UR}],$$

$$SS_{NC} = \frac{\sum_{j=1}^{b}\sum_{p=1}^{b} y_{N.j(..p.)}^2}{b^2} - \frac{\sum_{i=1}^{b} y_{N.j(....)}^2}{b^3}; SS_{NC} \in [SS_{LC}, SS_{UC}],$$

$$SS_{NS} = \frac{\sum_{q=1}^{b^2} y_{N..(...q)}^2}{b^2} - CF_N; \\ SS_{NS} \in [SS_{LS}, SS_{US}],$$

$$SS_{NE} = SS_{NT} - SS_{NRB} - SS_{NCB} - SS_{NTr} - SS_{NR} - SS_{NC} - SS_{NS}; \\ SS_{NE} \in [SS_{LE}, SS_{UE}].$$

Table 2 presents the NANOVA table for the NSSD-II.

Table 2
NANOVA Table for the NSSD-II

S. O. V	NSS	Ndf	NMS	F_N
Row Blocks	SS_{NRB}	$b-1$	$MS_{NRB}=\dfrac{SS_{NRB}}{b-1}$	$\dfrac{MS_{NRB}}{MS_{NE}}$
Column Blocks	SS_{NCB}	$b-1$	$MS_{NCB}=\dfrac{SS_{NCB}}{b-1}$	$\dfrac{MS_{NCB}}{MS_{NE}}$
Treatments	SS_{NTr}	b^2-1	$MS_{NTr}=\dfrac{SS_{NTr}}{b^2-1}$	$\dfrac{MS_{NTr}}{MS_{NE}}$
Rows within RB	SS_{NR}	$b(b-1)$	$MS_{NR}=\dfrac{SS_{NR}}{b(b-1)}$	$\dfrac{MS_{Row}}{MS_{NE}}$
Columns within CB	SS_{NC}	$b(b-1)$	$MS_{NC}=\dfrac{SS_{NC}}{b(b-1)}$	$\dfrac{MS_{NC}}{MS_{NE}}$
Squares	SS_{NSq}	b^2-1	$MS_{NSq}=\dfrac{SS_{NSq}}{b^2-1}$	$\dfrac{MS_{NSq}}{MS_{NE}}$
Error	SS_{NE}	(b^2-1) (b^2-3)	$MS_{NE}=\dfrac{SS_{NE}}{(b^2-1)(b^2-3)}$	
Total	SS_{NT}	b^4-1		

3.3. *Neutrosophic Sudoku square design - Type III (NSSD-III)*

This model considers neutrosophic horizontal square effects (or neutrosophic vertical square effects) nested within neutrosophic row block effects (or neutrosophic column block effects). The statistical NSSD-III model is defined as

$$y_{Nij(klpqu)}=\mu_N+\alpha_{Ni}+\beta_{Nj}+\tau_{Nk}+r_{Nl}+c_{Np}+s(\alpha)_{Nq(i)}+\delta(\beta)_{Nu(j)}+ \varepsilon_{Nij(klpqu)},\ \begin{cases} i=1,2,\dots,b\\ j=1,2,\dots,b\\ k=1,2,\dots,b^2\\ l=1,2,\dots,b^2\\ p=1,2,\dots,b^2\\ q=1,2,\dots,b\\ u=1,2,\dots,b \end{cases} \tag{4}$$

Again, we can write the neutrosophic form of $y_{Nij(klpqu)}$ as

$$y_{Nij(klpqu)}=y_{Lij(klpqu)}+y_{Uij(klpqu)}I_N;\ I_N\in[I_L,I_U], \tag{5}$$

where μ_N is the neutrosophic common mean, α_{Ni} is the i^{th} neutrosophic row block effect, β_{Nj} is the j^{th} neutrosophic column block effect, τ_{Nk} is the k^{th} neutrosophic treatment effect, r_{Nl} is the l^{th} neutrosophic row effect, c_{Np} is the p^{th} neutrosophic column effect, $s(\alpha)_{Nq(i)}$ is the q^{th} neutrosophic horizontal square effect nested in i^{th} neutrosophic row block, $\delta(\beta)_{Nu(j)}$ is the u^{th} neutrosophic vertical square effect nested in the j^{th} neutrosophic column block, and $\varepsilon_{Nij(klpqu)}$ is a neutrosophic centered random error with variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$.

