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Stress-strength reliability analysis of series systems with redundancy : A study utilizing Rayleigh distribution and estimation techniques

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Abstract

The Rayleigh distribution is among the numerous distributions employed as a model for lifetime events, finding appli-cations in survival analysis, reliability, and communications engineering. This study focuses on investigating the stress-strength reliability of a series system with redundancy at both component and system levels, utilizing the Rayleigh distribution with varying parameters. Estimators for model parameters are obtained through the Maximum Likelihood method and Bayesian procedures. A Monte Carlo simulation study is conducted to compare the different estimates. Additionally, a real-world application is presented to demonstrate the validity of our proposed model.

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Keywords: *Series system, Cold standby redundancy system, Rayleigh distribution, Bayes estimator, Maximum likelihood estimator, Simulation study.*

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1. Introduction

[1] introduced the Rayleigh distribution as a special case of Weibull distribution with shape parameter equal 2, and probability density function (pdf) is defined as

$$f(x;\sigma) = \frac{x}{\sigma^2}\, e^{-\frac{x^2}{\sigma^2}} \quad x > 0, \sigma > 0.$$

The main reason of choosing Rayleigh distribution is it has many applications in many fields such as reliability, survival analysis, clinical study etc. The main aim of this study is to investigate the stress-strength reliability of series system with cold standby redundancy at the components and system levels. In many fields the stress-strength reliability model $R = P[Y < X]$ is defined as a reliability system, when X and Y are strength and stress random variables respectively. In this case R can be used to measure the system performance. The system fails when the stress Y exceeds its strength X. The stress-strength model with single or multi-components case investigated by many authors such as [2], [3], [4], [5] , [6] and [7]. From the important concepts of the reliability theory is a coherent system for more details see [8]. To increase the efficiency of the system, add redundancy components to the system. [9] investigated the stress-strength reliability of standby redundancy system which plays an important role in enhancing reliability system. This type of reliability system when X and Y are independent is studied by many authors see [10]and [11]. In the context of redundancy system there three types of redundancies are hot standby (active), cold standby (standby redundancy) and warm standby for more details see [12], [13], [14] and [15].

Definition 1: Hot standby, available redundancy components are put in parallel with the original components and function simultaneously with them.

Definition 2: Cold standby, redundancy components are put in standby and start functioning when the original components fail.

Definition 3: Warm standby is a redundancy type between the hot standby and the cold standby.

Definition 4: Structure function φ of n-components system is defined as$\varphi: \{0,1\}^n \to \{0,1\}$ where 0, 1 mean a component fails or work respectively.

Definition 5: The System is said to be coherent if the structure function is non-decreasing at each component.

To improve a coherent system, we can add n standby redundancy system. Standby system can add at system level or component level to increase the system reliability for more details [16], [17], [18], [19], [20], [21], [22], [23] and [24]. The investigation of stress-strength model for cold, worm and active standby system does not consider by many authors except some authors such that [25] study the performance of the stress-strength model and mean remaining

strength of series system with cold standby redundancy at system and component levels using exponential distribution [26] study reliability estimation of a N-M-cold-standby redundancy system in a multicomponent stress–strength model with generalized half-logistic distribution. [27] study the estimation of the stress-strength reliability for parallel system consists of active and cold standby components. Our contribution to this study is to investigate the Bayesian and Non-Bayesian estimation for Stress-strength of series system with cold standby redundancy based on Rayleigh distribution at a system and component levels. In context of estimation of the Rayleigh distribution parameters see [28]. The main reason for choosing the Rayleigh distribution is it has many applications in communications, physical sciences and engineering. This paper is organized as follows in section 2 describe our proposed models. In section 3 we derived maximum likelihood estimators for our proposed models. Bayesian estimates of our proposed model using Markov chain Monte -Carlo method (MCMC) is obtained in section 4. In section 5 simulation study is performed to compare the behavior of our estimators. Section 6 we introduced the real application to show the validity of our models. Finally, in section 7, we introduce the results and conclusions of our paper.

