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Modelling stock data using a geometric BINAR(1) model

Yuvraj Sunecher *
Department of Accounting, Finance and Economics
University of Technology Mauritius
Pointe-Aux-Sables
Mauritius

Naushad Mamode Khan †
Department of Economics and Statistics
University of Mauritius
Reduit
Mauritius

Abstract

The intricate cross-correlation between the counting series makes the non-stationary bivariate integer-valued autoregressive of order 1 (BINAR(1)) model with a full correlation structure very challenging to model. This paper offers a novel full correlation non-stationary BINAR(1) model with geometric marginals (BINAR(1)GEOM). The regression and dependency parameters are estimated using the Generalized Quasi-Likelihood (GQL) estimation approach, which is contrasted with several well-known estimation techniques: the Generalized Method of Moments (GMM) and Generalized Least Squares (GLS). A numerical experiment is conducted to compare the performance of these estimation methods and the proposed model is tested on some stock transactions in the Mauritian market.

Subject Classification (2010): *65C60, 62J12, 62H12, 62J20, 62J10.*

Keywords: *Stock transactions, Geometric, BINAR(1), GQL, GMM, GLS.*

1. Introduction

Pedeli and Karlis [4, 5] first extended the traditional INAR(1) model of McKenzie [3] according to the mechanism of binomial thinning (Steutel and Van Harn [9]) to produce two constrained BINAR(1) models, namely the

* *E-mail:* `ysunecher@utm.ac.mu (Corresponding Author)`
† *E-mail:* `n.mamodekhan@uom.ac.mu`

CBINAR(1)P and CBINAR(1) NB. The latter models were derived using the stationary moment assumption. Likewise, given the same stationarity requirement, the CBINAR(1)P model was extended to an unconstrained BINAR(1) model (UBINAR(1)P) by Pedeli and Karlis [6].

Few BINAR(1) models have been developed for non-stationary time series of counts. An NSCBINAR(1)P model was developed by Mamodekhan, Sunecher, and Jowaheer [2] and an NSCBINAR(1)NB model was developed by Sunecher, Mamodekhan, and Jowaheer [11] under the same assumptions. Other BINAR(1) models in literature are the BINAR(1)PWE model (Sajjadnia et al. [8]) and the BINAR(1)PL model [1]. The development of an unconstrained BINAR(1) model that can cater for over-dispersed data has not yet been done. Hence, we propose an NSUBINAR(1)GEOM model in this study.

Hence, the paper is organized as follows: The BINAR(1) model with Geometric marginals is developed in section 2 by deriving its moments. Along with the GLS and GMM techniques, the GQL methodology is derived in section 3 using the non-stationarity bivariate set-up. The simulation experiment is presented in section 4, where BINAR(1) data with Geometric marginals are simulated and the model parameters are estimated using the GQL, GMM, and GLS techniques. In section 5, the model is applied on stock data in Mauritius. The final section provides the conclusion.

2. Model Construction

Assume $X_t^{[k]}$ follows the Geometric distribution with the following parameters:

$$X_t^{[k]} \sim \text{GEOM}\left(\frac{\theta_t^{[k]}}{1+\theta_t^{[k]}}\right), \text{ where}$$

$$E(X_t^{[k]}) = \theta_t^{[k]} \tag{1}$$

and

$$\text{Var}(X_t^{[k]}) = \theta_t^{[k]}\,(1+\theta_t^{[k]}) \tag{2}$$

We further assume that $\theta_t^{[k]} = \exp(m_t^{'}\varphi^{[k]})$ with $m_t = [m_{t1}, m_{t2}, \ldots, m_{tj}, \ldots, m_{tp}]'$ is a (p x 1) vector of explanatory variables impacting both $X_t^{[1]}$ and $X_t^{[2]}$ with regressors $\varphi^{[k]} = [\varphi_1^{[k]}, \varphi_2^{[k]}, \ldots, \varphi_j^{[k]}, \ldots, \varphi_p^{[k]}]^{'}$.