The null and alternative hypotheses for NSSD-III are

$$H_{N0}: \alpha_{Ni} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \alpha_{Ni} \neq 0, i = 1.2.,,,.b,$$
$$H_{N0}: \beta_{Nj} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \beta_{Nj} \neq 0, j = 1.2.,,,.b,$$
$$H_{N0}: \tau_{Nk} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \tau_{Nk} \neq 0, k = 1.2.,,,.b^2,$$
$$H_{N0}: r_{Nl} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ r_{Nl} \neq 0, l = 1.2.,,,.b^2,$$
$$H_{N0}: c_{Np} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ c_{Np} \neq 0, p = 1.2.,,,.b^2,$$
$$H_{N0}: s(\alpha)_{Nq(i)} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ s(\alpha)_{Nq(i)} \neq 0, i = 1.2.,,,.b, \\ q = 1.2.,,,.b,$$
$$H_{N0}: \delta(\beta)_{Nu(j)} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \delta(\beta)_{Nu(j)} \neq 0, j = 1.2.,,,.b, \\ u = 1.2.,,,.b.$$

Table 3
NANOVA Table for the NSSD-III

S. O. V	NSS	Ndf	NMS	F_N
Row Blocks	SS_{NRB}	$b-1$	$MS_{NRB} = \dfrac{SS_{NRB}}{b-1}$	$\dfrac{MS_{NRB}}{MS_{NE}}$
Column Blocks	SS_{NCB}	$b-1$	$MS_{NCB} = \dfrac{SS_{NCB}}{b-1}$	$\dfrac{MS_{NCB}}{MS_{NE}}$
Treatments	SS_{NTr}	b^2-1	$MS_{NTr} = \dfrac{SS_{NTr}}{b^2-1}$	$\dfrac{MS_{NTr}}{MS_{NE}}$
Rows	SS_{NR}	b^2-1	$MS_{NR} = \dfrac{SS_{NR}}{b^2-1}$	$\dfrac{MS_{NR}}{MS_{NE}}$
Columns	SS_{NC}	b^2-1	$MS_{NC} = \dfrac{SS_{NC}}{b^2-1}$	$\dfrac{MS_{NC}}{MS_{NE}}$
Squares within RB	SS_{NHS}	$b(b-1)$	$MS_{NHSq} = \dfrac{SS_{NHS}}{b(b-1)}$	$\dfrac{MS_{NHS}}{MS_{NE}}$
Squares within CB	SS_{NVS}	$b(b-1)$	$MS_{NVSq} = \dfrac{SS_{NVS}}{b(b-1)}$	$\dfrac{MS_{NVS}}{MS_{NE}}$
Error	SS_{NE}	$(b^2-1)(b^2-4)$	$MS_{NE} = \dfrac{SS_{NE}}{(b^2-1)(b^2-4)}$	
Total	SS_{NT}	b^2-1		

Neutrosophic sums of squares for NSSD-III are calculated as following:

$$CF_N = \frac{\left(\sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b^2}\sum_{p=1}^{b^2}\sum_{q=1}^{b}\sum_{u=1}^{b} y_{Nij(klpqu)}\right)^2}{b^4},$$
$$SS_{NT} = \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b^2}\sum_{p=1}^{b^2}\sum_{q=1}^{b}\sum_{u=1}^{b} y_{Nij(klpqu)}^2 - CF_N;$$

$$SS_{NT} \in [SS_{LT}, SS_{UT}],$$

$$SS_{NRB} = \frac{\sum_{i=1}^{b} y_{Ni.(.....)}^2}{b^3} - CF_N;\ SS_{NRB} \in [SS_{LRB}, SS_{URB}],$$

$$SS_{NCB} = \frac{\sum_{j=1}^{b} y_{N.j(.....)}^2}{b^3} - CF_N;$$

$$SS_{NCB} \in [SS_{LCB}, SS_{UCB}],$$

$$SS_{NTr} = \frac{\sum_{k=1}^{b^2} y_{N..(k....)}^2}{b^2} - CF_N;\ SS_{NTr} \in [SS_{LTr}, SS_{UTr}],$$

$$SS_{NR} = \frac{\sum_{l=1}^{b^2} y_{N..(.l...)}^2}{b^3} - CF_N;\ SS_{NR} \in [SS_{LR}, SS_{UR}],$$