2. Model Description

In this section, we assume that a series system with independent and identically n-components with lifetime distribution. For this system, let $X_1 \dots X_n$ be the random sample from the lifetimes of strength components has Rayleigh distribution with parameterσ_1 i.e., $X_i \approx Ray(\sigma_1)$, $i = 1 \dots n$ and let $Y_1 \dots Y_n$ be the random sample from lifetimes standby strength components in the series system has Rayleigh distribution with parameter σ_2 i.e. $Y_i \approx Ray(\sigma_2)$, $i = 1 \dots n$. The standby redundancy component can be added to the system at two levels, first at a component level, second at the system level. Let U be the common stress variable and it is also having Rayleigh distribution with parameter θ i.e., $U \approx Ray(\theta)$. In the context of the series system at component level, the standby redundancy components added for each component of the series system. Now the total lifetime of each strength component is $W_i = X_i + Y_i$, $i = 1 \dots n$. Then the probability density function (pdf) and the cumulative distribution function (cdf) of W_i, $i = 1 \dots n$ are given by,

$$f_{W_i}(w) = \int_o^w f_X(z-y) f_Y(y) dy,$$

when, $\sigma_1 = \sigma_2 = \sigma$.

$$f_{W_i}(w) = \frac{e^{-\frac{w^2}{2\sigma^2}}(2w\sigma + e^{\frac{w^2}{4\sigma^2}}\sqrt{\pi}(w^2 - 2\sigma^2)\mathrm{Erf}[\frac{w}{2\sigma}])}{4\sigma^3},$$

and

$$F_{W_i}(w) = 1 - e^{-\frac{w^2}{2\sigma^2}} - \frac{e^{-\frac{w^2}{4\sigma^2}}\sqrt{\pi} w \mathrm{Erf}[\frac{w}{2\sigma}]}{2\sigma},$$

where, $w > 0, \sigma > 0$ and $Erf(.)$ is the error function.

When, $\sigma_1 \neq \sigma_2$, then,

$$f_{W_i}(w) = \frac{e^{-\frac{1}{2}w^2(\frac{1}{\sigma_1^2}+\frac{1}{\sigma_2^2})}}{2(\sigma_1^2+\sigma_2^2)^{5/2}} \left(2e^{\frac{w^2}{2\sigma_2^2}} w\sigma_1^2\sqrt{\sigma_1^2+\sigma_2^2} + 2e^{\frac{w^2}{2\sigma_1^2}} w\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2} - e^{\frac{w^2\left(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}\right)}{2\sigma_1^2}} \sqrt{2\pi}\left(\mathrm{Erf}\left[\frac{w\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}\right] + \mathrm{Erf}\left[\frac{w\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}\right]\right)\sigma_1\sigma_2(-w^2+\sigma_1^2+\sigma_2^2)\right), \tag{1}$$

and,

$$F_{W_i}(w) = \frac{1}{2(\sigma_1^2+\sigma_2^2)^{3/2}} \left(-e^{-\frac{w^2}{2(\sigma_1^2+\sigma_2^2)}}\sqrt{2\pi}w\left(\mathrm{Erf}\left[\frac{w\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}\right] + \mathrm{Erf}\left[\frac{w\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}\right]\right)\right.$$
$$\left.\sigma_1\sigma_2 + 2\left(1-e^{-\frac{w^2}{2\sigma_1^2}}\right)\sigma_1^2\sqrt{\sigma_1^2+\sigma_2^2} + 2\left(1-e^{-\frac{w^2}{2\sigma_2^2}}\right)\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}\right). \tag{2}$$

Definition 6: (Stress-Strength Model for the series system at component level)

If U is the common stress variable in the series system which consist of n-components W_i, $i = 1 \ldots n$. Then the stress-strength reliability model of this system is

$$R_{COMP} = P\left(W_{(1)} > U\right)$$
$$= \int_0^\infty \left(1 - F_{W_i}(u)\right)^n f_U(u)du$$
$$= \sum_{i=0}^n (-1)^i \binom{n}{i} \int_0^\infty \left(F_{W_i}(u)\right)^{n-1} f_U(u)du, \tag{3}$$

where, $W_{(1)} = \min(W_1 \ldots W_n)$.