The full correlation structure BINAR(1) model is given as:

$$X_t^{[1]} = \alpha_{11} * X_{t-1}^{[1]} + \alpha_{12} * X_{t-1}^{[2]} + Z_t^{[1]} \tag{3}$$

$$X_t^{[2]} = \alpha_{21} * X_{t-1}^{[1]} + \alpha_{22} * X_{t-1}^{[2]} + Z_t^{[2]} \tag{4}$$

where * denotes the binomial thinning operator [9] with

$$\alpha_{ij} * X_{t-1}^{[j]} = \sum_{n=1}^{X_{t-1}^{[j]}} Y_n \tag{5}$$

$$\text{Where } Y_n \sim \text{Geom}\left(\frac{\alpha_{ij}}{1+\alpha_{ij}}\right).$$

Under the condition that $E(X_t^{[k]}) = \theta_t^{[k]}$ and $Var(X_t^{[k]}) = \theta_t^{[k]}(1+\theta_t^{[k]})$, from equation 3-4,we have

$$E\left(Z_t^{[k]}\right) = \lambda_t^{[k]} = \theta_t^{[k]} - \alpha_{kk}\theta_{t-1}^{[k]} - \theta_{kk'}\theta_{t-1}^{[k']} \quad (6)$$

$$ar\left(Z_t^{[k]}\right) = \theta_t^{[k]}\left(1+\theta_t^{[k]}\right) - \theta_{kk}(1+\theta_{kk})\theta_{t-1}^{[k]} - \theta_{kk}^2\,\theta_{t-1}^{[k]}\left(1+\theta_{t-1}^{[k]}\right)$$

$$V - \alpha_{kk'}(1+\alpha_{kk'})\theta_{t-1}^{[k']}$$

$$\alpha_{kk'}^2\,\alpha_{t-1}^{[k']}\left(1+\theta_{t-1}^{[k']}\right) - 2\,\alpha_{kk}\,\alpha_{kk'}\,Cov\left(X_{t-1}^{[k]}, X_{t-1}^{[k']}\right) \quad (7)$$

for k, $k' \in 1, 2$.

$$Cov\left(X_t^{[1]}, X_t^{[2]}\right) = (\alpha_{11}\alpha_{22} + \alpha_{12}\alpha_{21})\,Cov\left(X_{t-1}^{[1]}, X_{t-1}^{[2]}\right) + \alpha_{11}\alpha_{21}\left(\theta_{t-1}^{[1]} + \theta_{t-1}^{[1]2}\right)$$

$$+ \alpha_{12}\alpha_{22}\,\theta_{t-1}^{[2]}\,\theta_{t-1}^{[2]2} + [\,\kappa_{12,t}]\sqrt{Var\left(Z_t^{[1]}\right)}\sqrt{Var\left(Z_t^{[2]}\right)} \quad (8)$$

3. Estimation Methods

In this section, three estimation methods, namely GLS, GQL and GMM are presented.

3.1 *Generalized Least-Squares*

Based on equations (3) and (4), the GLS equations are:

$$e_{k1t} = X_t^{[k]} - E\left(X_t^{[k]} | X_{t-1}^{[1]}, X_{t-1}^{[2]}\right),\ \ e_{k2t} = \left[X_t^{[k]} - E\left(X_t^{[k]} | X_{t-1}^{[1]}, X_{t-1}^{[2]}\right)\right]^2 - V\left(X_{t-1}^{[k]} \,\middle|\, X_{t-1}^{[1]}, X_{t-1}^{[2]}\right), \quad (9)$$

$$e_{3t} = e_{11t}e_{21t} - Cov\left(X_t^{[1]},\ X_t^{[2]} | X_{t-1}^{[1]}, X_{t-1}^{[2]}\right), \quad (10)$$

$$S_{k1} = \textstyle\sum_{t=1}^{T} e_{k1t}^2,\ S_{k2} = \sum_{t=1}^{T} e_{k2t}^2 \text{ and } S_3 = \sum_{t=1}^{T} e_{3t}^2 \quad (11)$$

with their respective score functions given as:

$$\sum_{t=1}^{T}\frac{\delta S_{11}}{\delta\alpha_{11}} = \sum_{t=1}^{T} 2\left[x_t^{[1]} - \left[\alpha_{11}\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right) + \alpha_{12}\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right) + \theta_t^{[1]}\right]\right]\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right), \quad (12)$$

