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Option valuation using Garch-type models : Empirical evidence using USD/INR data

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Abstract

The aim of this paper[1] is to evaluate the option pricing performance of three GARCH-type models against the Black-Scholes model. The standard GARCH model and the asymmetric GJR-GARCH and EGARCH models are employed with the Gaussian innovation distribution. The models are assessed on two sample sets of USD/INR currency European call options data obtained from the National Stock Exchange of India. A Monte Carlo simulation approach is implemented on the estimated GARCH models for the valuation of European call options. An accuracy analysis of the option prices is then performed across different moneyness and maturity using relative loss functions. The three GARCH models performed better than the Black-Scholes model in all samples of USD/INR for all moneyness and maturity. From an overall standpoint, the GJR-GARCH model displayed the best option pricing accuracy.

Subject Classification (2010): *62P05, 91G20.*

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1. Introduction

Options trading has gained high momentum in recent years, with the US and Europe normally accounting for the highest volume of options

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traded. However, emerging economies such as India has also experienced spectacular growth in the financial derivatives market. The volume of stock options traded in the National Stock Exchange of India (NSE) increased from 80 million in 2013 to 677 million in 2021 with booms in turnover from 38 million (crores) to 1.6 billion (crores). India has also a booming foreign exchange market. The USD/ INR currency pair are the currencies that are mostly traded in India with the total volume of currency option contracts traded daily at the NSE in 2019 revolving around 2.6 million. The country is a major recipient of foreign portfolio investment and ranked among the top four in the MSCI Emerging Markets Index [1]. Investors are quite concerned with a volatile rupee which increases portfolio risk. Investors are therefore increasingly interested in currency options trading that serves to hedge against potential losses and at the same time ensuring payoff, and for arbitrage purposes. Given the importance of option trading in the financial derivatives market, any option mispricing might prove disastrous to investors considering the volume of derivatives traded in the market. It is necessary to have models that can compute a fair option price and at the same time it should be flexible and simple enough to implement.

Over decades, practitioners have been relying on the Black-Scholes model (BSM) proposed in [2] for estimating a fair price for option contracts. The model was developed under several assumptions which include amongst others, a constant and risk-free interest rate through the option lifetime, no dividends payment on stock, and zero transaction cost in purchasing or selling options [3]. Another crucial assumption is that the underlying asset price follows a Geometric Brownian motion process with a constant volatility. Many empirical studies and crash events of the financial markets have exposed the mispricing of the BSM model due to the misspecification of the asset dynamics [4]. Several researchers have criticized the assumption of constant volatility which is incompatible with the actual volatility dynamics prevailing in the stock market as characterized by the volatility smile phenomenon.

Challenging the constant volatility assumption, a substantial number of studies in the option pricing research domain focused on continuous stochastic volatility modelling [5, 6, 7]. Although continuous stochastic volatility models have demonstrated superior performance over the BSM, they were still deficient in their efficiency to capture common stylized facts in the financial market, and they were cumbersome to implement. Volatility clustering, leptokurtosis, the leverage effect, and

heteroscedasticity are among the well-known stylized features in the financial market that affect option pricing [8].

During the last decades, many GARCH-type models with varying specifications have been developed. For instance, the asymmetric GJR-GARCH and EGARCH variants were proposed to address the limitation of symmetric GARCH model in capturing the leverage effect. The work proposed by Duan in 1995 [9] was the pioneer in employing GARCH models for option pricing. The methodology used the locally risk neutral valuation relationship, where it is assumed that investors are risk neutral in the option pricing process. This approach has been efficiently applied by several past studies.

Empirical studies on the evaluation of model performance for option pricing have been mostly carried out for pricing options on financial stocks [10, 11, 12]. GARCH models generally provide more accurate option prices than the benchmark BSM model, though minor nuances were noted across different levels of maturity and moneyness. Asymmetric GARCH variants exhibit superior option pricing accuracy which is accounted for their ability to capture the leverage effect [13, 14, 15, 16]. Researchers have developed more complex GARCH models that were applied to option pricing such as the GARCH-in-jump [17], the GARCH with Student's t-distribution [18] and the model in [19] where a filtering technique is proposed to get the latent conditional variance. However, as complexity of a model increases, the implementation becomes more challenging and computationally costly. A right tradeoff between the model performance and complexity is necessary. As recommended by the study in [13], a simple parsimonious GARCH model that can capture the leverage effect can perform efficiently in option pricing.

The study by Venter and Mare in 2020 in [20] studied the option pricing performance of the symmetric GARCH and the asymmetric AGARCH and GJR-GARCH models applied to FTSE/JSE top 40 index from January 2010 to April 2019. They concluded that the asymmetric GARCH variants performed best. A similar study is performed in [21] where the standard GARCH is assessed against the asymmetric EGARCH and GJR-GARCH models on a sample of SET50 option data for the period 2015 to 2016. The findings concluded on the superiority of the asymmetric GARCH models in the valuation of option prices.