$$SS_{NC} = \frac{\sum_{p=1}^{b^2} y_{N..(..p..)}^2}{b^2} - CF_N;\ SS_{NC} \in [SS_{LC}, SS_{UC}],$$

$$SS_{NHS} = \frac{\sum_{i=1}^{b}\sum_{q=1}^{b} y_{Ni.(...q.)}^2}{b^2} - \frac{\sum_{i=1}^{b} y_{Ni.(.....)}^2}{b^3};$$

$$SS_{NHS} \in [SS_{LHS}, SS_{UHS}],$$

$$SS_{NVS} = \frac{\sum_{j=1}^{b}\sum_{u=1}^{b} y_{N.j(....u)}^2}{b^2} - \frac{\sum_{j=1}^{b} y_{N.j(.....)}^2}{b^3};\ SS_{NVS} \in [SS_{LVS}, SS_{UVS}],$$

$$SS_{NE} = SS_{NT} - SS_{NRB} - SS_{NCB} - SS_{NTr} - SS_{NR} - SS_{NC} - SS_{NHS} - SS_{NVS};$$

$$SS_{NE} \in [SS_{LE}, SS_{UE}].$$

Table 3 presents the NANOVA table for the NSSD-III.

3.4. *Neutrosophic Sudoku square design - Type IV (NSSD-IV)*

In this model, the neutrosophic row effects and the neutrosophic horizontal square effects (or, alternatively, the neutrosophic column effects and the neutrosophic vertical square effects) are embedded within the neutrosophic row block effects (or neutrosophic column block effects). This configuration defines the statistical NSSD-IV model.

$$y_{Nij(klpqu)} = \mu_N + \alpha_{Ni} + \beta_{Nj} + \tau_{Nk} + r(\alpha)_{Nl(i)} + c(\beta)_{Np(j)} + s(\alpha)_{Nq(i)} + \delta(\beta)_{Nu(j)} + \varepsilon_{Nij(klpqu)},\ \begin{cases} i = 1,2,\dots,b \\ j = 1,2,\dots,b \\ k = 1,2,\dots,b^2 \\ l = 1,2,\dots,b \\ p = 1,2,\dots,b \\ q = 1,2,\dots,b \\ u = 1,2,\dots,b \end{cases} \tag{6}$$

The neutrosophic form of $y_{Nij(klpqu)}$ is the same as in (5), where μ_N is the neutrosophic common mean, α_{Ni} is the i^{th} neutrosophic row block effect, β_{Nj} is the j^{th} neutrosophic column block effect, τ_{Nk} is the k^{th} neutrosophic treatment effect, $r(\alpha)_{Nl(i)}$ is the l^{th} neutrosophic row effect nested in i^{th} neutrosophic row block, $c(\beta)_{Np(j)}$ is the p^{th} neutrosophic column effect nested in j^{th} neutrosophic column block, $s(\alpha)_{Nq(i)}$ is the q^{th} neutrosophic horizontal square effect nested in i^{th} row block, $\delta(\beta)_{Nu(j)}$ is the u^{th} vertical square effect nested in j^{th} column block, and $\varepsilon_{Nij(klpqu)}$ is a neutrosophic centered random error with variance $\sigma_N^2 \in [\sigma_L^2, \sigma_U^2]$.

The null and alternative hypotheses for NSSD-IV are

$$H_{N0}: \alpha_{Ni} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \alpha_{Ni} \neq 0, i = 1.2.,,,.b,$$

$$H_{N0}: \beta_{Nj} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \beta_{Nj} \neq 0, j = 1.2.,,,.b,$$

$$H_{N0}: \tau_{Nk} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \tau_{Nk} \neq 0, k = 1.2.,,,.b^2,$$

$$H_{N0}: r(\alpha)_{Nl(i)} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ r(\alpha)_{Nl(i)} \neq 0, \quad i = 1.2.,,,.b, l = 1.2.,,,.b,$$

$$H_{N0}: c(\beta)_{Np(j)} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ c(\beta)_{Np(j)} \neq 0, \quad j = 1.2.,,,.b, p = 1.2.,,,.b,$$

$$H_{N0}: s(\alpha)_{Nq(i)} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ s(\alpha)_{Nq(i)} \neq 0, \quad i = 1.2.,,,.b, q = 1.2.,,,.b,$$

$$H_{N0}: \delta(\beta)_{Nu(j)} = 0 \ \ vs \ \ H_{N1}: at\ least\ one\ \delta(\beta)_{Nu(j)} \neq 0, \quad j = 1.2.,,,.b, u = 1.2.,,,.b.$$