$$\sum_{t=1}^{T}\frac{\delta S_{11}}{\delta\alpha_{12}} = \sum_{t=1}^{T} 2\left[x_t^{[1]} - \left[\alpha_{11}\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right) + \alpha_{12}\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right) + \theta_t^{[1]}\right]\right]\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right), \quad (13)$$

$$\sum_{t=1}^{T}\frac{\delta S_{21}}{\delta\alpha_{21}} = \sum_{t=1}^{T} 2\left[x_t^{[2]} - \left[\alpha_{21}\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right) + \alpha_{22}\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right) + \theta_t^{[2]}\right]\right]\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right), \quad (14)$$

$$\sum_{t=1}^{T}\frac{\delta S_{21}}{\delta\alpha_{22}} = \sum_{t=1}^{T} 2\left[x_t^{[2]} - \left[\alpha_{21}\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right) + \alpha_{22}\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right) + \theta_t^{[2]}\right]\right]\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right), \quad (15)$$

and given $\theta_t^{[k]} = \exp\left(m_{t1}\varphi_1^{[k]} + m_{t2}\varphi_2^{[k]} + \ldots + m_{tj}\varphi_j^{[k]} + \ldots + m_{tp}\varphi_p^{[k]}\right)$,

$$\sum_{t=1}^{T}\frac{\delta S_{11}}{\delta\varphi_j^{[1]}} = \sum_{t=1}^{T} 2\left[x_t^{[1]} - \left[\alpha_{11}\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right) + \alpha_{12}\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right) + \theta_t^{[1]}\right]\right]$$

$$X\left(\alpha_{11}\,\theta_{t-1}^{[1]}\,m_{t-1,j} - \theta_t^{[1]}m_{t,j}\right) \quad (16)$$

$$\sum_{t=1}^{T} \frac{\delta S_{21}}{\delta \varphi_j^{[2]}} = \sum_{t=1}^{T} 2\,[\,x_t^{[2]} - [\alpha_{21}\left(X_{t-1}^{[1]} - \theta_{t-1}^{[1]}\right) + \alpha_{22}\left(X_{t-1}^{[2]} - \theta_{t-1}^{[2]}\right) + \theta_t^{[2]}]]$$

$$X\,(\alpha_{22}\,\theta_{t-1}^{[2]}\,m_{t-1,j} - \theta_t^{[2]} m_{t,j}\,) \tag{17}$$

The *R optim* function is used to solve for the above unknown parameters.

3.2. *Generalized Quasi-Likelihood Technique*

The first GQL equation is as follows:

$$D_\varphi' \,\Sigma_\varphi^{-1}\,(g-\theta)=0 \tag{18}$$

with score vector $\boldsymbol{g} = [g_1, g_2, \ldots, g_t, \ldots, g_{t+h}, \ldots, g_T]$ with $g_t=[X_t^{[1]}, X_t^{[2]}]$ and $\theta = [\theta_1, \theta_2, \ldots, \theta_t, \ldots, \theta_{t+h}, \ldots, \theta_T]$ with corresponding mean $\theta_t = [\theta_t^{[1]}, \theta_t^{[2]}]$ for t=1,2,...,T.

The covariance matrix Σ_φ is a (2T x 2T).

$$D_t = \begin{pmatrix} \frac{\delta\theta_t^{[1]}}{\delta\varphi^{[1]}} & 0 \\ 0 & \frac{\delta\theta_t^{[2]}}{\delta\varphi^{[2]}} \end{pmatrix}$$

where $\frac{\delta\mu_t^{[k]}}{\delta\beta_j^{[k]}} = \mu_t^{[k]}\, x_{tj}^{[']}$.

To estimate the regression coefficients, the Newton-Raphson iterative technique is used as follows:

$$\begin{pmatrix} \varphi_{r+1}^{[1]} \\ \varphi_{r+1}^{[2]} \end{pmatrix} = \begin{pmatrix} \varphi_r^{[1]} \\ \varphi_r^{[2]} \end{pmatrix} + [D_\varphi' \,\varepsilon_\varphi^{-1}\, D_\varphi]_r^{-1}\; [D_\varphi' \,\varepsilon_\varphi^{-1} \left(g-\theta\right)]_r \tag{19}$$

To estimate the dependence parameters $\omega=[\alpha_{11},\alpha_{12},\alpha_{21},\alpha_{22}]$, a second GQL is used as follows:

$$D_\omega' \,\varepsilon_\omega^{-1}\,(X_\omega - \theta_\omega)=0 \tag{20}$$

with $\theta_\omega = E(X_\omega)$.