While researchers have successfully implemented different GARCH models in stock option pricing, fewer studies performed empirical investigation on GARCH models for pricing alternative market derivatives such as foreign exchange. The research work in [22] was the pioneering

study in employing the standard GARCH model to price foreign exchange options. The study in [23] compared the performance of standard GARCH and the BSM model when pricing options on currencies written on Japanese Yen and GBP amongst others. They found that the BSM performed much better than the standard GARCH model. Similar findings are shared by [24] and [25] whereby the BSM outperformed GARCH models in their currency option valuation experiments.

This paper contributes to the literature by proposing the application of GARCH-type models for the pricing of currency European call options in the Indian derivatives market. The aim of the paper is to assess the option pricing performance of the symmetric standard GARCH model and the asymmetric GJR-GARCH and EGARCH models. The comparative analysis also includes the benchmark Black-Scholes model. The accuracy of the valuation models is assessed against actual option market prices across different moneyness and maturity of options.

2. Methodology

2.1 *GARCH-type Models*

Under the physical probability measure $\mathbb{P}$, the logarithmic return of the underlying asset for the standard GARCH model [9] is given by

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \varepsilon_t \tag{1}$$

where r is the risk-free interest rate, λ is the asset risk premium[2], σ_t is the conditional standard deviation and ε_t follows a normal distribution with mean zero and variance σ_t^2.

The standard GARCH (p, q) process in [26] models the conditional variance σ_t^2 using the expression

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2. \tag{2}$$

[2] The asset risk premium (market price of risk), λ, is given by the following equation: $\lambda = \frac{\mu - r}{\sigma}$, where μis the return on the traded security, σ is the volatility and r is the risk-free interest rate. Rearranging the equation given above in the form: $\mu - r = \lambda\sigma$ is analogous to the capital asset pricing model which relates the expected excess return on an asset to its risk [27].

2.1.1 Standard GARCH (1, 1)

The standard GARCH (1, 1) can be expressed as follows

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{3}$$

with the conditions

$$\alpha_0 > 0, \alpha_1 \geq 0, \beta_1 \geq 0, \alpha_1 + \beta_1 \leq 1, \tag{4}$$

where α_0 is a combination of the long-term variance V_L and the weight ω such that $\alpha_0 = \omega V_L$, β_1 is the weight given to the lagged variance and α_1 determines the kurtosis.

2.1.2 GJR- GARCH (1, 1)

This model incorporates the leverage parameter that measures negativity in shocks. The paper [28] proposed an indicator function in the form of $\mathbb{1}_{\{\varepsilon_{t-1<0}\}}$ which is meant to capture asymmetry of positive and negative shocks. When shocks are negative, the function, $\mathbb{1}_{\{\varepsilon_{t-1<0}\}}$ takes a value of one otherwise it will take a value of zero if the shocks are positive.

Under the physical measure, the log return of underlying asset takes the form:

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = r + \lambda\sigma_t - \frac{1}{2}\sigma_t^2 + \sigma_t \varepsilon_t \tag{5}$$

The conditional variance σ_t^2 for the GJR-GARCH(1,1) takes the form

$$\sigma_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \gamma \mathbb{1}_{\{\varepsilon_{t-1<0}\}} \varepsilon_{t-1}^2 \tag{6}$$

with the conditions

$$\alpha_0, \alpha_1 > 0,\ \beta_1 \geq 0,\ \alpha_1 + \gamma \geq 0, \gamma < 2(1 - \alpha_1 - \beta_1),$$

The coefficient γ determines the leverage effect. The higher the value of γ, the higher is the impact of negative shocks on the conditional variance.

2.1.3 EGARCH (1, 1)

The EGARCH model, developed in [29], is also an asymmetric GARCH model that can capture the leverage effect represented by the parameter γ. The EGARCH model assumes the same specification for the log returns as for the GJR-GARCH model, given in equation (5). The conditional variance for the EGARCH(1,1) is given by

$$\ln \sigma_t^2 = \alpha_0 + \beta_1 \ln \sigma_{t-1}^2 + \alpha_1 (|\varepsilon_{t-1}| + \gamma \varepsilon_{t-1}) \tag{7}$$

In the EGARCH specification, β_1 represents the persistence in conditional volatility. The higher the value of β_1 implies a stronger clustering of volatility. The coefficient α_1 denotes the kurtosis of the distribution which signifies that the higher the value of α_1, the deeper is the volatility smile.