Table 4
NANOVA Table for the NSSD-IV

S. O. V	NSS	Ndf	NMS	F_N
Row Blocks	SS_{NRB}	$b-1$	$MS_{NRB} = \frac{SS_{NRB}}{b-1}$	$\frac{MS_{RB}}{MS_{NE}}$
Column Blocks	SS_{NCB}	$b-1$	$MS_{NCB} = \frac{SS_{NCB}}{b-1}$	$\frac{MS_{CB}}{MS_{NE}}$
Treatments	SS_{NTr}	b^2-1	$MS_{NTr} = \frac{SS_{NTr}}{b^2-1}$	$\frac{MS_{NTr}}{MS_{NE}}$
Rows within RB	SS_{NR}	$b(b-1)$	$MS_{NR} = \frac{SS_{NR}}{b(b-1)}$	$\frac{MS_{NR}}{MS_{NE}}$
Columns within CB	SS_{NC}	$b(b-1)$	$MS_{NC} = \frac{SS_{NC}}{b(b-1)}$	$\frac{MS_{NC}}{MS_{NE}}$
Squares within RB	SS_{NHS}	$b(b-1)$	$MS_{NHSq} = \frac{SS_{NHS}}{b(b-1)}$	$\frac{MS_{NHS}}{MS_{NE}}$
Squares within CB	SS_{NVS}	$b(b-1)$	$MS_{NVSq} = \frac{SS_{NVS}}{b(b-1)}$	$\frac{MS_{NVS}}{MS_{NE}}$
Error	SS_{NE}	$(b-1)$ (b^3+b^2-4b-2)	$MS_{NE} = \frac{SS_{NE}}{(b-1)(b^3+b^2-4b-2)}$	
Total	SS_{NT}	b^4-1		

Neutrosophic sums of squares for NSSD-IV are calculated as following:

$$CF_N = \frac{\left(\sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b}\sum_{p=1}^{b}\sum_{q=1}^{b}\sum_{u=1}^{b} y_{Nij(klpqu)}\right)^2}{b^4},$$

$$SS_{NT} = \sum_{i=1}^{b}\sum_{j=1}^{b}\sum_{k=1}^{b^2}\sum_{l=1}^{b}\sum_{p=1}^{b}\sum_{q=1}^{b}\sum_{u=1}^{b} y_{Nij(klpqu)}^2 - CF_N;$$
$$SS_{NT} \in [SS_{LT}, SS_{UT}],$$

$$SS_{NRB} = \frac{\sum_{i=1}^{b} y_{Ni.(.....)}^2}{b^3} - CF_N;\ SS_{NRB} \in [SS_{LRB}, SS_{URB}],$$

$$SS_{NCB} = \frac{\sum_{j=1}^{b} y_{N.j(.....)}^2}{b^3} - CF_N;\ SS_{NCB} \in [SS_{LCB}, SS_{UCB}],$$

$$SS_{NTr} = \frac{\sum_{k=1}^{b^2} y_{N..(k....)}^2}{b^2} - CF_N;\ SS_{NTr} \in [SS_{LTr}, SS_{UTr}],$$

$$SS_{NR} = \frac{\sum_{i=1}^{b}\sum_{l=1}^{b} y_{Ni.(.l...)}^2}{b^3} - \frac{\sum_{i=1}^{b} y_{Ni.(.....)}^2}{b^3};\ SS_{NR} \in [SS_{LR}, SS_{UR}],$$

$$SS_{NC} = \frac{\sum_{j=1}^{b}\sum_{p=1}^{b} y_{N.j(..p..)}^2}{b^2} - \frac{\sum_{j=1}^{b} y_{N.j(.....)}^2}{b^3};\ SS_{NC} \in [SS_{LC}, SS_{UC}],$$

$$SS_{NHS} = \frac{\sum_{i=1}^{b}\sum_{q=1}^{b} y_{Ni.(...q.)}^2}{b^2} - \frac{\sum_{i=1}^{b} y_{Ni.(.....)}^2}{b^3};\ SS_{NHS} \in [SS_{LHS}, SS_{UHS}],$$

$$SS_{NVS} = \frac{\sum_{j=1}^{b}\sum_{u=1}^{b} y_{N.j(....u)}^2}{b^2} - \frac{\sum_{j=1}^{b} y_{N.j(.....)}^2}{b^3};\ SS_{NVS} \in [SS_{LVS}, SS_{UVS}],$$

$$SS_{NE} = SS_{NT} - SS_{NRB} - SS_{NCB} - SS_{NTr} - SS_{NR} - SS_{NC} - SS_{NHS} - SS_{NVS};$$
$$SS_{NE} \in [SS_{LE}, SS_{UE}].$$

Table 4 presents the NANOVA table for the NSSD-IV.