Σ_ω. have the following elements: Var $(X_t^{[1]} X_t^{[2]} \mid X_{t-1}^{[1]}, X_{t-2}^{[2]})$ along the diagonal and

Cov $(X_t^{[1]} X_t^{[2]}, X_{t+h}^{[1]}, X_{t+h}^{[2]} \mid X_{t-1}^{[1]} X_{t-1}^{[2]}, X_{t+h-1}^{[1]}, X_{t+h-1}^{[2]})$ in the off-diagonal entries.

The entries of the derivative matrix D_ω are derived as follows:

$$E\,(X_t^{[1]} X_t^{[2]} \mid X_{t-1}^{[1]}, X_{t-2}^{[2]})$$

$$= [\kappa_{12,t}\,[\,\theta_t^{[1]}(1+\theta_t^{[1]}) - \alpha_{11}(1+\alpha_{11})\,\theta_{t-1}^{[1]} + \alpha_{11}^2\;\theta_{t-1}^{[1]}(1+\theta_{t-1}^{[1]}) - \alpha_{12}(1+\alpha_{12})\,\theta_{t-1}^{[2]}$$

$$- \alpha_{12}^2\;\theta_{t-1}^{[2]}(1+\theta_{t-1}^{[2]})]\,[\theta_t^{[2]}(1+\theta_t^{[2]}) - \alpha_{21}(1+\alpha_{21})\,\theta_{t-1}^{[1]} - \alpha_{21}^2\;\theta_{t-1}^{[1]}(1+\theta_{t-1}^{[1]})$$

$$- \alpha_{22}(1+\alpha_{22})\,\theta_{t-1}^{[2]} - \alpha_{22}^2\;\theta_{t-1}^{[2]}(1+\theta_{t-1}^{[2]})] + (\,\alpha_{11}X_{t-1}^{[1]} + \alpha_{12}X_{t-1}^{[2]} + \theta_t^{[1]} - \alpha_{11}\theta_{t-1}^{[1]}$$

$$- \alpha_{12}\theta_{t-1}^{[2]})\;X\;(\,\alpha_{21}X_{t-1}^{[1]} + \alpha_{22}X_{t-1}^{[2]} + \theta_t^{[2]} - \alpha_{21}\theta_{t-1}^{[2]} - \alpha_{22}\theta_{t-1}^{[1]})$$

$$[\kappa_{12,t}=\frac{Cov\left(X_t^{[1]}X_t^{[2]}\right)-(\alpha_{11}\alpha_{22}-\alpha_{12}\alpha_{21})Cov\left(X_{t-1}^{[1]}X_{t-1}^{[2]}\right)-\alpha_{11}\alpha_{21}\left(\theta_{t-1}^{[1]}+\theta_{t-1}^{[1]2}\right)-\alpha_{22}\alpha_{12}\ (\theta_{t-1}^{[2]}+\theta_{t-1}^{[2]2})}{\sqrt{Var\ (Z_t^{[1]})}\sqrt{Var\ (Z_t^{[2]})}} \quad (22)$$

The second GQL is solved using the Newton-Raphson iterative technique as follows:

$$\begin{pmatrix}\omega_{r+1}^{[1]}\\ \omega_{r+1}^{[2]}\end{pmatrix}=\begin{pmatrix}\omega_{r}^{[1]}\\ \omega_{r}^{[2]}\end{pmatrix}+[D_\omega^{'}\ \varepsilon_\omega^{-1}\ D_\omega]_r^{-1}\ [D_\omega^{'}\ \varepsilon_\omega^{-1}\ (X_\omega-\theta_\omega)]_r \quad (23)$$

As for the GMM approach, the estimating equations (20) and (23) are referred with the same corresponding score vector but their respective auto-covariance structure is simply computed empirically as explained in Mamodekhan et al. [2]. As for $k_{12,t}$, its estimating procedure is similar to equation (22).