2.1.4 The innovation distribution

The innovation term ε_t is assumed to follow the normal distribution with mean 0 and variance σ^2. The probability density function is given by

$$f_{\varepsilon_t}(x_t) = \frac{1}{\sigma_t \sqrt{2\pi}} e^{-\frac{x_t^2}{2\sigma_t^2}} . \tag{8}$$

2.1.5 Parameter Estimation

The maximum likelihood method (MLE) is used to estimate the parameters of the GARCH-type models under the physical measure $\mathbb{P}$ using historical returns only. This is feasible since there is no separate parameter for the risk neutral process.

The log-likelihood function for the GARCH-type models is given by

$$\ln L_R = -\frac{N}{2} \ln 2\pi - \frac{1}{2} \sum_{t=1}^{N} \left(\ln \sigma_t^2 + \frac{\left[\ln \frac{S_t}{S_{t-1}} - r - \lambda \sigma_t + \frac{1}{2} \sigma_t^2 \right]^2}{\sigma_t^2} \right) \tag{9}$$

Where *N*is the number of historical returns in the estimation period (Hao and Zhang, 2013).

2.1.6 Model Selection

The model selection to determine the best fitting model is performed using the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) metrics. The model displaying the lowest AIC and BIC values will have the best fitting accuracy. The AIC and BIC metrices are computed as

$$AIC = -2 \ln L + 2k \tag{10}$$

$$BIC = -2 \ln L + k \ln N \tag{11}$$

where L is the maximized value of the likelihood function; k is the number of estimated parameters and N is the sample size. The BIC approach adds a penalty based on the number of model parameters, in favor of the principle of parsimony.

2.2 *Option Pricing Models*

This paper aims to perform a comparative analysis of the accuracy of the three GARCH-type models in the valuation of European options. The experiment will also compute prices using the Black-Scholes model.

2.2.1 The Black-Scholes Model

The Black-Scholes model is used as a benchmark in this analysis mainly because of the convenience of implementing the model, and the easy computation of option prices and replication of observed prices given implied volatilities. Consider a European call option with strike price K and maturity time T, defined on an underlying asset whose price at t is S_t and volatility is σ.

Using the Black-Scholes model, the analytical price C_t of the European call option at time $0 < t < T$ is given as follows

$$C_t = N(d_1)S_t - N(d_2)Ke^{-r(T-t)} \tag{12}$$

where

$$d_1 = \frac{\ln\left(\frac{S_t}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \text{ and } d_2 = d_1 - \sigma\sqrt{T-t}, \tag{13}$$

where r is the risk-free rate.

2.2.2 GARCH Models for option pricing

Under the risk neutral measure $\mathbb{Q}$, the price of an option is equal to the expectation of the pay-off discounted at the risk-free rate. Even though the GARCH models are calibrated under the physical measure $\mathbb{P}$ as specified in equation (1), it is necessary to implement the risk neutral component for the underlying asset. Under the measure $\mathbb{Q}$, the methodology in [9] proposed the following dynamics for the log returns

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = r - \frac{1}{2}\sigma_t^2 + \xi_t \tag{14}$$

where $\xi_t \sim N(0, \sigma_t^2)$. A modified risk neutral valuation relationship for the GARCH (p, q) is recommended, whereby

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i (\xi_{t-i} - \lambda \sigma_{t-i})^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-i}^2. \tag{15}$$

2.2.2.1 The standard GARCH (1, 1)

Under the risk neutral measure $\mathbb{Q}$, the GARCH (1, 1) takes the following form

$$\sigma_t^2 = \alpha_0 + \alpha_1 (\xi_{t-1} - \lambda \sigma_{t-1})^2 + \beta_1 \sigma_{t-1}^2, \tag{16}$$

where $\xi_t \sim N(0, \sigma_t^2)$ and λ is the equity risk premium. Moreover, the stationarity constraints of the standard GARCH (1,1) model are defined by:

$$\alpha_1 (1 + \lambda^2) + \beta_1 < 1 \tag{17}$$

2.2.2.2 GJR-GARCH (1, 1)

Using the risk neutral valuation relationship, the price process is given by:

$$\ln\left(\frac{S_t}{S_{t-1}}\right) = r - \frac{1}{2}\sigma_t^2 + \sigma_t \xi_t. \tag{18}$$

where $\xi_t \sim N(0, \sigma_t^2)$. The conditional variance takes the following form when it is modelled using GJR-GARCH (1, 1) under the $\mathbb{Q}$ -measure

$$\sigma_t^2 = \alpha_0 + (\xi_{t-1} - \lambda \sigma_{t-1})^2 [\alpha_1 + \gamma \mathbb{1}_{\{(\xi_{t-1} - \lambda \sigma_{t-1}) < 0\}}] + \beta_1 \sigma_{t-1}^2. \tag{19}$$