4. Numerical example

The purpose of this example is to test the efficiency of our proposed analysis method. Accordingly, we conducted an analysis of the neutrosophic data using the proposed design. The nine treatments are denoted by 1, 2, ..., 9 in nine rows and nine columns, as shown in Table 9 (In the Appendix).

The following steps summarize how one would derive and compute the suitable F_N-test statistics for NSSD(I-IV) models.

Step 1: Determine the neutrosophic test hypotheses.

For example, the neutrosophic test hypotheses for testing the effect of treatments for NSSD-I can be determined as follows:

H_{N0}: The average values across the nine treatments show no significant difference

H_{N1}: At least one of the nine treatments has a mean that differs from the others.

Step 2: Calculate NANOVA Tables 5-8 for NSSD(I-IV).

Step 3: Determine the neutrosophic decomposition of F_N- tests.

For example, the neutrosophic of F_N- test of treatments for NSSD-I is $0.312 + 0.313I_N;\ I_N \in [0,0.003]$, where 0.312 represents F- test under CS and 0.313 represents indeterminate part with the degree of indeterminacy 0.003.

Step 4: Calculate $p_N - value$ at the level of significance $\alpha = 0.05$.

For example, from the NANOVA Table 6, $p_N - value$ (treatments) is [0.952, 0.951]. Thus, we accept the null hypothesis H_{N0} because $p_N - value = [0.952, 0.951] > 0.05$. i.e., the means of the nine treatments are all equal.

5. Comparison analysis

In this section, we would like to evaluate the proposed designs by comparing its performance with previously familiar ones.

Comparison by adopting the neutrosophic F_N

We noted that F_N-test statistics is neutrosophically written as $F_N = F_L + F_U I_N;\ I_N \in [I_L, I_U]$, which obviously consists of determinate and indeterminate parts. Specifically, the determinate part F_L represents F-test value under CS, whereas $F_U I_N$ is the indeterminate part. Notably, F_N reduces to classical F when $I_N = 0$. It can also be seen that the proposed NS designs provide F_N-test values in an interval of indeterminacy with a measure of uncertainty.

For example, the neutrosophic F_N-test for treatment effects in Table 5 is $0.312 + 0.313I_N;\ I_N \in [0,0.003]$. This reflects the fact that in NSSD-I, F_N ranges between 0.312 and 0.313 with a recorded degree of indeterminacy of 0.003.

In short, we can assure that all proposed NS designs outperform their classical counterparts, and are more informative and flexible.

Table 5
ANOVA Table for the NSSD-I

S. O. V	Ndf	NSS	NMS	F_N	Neutrosophic form F_N	$p_N - value$
Row Blocks	2	[206.365, 197.233]	[103.183, 98.617]	[0.967, 0.933]	$0.967 - 0.933I_N;\ I_N \in [0, 0.036]$	[0.388, 0.401]
Column Blocks	2	[12.314, 4.392]	[6.157, 2.196]	[0.058, 0.021]	$0.058 - 0.021I_N;\ I_N \in [0, 1.762]$	[0.944, 0.979]
Rows	8	[957.883, 954.691]	[119.735, 119.336]	[1.122, 1.129]	$1.122 + 1.129I_N;\ I_N \in [0, 0.006]$	[0.368, 0.363]
Columns	8	[392.503, 383.568]	[49.063, 47.946]	[0.460, 0.454]	$0.460 - 0.454I_N;\ I_N \in [0, 0.013]$	[0.877, 0.882]
Treatments	8	[266.637, 265.027]	[33.330, 33.128]	[0.312, 0.313]	$0.312 + 0.313I_N;\ I_N \in [0,0.003]$	[0.957, 0.957]
Squares	8	[295.470, 269.278]	[36.934, 33.660]	[0.346, 0.318]	$0.346 + 0.318I_N;\ I_N \in [0, 0.088]$	[0.943, 0.955]
Error	44	[4695.122, 4651.462]	[106.707, 105.715]	--	--	--