3.3. *Forecasting Equations*

The following is the derivation of the forecasting equations:

Utilizing the model from section 2, we have

$$X_{t+1}^{[1]}=\alpha_{11}*X_t^{[1]}+\alpha_{12}*X_t^{[2]}+Z_{t+1}^{[1]} \quad (24)$$

$$X_{t+1}^{[2]}=\alpha_{21}*X_t^{[1]}+\alpha_{22}*X_t^{[2]}+Z_{t+1}^{[2]} \quad (25)$$

and for given $X_t^{[k]}$, the forecasting equations are expressed as

$$E\left(X_{t+1}^{[1]}\ \middle|\ X_t^{[1]},X_t^{[2]}\right)=\theta_{t+1}^{[1]}+\alpha_{11}\ (X_t^{[1]}-\theta_t^{[1]})+\alpha_{12}\ (X_t^{[2]}-\theta_t^{[2]}) \quad (26)$$

$$E\left(X_{t+1}^{[2]}\ \middle|\ X_t^{[1]},X_t^{[2]}\right)=\theta_{t+1}^{[2]}+\alpha_{21}\ (X_t^{[1]}-\theta_t^{[1]})+\alpha_{22}\ (X_t^{[2]}-\theta_t^{[2]}) \quad (26)$$

4. Simulation Study

BINAR(1) time series data with Geometric marginals are generated using the following covariate design:

$$y_{t1}=\begin{cases}-\cos(3\pi t)+0.015\ (t=1,\ldots,T/5)\\ \sin(3\pi t)+0.055\ (t=T/5+1,\ldots,3T/5)\\ \cos(3\pi t)+0.104\ (t=3T/5,\ldots,T)\end{cases}$$

$$y_{t2}=\begin{cases}(1T) & (t=1,\ldots,T/4)\\ -1/T & (t=T/4+1,\ldots,3T/4)\\ t & (t=3T/4\ ,\ldots,T)\end{cases}$$

where $\theta_t^{[k]}=\exp(\ y_{t1}\ \varphi_1^{[k]}+y_{t2}\ \varphi_2^{[k]})$. $Z_t^{[k]}$ is generated using the inverse transformation method as in Ristic, Bakouch and Nastic [7] for $[\alpha_{12},\alpha_{21}]$ = [0.5,0.5], $[\alpha_{11},\alpha_{22}]$ = [0.9,0.9], [0.3,0.3], $\varphi^{[1]}$=0.5 and $\varphi^{[1]}$=0.9 for *t* = 1,2, ..., *T* = 100, 500, 1000. The simulated mean estimates after 5000 Monte Carlo replications are shown in Table 1-2:

Table 1
Estimates of the Model Parameters

T	Methods	$\varphi_1^{[1]}$	$\varphi_2^{[1]}$	$\varphi_1^{[2]}$	$\varphi_2^{[2]}$	α_{11}	α_{22}	α_{12}	α_{21}	$\kappa_{12,1}$
100	GQL	0.482	0.487	0.887	0.884	0.881	0.884	0.485	0.484	0.981
		(0.091)	(0.093)	(0.097)	(0.094)	(0.116)	(0.111)	(0.112)	(0.115)	
	GMM	0.472	0.475	0.879	0.871	0.872	0.871	0.474	0.475	0.970
		(0.125)	(0.112)	(0.126)	(0.129)	(0.146)	(0.147)	(0.150)	(0.142)	
	GLS	0.480	0.482	0.881	0.881	0.882	0.882	0.485	0.483	0.984
		(0.109)	(0.119)	(0.108)	(0.107)	(0.120)	(0.129)	(0.128)	(0.127)	
500	GQL	0.492	0.491	0.895	0.891	0.894	0.892	0.494	0.494	0.990
		(0.051)	(0.052)	(0.054)	(0.058)	(0.064)	(0.062)	(0.062)	(0.063)	
	GMM	0.483	0.484	0.881	0.880	0.882	0.882	0.486	0.484	0.983
		(0.092)	(0.093)	(0.101)	(0.092)	(0.115)	(0.104)	(0.112)	(0.116)	
	GLS	0.487	0.485	0.886	0.889	0.888	0.888	0.491	0.492	0.988
		(0.079)	(0.068)	(0.076)	(0.065)	(0.084)	(0.082)	(0.079)	(0.081)	
1000	GQL	0.499	0.498	0.896	0.898	0.898	0.899	0.502	0.501	0.997
		(0.012)	(0.017)	(0.019)	(0.014)	(0.027)	(0.022)	(0.020)	(0.023)	
	GMM	0.488	0.487	0.884	0.889	0.885	0.888	0.489	0.488	0.986
		(0.054)	(0.042)	(0.053)	(0.050)	(0.066)	(0.058)	(0.069)	(0.063)	
	GLS	0.491	0.490	0.892	0.891	0.894	0.891	0.502	0.503	0.993
		(0.026)	(0.020)	(0.038)	(0.035)	(0.031)	(0.034)	(0.042)	(0.035)	