The stationarity constraint of the GJR-GARCH process is given by

$$\alpha_1 (1 + \lambda^2) + \beta_1 + \gamma \left[\frac{\lambda}{\sqrt{2\pi}} \exp\left(-\frac{\lambda^2}{2} \right) + (1 + \lambda^2) \Phi(\lambda) \right] < 1. \tag{20}$$

2.2.2.3 EGARCH (1, 1)

The price process under the risk neutrality criteria for the EGARCH specification is the same as in equation (18). The conditional variance for the EGARCH(1,1) takes the following form under the $\mathbb{Q}$ -measure

$$\ln \sigma_t^2 = \alpha_0 + \beta_1 \ln \sigma_{t-1}^2 + \alpha_1 [(|\xi_{t-1} - \lambda|) + \gamma (\xi_{t-1} - \lambda)] \tag{21}$$

where $\xi_t \sim N(0, \sigma_t^2)$. The condition $\beta_1 < 1$ prevents the conditional variance from taking extreme values with time. Furthermore, the condition $|\beta_1| < 1$ must be satisfied for stationarity. Hence a strictly stationary solution is given by

$$\ln \sigma_t^2 = \frac{\alpha_0}{1-\beta_1} + \alpha_1 \sum_{k=1}^{+\infty} \beta_1^{k-1} (|\varepsilon_{t-k}| + \gamma \varepsilon_{t-k}). \tag{22}$$

2.2.3 Monte Carlo Simulation

The GARCH models do not offer closed-form solution for option pricing but their discrete-time structure renders option pricing straight forward with the Monte Carlo simulation method as in [20]. For each observation in the out-sample dataset, 50,000 simulations are done to generate 50,000 paths to compute the option price. Under the no-arbitrage assumption, the price of an option equals to its discounted expected payoff at maturity. For instance, the numerical price of a European call is given by

$$c_0 = \exp\left(-\frac{rT}{365}\right) \times \max(S_T - K, 0). \tag{23}$$

Once the GARCH models are estimated, we employ the estimated model parameters, the current value of the underlying asset, S_0 and the risk-free interest rate r to price the European call options. We assume that the initial risk neutral variance σ_0 and the initial value of ξ_0 to be the unconditional variance and zero respectively. Algorithms I and II for the Monte Carlo simulation are employed for the symmetric GARCH model and the asymmetric GARCH models respectively.

Algorithm I - Standard GARCH option pricing model using Monte Carlo

1. Generate $(Z_{1,1}, \ldots, Z_{M,T}) \sim N(0,1)$ where M is the number of simulations and T is the number of days in the simulation period.
2. Use the GARCH(1,1) specification under the $\mathbb{Q}$ -measure to compute

   $$\sigma_{j,t}^2 = \alpha_0 + \alpha_1 (\xi_{j,t-1} - \lambda \sigma_{j,t-1})^2 + \beta_1 \sigma_{j,t-1}^2$$

   where $\xi_{j,t} = \sigma_{j,t} \times Z_{j,t}$, for $t = 1, \ldots, T$.
3. Compute $S_{j,t} = S_{j,t-1} \exp\left(r - \frac{1}{2}\sigma_t^2 + \xi_t\right)$, for $t = 1, \ldots, T$.

4. Repeat steps 2. and 3. M times, that is for $j = 1, \ldots, M$.
5. The option price is equal to the discounted payoff function (f) applied to the underlying:

$$V_0 = \exp\left(-\frac{rT}{365}\right)\frac{1}{M}\sum_{j=1}^{M} f(S_{j,t}), \tag{24}$$

where $f(S_{j,t}) = \max(S_{j,t} - K, 0)$.

$$\sigma_t^2 = \alpha_0 + (\xi_{t-1} - \lambda\sigma_{t-1})^2[\alpha_1 + \gamma \mathbb{1}_{\{(\xi_{t-1}-\lambda\sigma_{t-1})<0\}}] + \beta_1\sigma_{t-1}^2.$$

$$\ln\sigma_t^2 = \alpha_0 + \beta_1 \ln\sigma_{t-1}^2 + \alpha_1[(|\xi_{t-1} - \lambda|) + \gamma(\xi_{t-1} - \lambda)]$$

Algorithm II - GJR-GARCH & EGARCH option pricing models using Monte Carlo

The steps to compute option prices for the asymmetric GARCH using Monte Carlo simulation are same to those defined in Algorithm I, except that the conditional variance in step 2. is computed using the expressions in (19) and (21) for the GJR-GARCH(1, 1) and EGARCH(1,1) respectively.

2.2.4 Performance Metrics

The numerical experiments employ the Root Mean Squared Error (RMSE) and the Mean Absolute Percentage Error (MAPE) to assess the performance of the GARCH models in the valuation of options [30]. The two metrics are defined as follows

Root Mean Squared Error (RMSE %)

The RMSE measures the relative error of the model option price with respect to the current market price.