Total	80	[6826.294, 6725.650]	--	--	--	--

Table 6
ANOVA Table for the NSSD-II

S. O. V	Ndf	NSS	NMS	F_N	Neutrosophic form F_{N}	$p_N - value$
Row Blocks	2	[206.365, 197.233]	[103.183, 98.617]	[1.008, 0.975]	$1.008 - 0.975I_N;\ I_N \in [0, 0.034]$	[0.373, 0.384]
Column Blocks	2	[12.314, 4.392]	[6.157, 2.196]	[0.060, 0.022]	$0.060 - 0.022I_N;\ I_N \in [0, 1.727]$	[0.942, 0.978]
Rows	6	[751.518, 757.457]	[125.253, 126.243]	[1.224, 1.249]	$1.224 + 1.249I_N;\ I_N \in [0, 0.020]$	[0.311, 0.299]
Columns	6	[380.190, 379.175]	[63.365, 63.196]	[0.619, 0.625]	$0.619 + 0.625I_N;\ I_N \in [0, 0.010]$	[0.714, 0.709]
Treatments	8	[266.637, 265.027]	[33.330, 33.128]	[0.326, 0.328]	$0.326 + 0.328I_N;\ I_N \in [0,0.006]$	[0.952, 0.951]
Squares	8	[295.470, 269.278]	[36.934, 33.660]	[0.361, 0.333]	$0.361 - 0.333I_N;\ I_N \in [0, 0.084]$	[0.936, 0.949]
Error	48	[4913.801, 4853.087]	[102.371, 101.106]	--	--	--
Total	80	[6826.294, 6725.650]	--	--	--	--

Table 7
ANOVA Table for the NSSD-III

S. O. V	Ndf	NSS	NMS	F_N	Neutrosophic form F_{N}	$p_N - value$
Row Blocks	2	[206.365, 197.233]	[103.183, 98.617]	[0.894, 0.861]	$0.894 - 0.861I_N;\ I_N \in [0, 0.038]$	[0.417, 0.431]
Column Blocks	2	[12.314, 4.392]	[6.157, 2.196]	[0.053, 0.019]	$0.053 - 0.019I_N;\ I_N \in [0, 1.789]$	[0.948, 0.981]
Rows	8	[957.883, 954.691]	[119.735, 119.336]	[1.037, 1.041]	$1.037 + 1.041I_N;\ I_N \in [0, 0.004]$	[0.425, 0.422]
Columns	8	[392.503, 383.568]	[49.063, 47.946]	[0.425, 0.418]	$0.425 - 0.418I_N;\ I_N \in [0, 0.017]$	[0.899, 0.903]
Treatments	8	[266.637, 265.027]	[33.330, 33.128]	[0.289, 0.289]	$0.289 + 0.289I_N;\ I_N \in [0,0.001]$	[0.966, 0.966]
Squares within RB	6	[89.104, 72.044]	[14.851, 12.007]	[0.129, 0.105]	$0.129 - 0.105I_N;\ I_N \in [0, 0.229]$	[0.992, 0.995]
Squares within CB	6	[283.156, 264.886]	[47.193, 44.148]	[0.409, 0.385]	$0.409 - 0.385I_N;\ I_N \in [0, 0.062]$	[0.869, 0.884]
Error	40	[4618.332, 4583.809]	[115.458, 114.595]	--	--	--
Total	80	[6826.294, 6725.650]	--	--	--	--