Table 2
Estimates of the Model Parameters

T	Methods	$\varphi_1^{[1]}$	$\varphi_2^{[1]}$	$\varphi_1^{[2]}$	$\varphi_2^{[2]}$	α_{11}	α_{22}	α_{12}	α_{21}	$\kappa_{12,1}$
100	GQL	0.482	0.487	0.880	0.889	0.281	0.283	0.485	0.483	0.981
		(0.098)	(0.091)	(0.093)	(0.092)	(0.116)	(0.116)	(0.113)	(0.119)	
	GMM	0.478	0.474	0.876	0.870	0.870	0.276	0.279	0.471	0.972
		(0.108)	(0.114)	(0.108)	(0.112)	(0.122)	(0.129)	(0.128)	(0.127)	
	GLS	0.482	0.481	0.880	0.881	0.281	0.283	0.515	0.513	0.980
		(0.108)	(0.114)	(0.108)	(0.112)	(0.122)	(0.129)	(0.128)	(0.127)	
500	GQL	0.492	0.494	0.891	0.893	0.296	0.293	0.492	0.491	0.991
		(0.059)	(0.058)	(0.056)	(0.058)	(0.061)	(0.062)	(0.067)	(0.068)	
	GMM	0.482	0.481	0.882	0.880	0.282	0.283	0.488	0.482	0.983
		(0.112)	(0.109)	(0.110)	(0.112)	(0.110)	(0.117)	(0.111)	(0.124)	
	GLS	0.486	0.486	0.889	0.886	0.285	0.285	0.490	0.491	0.985
		(0.077)	(0.077)	(0.070)	(0.061)	(0.075)	(0.088)	(0.087)	(0.081)	
1000	GQL	0.495	0.499	0.898	0.899	0.298	0.298	0.501	0.504	0.996
		(0.015)	(0.012)	(0.013)	(0.015)	(0.024)	(0.020)	(0.021)	(0.021)	
	GMM	0.485	0.489	0.887	0.889	0.288	0.287	0.485	0.487	0.988
		(0.052)	(0.051)	(0.056)	(0.051)	(0.062)	(0.061)	(0.054)	(0.052)	
	GLS	0.490	0.490	0.893	0.890	0.291	0.292	0.502	0.501	0.991
		(0.027)	(0.032)	(0.033)	(0.029)	(0.047)	(0.038)	(0.034)	(0.031)	

The GQL estimates are consistent and as the time points increases, there is a decrease in the standard errors, with GQL yielding lower standard errors than GLS and GMM, as also demonstrated in Sunecher et al. [10].

5. Application to Stock Series

Mauritius Commercial Bank (MCB) and State Bank of Mauritius Holding (SBMH) are known to have the highest turnovers at the Stock Exchange of Mauritius (SEM). Some specifications on the data: The stock data are accumulated from August 3-October 16, 2015 and since our interest is on the number of ordinary transactions, we consider the number transactions started at 09.00 and ended at 13 30. The data are aggregated into 30 minutes intervals of time and 495 paired observations were collected. The different plots are displayed in Figures 1-4:

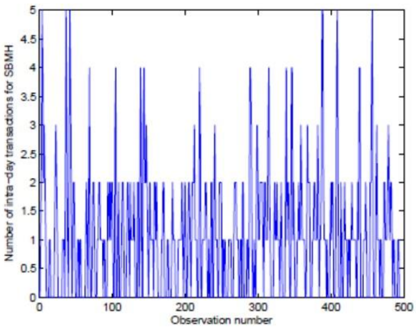

Figure 1
SBMH

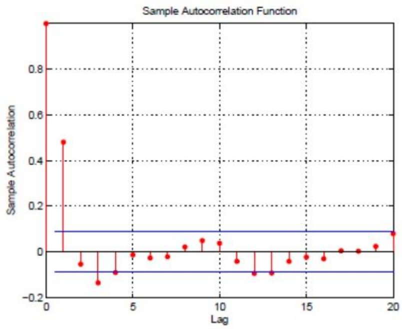

Figure 2
ACF plot for SBMH

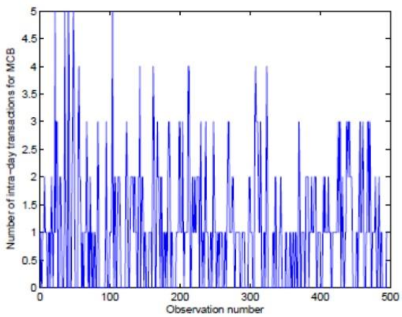

Figure 3
MCB

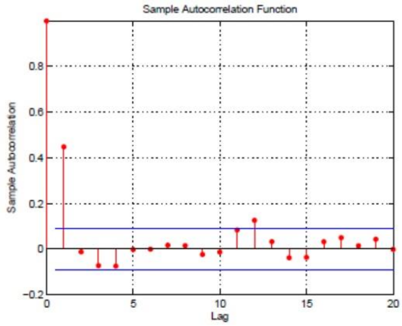

Figure 4
ACF plot for MCB

The sample statistics are summarized in Table 3:

Table 3
Summary Statistics for MCB and SBMH

	Mean	Variance	Cross Correlation
MCB	1.0667	1.1474	0.1562
SBMH	1.099	1.2311	

The variables that we take into account that impact these two series are the Friday Effect (y_{t2}), which indicates that Friday is one day of the week and 0 on the other days, the news effect (y_{t1}), which is coded as 1 for any news and 0 for no such news, and the time of day effect (y_{t3}), which indicates that trading is conducted in the afternoon (12 0013 30) and 0 for the remainder of the trading session. Using $\lambda_t^{[k]} = \exp(\varphi_0^{[k]} + y_{t1}\, \varphi_1^{[k]} + y_{t2}\, \varphi_2^{[k]} + y_{t3}\, \varphi_3^{[k]})$, we analyze the intra-day stock transactions of these two companies using the BINAR(1)GEOM model and the GQL estimates are given in Table 4:

Table 4
Number of Transactions for SBMH and MCB: GQL Estimates of the regression and dependence.

Time Series	φ_0	φ_1	φ_2	φ_3	α_{11}	α_{22}	α_{12}	α_{21}
$X_t^{[1]}$	0.2087	0.2145	0.0832	0.1082	0.3581	0.5183		
s.e	(0.1859)	(0.0378)	(0.0469)	(0.0531)	(0.0477)	(0.0511)		
$X_t^{[2]}$	0.2047	0.1341	0.1190	0.1256			0.5402	0.3307
s.e	(0.1698)	(0.0472)	(0.0440)	(0.0540)			(0.0501)	(0.0488)

We can observe from Table 4 that there is significant cross-correlation between the two series and the covariates have significant impact on the number of transactions. All the three covariates, namely the news effect, the Friday effect and the time effect leads to a rise in the quantity of transactions MCB and SBMH. Using the forecasting equations (26)-(27), we estimate the number of SBMH and MCB transactions one step ahead of time and the root mean square errors (RMSEs) are 0.1852 and 0.1686 respectively.

6. Conclusion

In this paper, we present the BINAR(1) model with Geometric marginals as well as its moments. As for the estimation of model parameters, we use the GQL approach and compare its performance with the CLS and GMM through a simulation study. The model is applied on stock transactions and the estimates of the covariates proved to be reliable.

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