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\left(\frac{C_n^{market} - C_n^{model}}{C_n^{market}}\right)^2} \tag{25}$$

Mean Absolute Percentage Error (MAPE %).

The Mean Percentage Absolute Error (MAPE) refers to the relative absolute ratio of the differences between the actual market price and the model option price to the current market price.

$$MAE = \frac{1}{N}\sum_{n=1}^{N}\left|\frac{C_n^{market} - C_n^{model}}{C_n^{market}}\right| \tag{26}$$

3. Numerical Results

3.1 *Data*

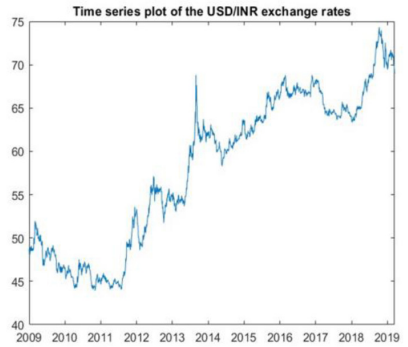
Daily stock closing prices of USD/INR exchange rate are obtained from the website of the National Stock Exchange of India (Nseindia.com). The historical data is used to estimate the GARCH model parameters. We consider two in-sample datasets in the numerical experiments of this study. The first sample of data is retrieved for the period 1st January 2009 to 15th March 2019 and comprises 2662 observations. The corresponding out-sample dataset consists of USD/INR currency-pair European call options traded on the 15th March 2019. This dataset has a total of 470 observations, after applying option filtering. The second in-sample dataset is retrieved for the period 1st January 2009 to 22nd March 2019 and contains 2667 observations. The corresponding out-sample dataset consists of USD/INR currency-pair European call options traded on the 22nd March 2019 and has a total of 510 observations.

3.1.1 Computation of log returns

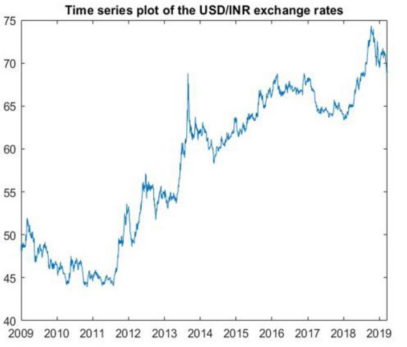
It is commonly known that the time series of financial prices exhibit non-stationarity. The non-stationarity of the USD/INR exchange rates is confirmed by plots (a) and (b) in Figure 1 which indicate that the price series show an increasing trend over several periods. To address the issue of non-stationarity, the price series data is converted into logarithmic returns

$$Y_t = \ln\left(\frac{S_t}{S_{t-1}}\right) \tag{27}$$

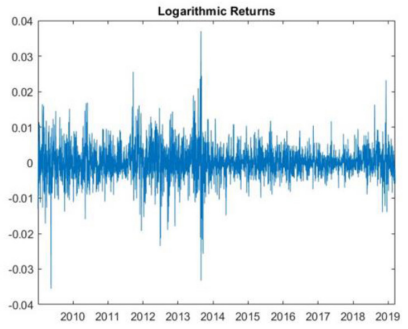
where Y_t is the continuously compounded log return and, S_t and S_{t-1}denote the daily closing price at time t and $t - 1$ respectively. As depicted in plots (c) and (d) in Figure 1, the logarithmic transformation has stabilized the series. The plots (c) and (d) moreover display the presence of volatility clustering. We can observe that large fluctuations tend to be followed by large fluctuations, of either sign, and small fluctuations tend to be followed by small fluctuations. Some preliminary analyses and statistical tests are



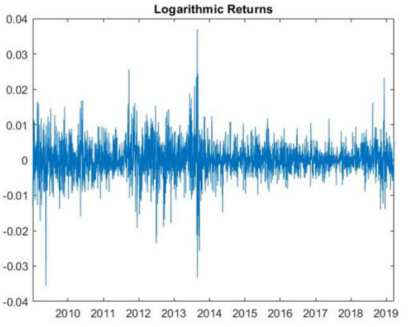
(a) Daily Prices of USD/INR (1st Jan 2009 -15th Mar 2019)



(b) Daily Prices of USD/INR (1st Jan 2009 -22nd Mar 2019)



(c) Daily Log Returns of USD/INR (1st Jan 2009 -15th Mar 2019)



(d) Daily Returns of USD/INR (1st Jan 2009-22nd Mar 2019)

Figure 1

Daily Prices and Returns for USD/INR exchange rates

performed on the datasets to confirm the findings of the graphical interpretation.