Table 8
ANOVA Table for the NSSD-IV

S. O. V	Ndf	NSS	NMS	F_N	Neutrosophic form F_N	$p_N - value$
Row Blocks	2	[206.365, 197.233]	[103.183, 98.617]	[0.939, 0.907]	$0.939 - 0.907I_N;\ I_N \in [0, 0.035]$	[0.399, 0.411]
Column Blocks	2	[12.314, 4.392]	[6.157, 2.196]	[0.056, 0.020]	$0.056 - 0.020I_N;\ I_N \in [0, 1.800]$	[0.946, 0.980]
Rows	8	[751.518, 757.457]	[125.253, 126.243]	[1.139, 1.161]	$1.139 + 1.161I_N;\ I_N \in [0, 0.019]$	[0.356, 0.344]
Columns	8	[380.190, 379.175]	[63.365, 63.196]	[0.576, 0.581]	$0.576 + 0.581I_N;\ I_N \in [0, 0.009]$	[0.747, 0.743]
Treatments	8	[266.637, 265.027]	[33.330, 33.128]	[0.303, 0.305]	$0.303 + 0.305I_N;\ I_N \in [0,0.007]$	[0.961, 0.960]
Squares within RB	6	[89.104, 72.044]	[14.851, 12.007]	[0.135, 0.110]	$0.135 - 0.110I_N;\ I_N \in [0, 0.227]$	[0.991, 0.995]
Squares within CB	6	[283.156, 264.886]	[47.193, 44.148]	[0.429, 0.406]	$0.429 - 0.406I_N;\ I_N \in [0,0.057]$	[0.855, 0.871]
Error	44	[4837.011, 4785.435]	--	--	--	--
Total	80	[6826.294, 6725.650]	--	--	--	--

6. Conclusion

This paper introduces a novel Sudoku square design within the framework of neutrosophic statistics (NS). Four variations of the proposed design were examined by defining its statistical model, formulating null hypotheses, constructing NANOVA tables, and deriving appropriate test statistics for comparison with conventional classical methods. Based on the simulation study, numerical example, and comparisons, it is concluded that the proposed NSSD(I-IV) serves as a generalization of the classical SSD. Moreover, the NSSD(I-IV) demonstrates effectiveness, flexibility, and practicality, particularly in applications where the data or specification limits are neutrosophic. We anticipate that this design will find wide application in future scientific research conducted under uncertain conditions.

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Appendix

Table 9
Data for the NSSD(I-IV)

	1	2	3	4	5	6	7	8	9
1	1 [59.33, 63.78]	2 [60.62, 63.31]	3 [48.64, 51.33]	4 [35.25, 39.08]	5 [49.52, 52.27]	6 [43.14, 46.74]	7 [41.06, 43.7]	8 [36.94, 40.61]	9 [45.18, 46.4]
2	4 [51.85, 55.01]	5 [35.52, 38.83]	6 [34.5, 35.87]	7 [52.52, 55.73]	8 [61.95, 64.17]	9 [43.13, 46.27]	1 [39.28, 42.35]	2 [51.55, 54.07]	3 [57.6, 62.54]
3	7 [36.07, 40.7]	8 [37.09, 40.63]	9 [40.2, 42.95]	1 [39.89, 42.04]	2 [21.92, 25.56]	3 [39.14, 41.56]	4 [47.66, 51.02]	5 [37.54, 39.58]	6 [47.73, 50.37]
4	2 [45.02, 47.09]	3 [38.16, 41.94]	4 [44.94, 48.44]	5 [49.45, 54.26]	6 [60.84, 62.65]	7 [25.75, 29.07]	8 [44.49, 47.29]	9 [36.57, 40.26]	1 [51.7, 54.64]
5	5 [39.27, 42.95]	6 [62.6, 66.63]	7 [59.93, 62.63]	8 [63.78, 66.44]	9 [35.6, 39.11]	1 [55.17, 57.6]	2 [58.9, 62.17]	3 [48.32, 50.09]	4 [40.54, 44.74]
6	8 [41.38, 43.15]	9 [43.95, 47]	1 [43.1, 45.81]	2 [46.99, 49.7]	3 [42.51, 45.49]	4 [55.16, 58.93]	5 [41.51, 43.28]	6 [58.64, 62.79]	7 [60.06, 61.4]
7	3 [55.25, 57.8]	4 [45.23, 47.18]	5 [65.1, 67.3]	6 [50.83, 55.14]	7 [38.95, 42.15]	8 [48.12, 52.73]	9 [50.14, 52.99]	1 [49.02, 51.63]	2 [38.44, 40.79]
8	6 [44.76, 47.28]	7 [47.26, 50.14]	8 [44.55, 47.78]	9 [56.35, 60.69]	1 [46.98, 48.49]	2 [28.88, 33.54]	3 [39.24, 41.02]	4 [60.12, 62.41]	5 [58.14, 60.14]
9	9 [36.78, 38.88]	1 [47.26, 49.69]	2 [53.43, 54.27]	3 [47.11, 49.74]	4 [50.72, 53.42]	5 [50.21, 52.88]	6 [33.22, 35.56]	7 [36.34, 41.39]	8 [52.63, 55.94]

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