3.2 *Empirical Properties*

3.2.1 Summary Statistics

Table 1 presents the summary measures of the two in-sample datasets of the USD/INR exchange rates. The daily returns display a highly positive value of excess kurtosis, revealing the presence of leptokurtosis. The mean of USD/INR log returns revolves around 0, and the minimum and maximum values deviate significantly from the mean value. This implies

the presence of extreme events and confirms the departure of the log returns from the normal distribution.

Table 1

Summary Measures for USD/INR Exchange rate returns

	USD/INR	
	1st Jan 2009 to 22nd Mar 2019	**1st Jan 2009 to 15th Mar 2019**
Mean	0.00013123	0.00013033
Standard Deviation	0.0105	0.0107
Minimum	-0.0355	-0.0355
Maximum	0.0370	0.0370
Skewness	0.5749	0.6786
Excess Kurtosis	15.8974	15.0957

3.2.2 Normality test

The Jarque-Bera test is conducted to further ascertain the non-normality of the log return series. The results in Table 2 show that the test returns p-values = 0.001, thus implying that the log return series are not normally distributed at the 5% significance level.

Table 2

Jarque-Bera Normality test

Jarque Bera Test			
Data Period	**X-Squared**	**P-value**	**Test conclusion**
1st Jan 2009 -22nd Mar 2019	35998	0.001	Not normally distributed
1st Jan 2009 -15th Mar 2019	36547	0.001	Not normally distributed

3.2.3 Stationary test

The ADF test is applied on the series to investigate their stationarity behavior. The results in Table 3 conclude that the log returns series data are stationary at the 5% significance level.

Table 3
ADF test

ADF Test			
Data Period	**Test Statistic**	**P-value**	**Conclusion**
1st Jan 2009 -22nd Mar 2019	-10.5821	0.001	Stationary
1st Jan 2009 -15th Mar 2019	-10.5252	0.001	Stationary

3.2.4 Conditional mean

The conditional mean process of the GARCH models is modelled using an ARMA (p, q) model in order to cater for the presence of autocorrelation [31]. Since the analysis of the autocorrelation function and the partial autocorrelation function does not conclude on the parameters p and q, we determine the optimal values of p and q by assessing several specifications of the ARMA (p, q) model. Using the AIC and BIC scores, we conclude that the ARMA(1,1) model provides the best fitting accuracy for the conditional mean process. The Ljung-Box Q test was applied on the residuals of the ARMA(1,1) and the results conclude on the absence of auto-correlation.

3.2.5 ARCH effects

The ARCH-LM test is performed and concludes on the presence of ARCH effects in all the residuals series. This finding therefore supports the use of the GARCH-type models.

3.3 *Estimation of the GARCH models*

The standard GARCH, the GJR-GARCH and the EGARCH models are implemented under the Gaussian innovation distribution to model the USD/INR exchange rate for the periods 1st Jan 2009 to 15th Mar 2019 and 1st Jan 2009 to 22nd Mar 2019. Table 4 displays the GARCH parameter estimates with the corresponding p-values within the parentheses.

The results show that for both datasets, the average conditional variance denoted by the parameter α_0 and the ARCH parameter α_1 are highly significant for the GARCH models. Furthermore, the GARCH parameter β_1 shows the largest weight among the parameters and is significant for all the GARCH models. This confirms the presence of the volatility clustering stylized feature. The persistence values are less than 1, which satisfies the stationary constraint $\alpha_1 + \beta_1 < 1$. The leverage parameter

Table 4
Parameter estimates for each GARCH model

	USD/INR exchange rate (1st Jan 2009 to 15th Mar 2019)			USD/INR exchange rate (1st Jan 2009 to 22nd Mar 2019)		
	GARCH	EGARCH	GJR-GARCH	GARCH	EGARCH	GJR-GARCH
α_0	3.5105e-06 (5.5181e-13)	-0.086717 (8.1697e-09)	3.5105e-06 (6.5763e-13)	3.5143e-06 (5.5667e-13)	-0.08725 (7.2215e-09)	3.5143e-06 (6.6337e-13)
α_1	0.2216 (5.4559e-23)	0.088353 (4.8353e-25)	0.2216 (5.4559e-23)	0.22164 (8.2525e-36)	0.088561 (4.2088e-25)	0.22164 (5.9697e-23)
β_1	0.62781 (4.0497e-227)	0.99174 (0)	0.62781 (6.5871e-221)	0.62781 (1.1786e-226)	0.99169 (0)	0.62781 (1.8393e-220)
γ	-	0.035667 (3.3109e-14)	2e-12 (1)	-	0.035919 (2.4258e-14)	2e-12 (1)
Persistence	0.84941	0.99174	0.62781	0.84945	0.99169	0.62781

γ is highly significant for the EGARCH model but insignificant for the GJR-GARCH model. Unlike the GJR-GARCH model, the presence of asymmetry for the EGARCH model can be confirmed. It can also be observed that the leverage parameter has positive values for both models implying that there is a negative correlation between shocks to returns and volatility.

The model selection for the daily returns is conducted using the Akaike Information Criteria (AIC) and the Bayesian Information Criterion (BIC). The model which generates the lowest AIC and BIC values is the best fitted model. Table 5 shows the results for the model selection criteria. The EGARCH(1, 1) performs better than its counterparts for the two different periods taken from the datasets. The second-best performing model is the GJR-GARCH(1, 1). The symmetric GARCH (1, 1) model is the

Table 5
Model selection criteria

	USD/INR exchange rates (1st Jan 2009 to 15th Mar 2019)			USD/INR exchange rates (1st Jan 2009 to 22nd Mar 2019)		
	GARCH	EGARCH	GJR-GARCH	GARCH	EGARCH	GJR-GARCH
AIC	-21274.8566	-21456.1524	-21272.8566	-21314.877	-21496.0605	-21312.877
BIC	-21257.1983	-21432.6081	-21249.3122	-21297.2131	-21472.5086	-21289.3251

worst fitted model. This may be due to its inability to capture the fat tails in the returns data.

3.4 *Option pricing*

3.4.1 The out-sample dataset

The option out-sample datasets are subjected to a systematic option filtering technique to obtain a proper sample for the model fitting [25]. European call options with trading volume less than 100 and options data with days to expiry less than 5 and more than 100 days are excluded. Furthermore, arbitrage check is also done whereby call options data not meeting the arbitrage restriction $C(t) \geq \max(0, S(t) - Ke^{-rT})$ are removed. Lastly, only option data whose moneyness ranges from 0.9 to 1.2 are retained. Moneyness, *M*, is the given by the ratio of the strike price to the underlying spot price, *K/S*.

3.4.2 Computation of option prices

Option prices are computed under the GARCH(1,1), the GJR-GARCH(1,1) and the EGARCH(1,1) specifications using the Monte Carlo simulation method. The conditional variance value for each GARCH model corresponding to the quote date of the option contracts is used as the initial volatility in the algorithm. The RMSE and MAPE metrics are calculated across the different maturities and moneyness for each of the models, to measure the pricing error relative to the quoted market option prices.

The numerical experiment classifies the options contracts by moneyness into three categories namely (1) in-the-money (ITM) if $0.9 \leq M \leq 0.97$, (2) near-the-money (NTM) if $0.98 \leq M \leq 1.01$ and (3) out-of-the-money (OTM) if $1.02 \leq M \leq 1.2$. The options are also categorized in terms of their expirations. Options having time to maturity between 30 to 36 days, between 37 to 65 days and between 65 and 100 days are classified in the short-term (ST) category, the mid-term (MT) category and the long-term (LT) category respectively.

Table 6 presents the RMSE and MAPE scores for USD/INR call options by models for different maturity terms (ST, MT and LT) and moneyness (ITM, NTM, and OTM), for the period of 15th March 2019. The comparative analysis also considers the prices computed using the Black-Scholes model.

Table 6

USD/INR call options valuation errors (15th March 2019)

Short term Maturity								
	Black-Scholes		GARCH (1, 1)		GJR-GARCH (1, 1)		EGARCH (1, 1)	
(%)	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
ITM	0.13914	0.132912	0.11830	0.11747	0.11943	0.11852	0.13391	0.13167
NTM	0.15395	0.149046	0.13945	0.13626	0.13906	0.13608	0.14956	0.14825
OTM	0.32742	0.163458	0.06470	0.03860	0.06907	0.04357	0.06920	0.04350

Medium term Maturity								
	Black-Scholes		GARCH (1, 1)		GJR-GARCH (1, 1)		EGARCH (1, 1)	
(%)	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
ITM	0.154604	0.152924	0.14128	0.14092	0.14155	0.14064	0.15464	0.14224
NTM	0.164061	0.153175	0.13233	0.12733	0.13116	0.12575	0.15739	0.15520
OTM	0.393662	0.291005	0.10085	0.09924	0.10087	0.09920	0.10183	0.09969

Long term Maturity								
	Black-Scholes		GARCH (1, 1)		GJR-GARCH (1, 1)		EGARCH (1, 1)	
(%)	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
ITM	0.19407	0.18798	0.16205	0.16181	0.16088	0.16064	0.18826	0.18008
NTM	0.18920	0.17082	0.14688	0.14576	0.14810	0.14398	0.17369	0.16356
OTM	0.36065	0.32292	0.13597	0.15487	0.13576	0.15432	0.16096	0.16870

Under the short-term category, the standard GARCH overall outperforms its counterparts, displaying the lowest valuation errors except for the near-the-money category whereby GJR-GARCH outperforms all the models. The symmetric GARCH is then followed by the asymmetric GJR-GARCH model. The EGARCH model is the most underperforming model of all the GARCH models. With regards to the same maturity category, the Black-Scholes is the least accurate model as it has the highest valuation errors.

For the medium-term and long-term maturity categories, it can be seen that the asymmetric GJR-GARCH model has the best performance in pricing the call options at all levels of moneyness. The leverage effect in the foreign exchange market is relevant given the noise traders speculation and hedging activities through the foreign exchange derivatives [32]. The

superiority of the GJR-GARCH in the medium and long-term maturity can be explained by the fact the asymmetric model has captured volatility persistence and leverage effect present in the USD/INR exchange rate returns. This finding is coherent with [33] regarding the capability of asymmetric model in capturing leverage effect. The second-best performing model is the symmetric GARCH model and the EGARCH is the worst performing GARCH-type model. The least accurate model is the Black-Scholes, yielding the largest valuation errors. Thus, the findings corroborate with past studies which found the superior performance of GARCH models over the classical Black-Scholes in option pricing [33, 34, 35, 36]. The study in [37] attempted to explain the difference in the performance of the GARCH-type models. They suggested that the EGARCH performs better in an economic downturn when it comes to option pricing unlike the simple GARCH which shows good performance in an upward moving economy. This seems to support the findings obtained in this study.

Table 7 displays the corresponding pricing error results for USD/INR call options for the period 22nd March 2019.

Table 7

USD/INR call options valuation errors (22nd March 2019)

Short term Maturity								
	Black-Scholes		**GARCH (1, 1)**		**GJR-GARCH (1, 1)**		**EGARCH (1, 1)**	
(%)	**RMSE**	**MAPE**	**RMSE**	**MAPE**	**RMSE**	**MAPE**	**RMSE**	**MAPE**
ITM	0.11803	0.11537	0.00726	0.00596	0.00803	0.00653	0.01976	0.01651
NTM	0.34863	0.31003	0.08538	0.06461	0.09214	0.07623	0.09820	0.07901
OTM	0.45486	0.30296	0.10691	0.10577	0.11475	0.11055	0.11855	0.11792
Medium term Maturity								
	Black-Scholes		**GARCH (1, 1)**		**GJR-GARCH (1, 1)**		**EGARCH (1, 1)**	
(%)	**RMSE**	**MAPE**	**RMSE**	**MAPE**	**RMSE**	**MAPE**	**RMSE**	**MAPE**
ITM	0.13949	0.13795	0.04500	0.04307	0.04468	0.04263	0.07809	0.07460
NTM	0.23108	0.22911	0.04595	0.03538	0.04125	0.03218	0.09876	0.09334
OTM	0.42993	0.33139	0.12107	0.11418	0.11967	0.11154	0.14223	0.12908

Contd...

Long term Maturity								
	Black-Scholes		GARCH (1, 1)		GJR-GARCH (1, 1)		EGARCH (1, 1)	
(%)	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE	RMSE	MAPE
ITM	0.15511	0.15383	0.08203	0.08052	0.08167	0.08011	0.08901	0.08451
NTM	0.24629	0.24287	0.06530	0.05743	0.06332	0.05564	0.09946	0.00940
OTM	0.28889	0.26426	0.12784	0.11527	0.12040	0.11534	0.17755	0.16778

Under the short-term category, the standard GARCH model outperforms all the other models across all moneyness. However, for the medium term and long-term category, the GJR-GARCH model provides the best performance across all moneyness, showing comparable results as the out-sample dataset for 15th March 2019.

Table 8 summarises the performance of the option pricing models in terms of their relative rankings for each maturity category.

Table 8

Ranking of the option pricing models

Rank	15th March 2019 dataset				22nd March 2019 dataset		
	STM	MTM	LTM		STM	MTM	LTM
1st	GARCH	GJR-GARCH	GJR-GARCH		GARCH	GJR-GARCH	GJR-GARCH
2nd	GJR-GARCH	GARCH	GARCH		GJR-GARCH	GARCH	GARCH
3rd	EGARCH	EGARCH	EGARCH		EGARCH	EGARCH	EGARCH
4th	Black-Scholes	Black-Scholes	Black-Scholes		Black-Scholes	Black-Scholes	Black-Scholes

4. Conclusion

This study explored the option pricing performance of the symmetric standard GARCH model and the asymmetric GJR-GARCH and EGARCH models, vis-à-vis the well-known Black-Scholes model. The GARCH models are complemented by the Monte Carlo simulation to generate European call option prices. The pricing accuracy of the models are gauged on two sample sets of the USD/INR currency European call options data quoted from the National Stock Exchange of India. The empirical findings show that the GARCH models outperformed the Black-Scholes model in all moneyness and maturity for the USD/INR currency pair Euro calls. From an overall standpoint, the asymmetric GJR-GARCH model provides the best option pricing accuracy.

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