For Rayleigh distribution we can't get the closed form of R_{COMP}, then we use the numerical integration for both cases, $\sigma_1 = \sigma_2 = \sigma$ and $\sigma_1 \neq \sigma_2$. In the context of the series system at a system level. The standby redundancy components consist of a duplicate series system for the main system. The total lifetime of these series system is $W_{(1)} = X_{(1)} + Y_{(1)}$ where $X_{(1)} \approx Ray(\frac{\sigma_1}{\sqrt{n}})$ and $Y_{(1)} \approx Ray(\frac{\sigma_2}{\sqrt{n}})$ then the pdf and cdf of $W_{(1)}$ are given by

$$f_{W_{(1)}}(w) = \int_o^w f_{X_{(1)}}(z-y) f_{Y_{(1)}}(y)dy.$$

When, $\sigma_1 = \sigma_2 = \sigma$

$$f_{W_{(1)}}(w) = \frac{e^{-\frac{nw^2}{2\sigma^2}}(2nw\sigma + e^{\frac{nw^2}{4\sigma^2}}\sqrt{n}\sqrt{\pi}(nw^2-2\sigma^2)\mathrm{Erf}[\frac{\sqrt{n}w}{2\sigma}])}{4\sigma^3},$$

$$F_{W_{(1)}}(w) = 1 - e^{-\frac{nw^2}{2\sigma^2}} - \frac{e^{-\frac{nw^2}{4\sigma^2}}\sqrt{n}\sqrt{\pi}w\mathrm{Erf}[\frac{\sqrt{n}w}{2\sigma}]}{2\sigma}.$$

When, $\sigma_1 \neq \sigma_2$, then,

$$f_{W_{(1)}}(w) = \frac{e^{-\frac{1}{2}nw^2(\frac{1}{\sigma_1^2}+\frac{3}{\sigma_2^2})}}{2(\sigma_1^2+\sigma_2^2)^{5/2}}(e^{\frac{1}{2}nw^2(\frac{1}{\sigma_1^2}+\frac{3}{\sigma_2^2}-\frac{1}{\sigma_1^2+\sigma_2^2})}\sqrt{n}\sqrt{2\pi}(\mathrm{Erf}[\frac{\sqrt{n}w\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}] +$$
$$\mathrm{Erf}[\frac{\sqrt{n}w\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])\sigma_1\sigma_2(nw^2-\sigma_1^2-\sigma_2^2) + 2e^{\frac{3nw^2}{2\sigma_2^2}}nw\sigma_1^2\sqrt{\sigma_1^2+\sigma_2^2} +$$
$$2e^{\frac{1}{2}nw^2(\frac{1}{\sigma_1^2}+\frac{2}{\sigma_2^2})}nw\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}) \quad (4)$$

$$F_{W_{(1)}}(w) = \frac{1}{2(\sigma_1^2+\sigma_2^2)^{3/2}}(-\mathrm{e}^{-\frac{w^2}{2(\sigma_1^2+\sigma_2^2)}}\sqrt{2\pi}w(\mathrm{Erf}[\frac{w\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}] + \mathrm{Erf}[\frac{w\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])$$
$$\sigma_1\sigma_2 + 2(1-\mathrm{e}^{-\frac{w^2}{2\sigma_1^2}})\sigma_1^2\sqrt{\sigma_1^2+\sigma_2^2} + 2(1-\mathrm{e}^{-\frac{w^2}{2\sigma_2^2}})\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}) \quad (5)$$

Definition 7: (Stress-Strength Model for the series system at a system level)

If U is the common stress variable, then the stress-strength model of this system is defined as

$$R_{SYSTEM} = P(W_{(1)} > U)$$
$$= \int_0^\infty (1-F_{W_i}(u))f_U(u)du$$

When, $\sigma_1 = \sigma_2 = \sigma$

$$R_{SYSTEM} = \frac{\sigma^2 + \frac{n(\sqrt{n}\theta^2\sqrt{\frac{2}{\theta^2}+\frac{n}{\sigma^2}}\sigma + 2(n\theta^2+\sigma^2)\mathrm{ArcCot}[\frac{\sqrt{\frac{2}{\theta^2}+\frac{n}{\sigma^2}}\sigma}{\sqrt{n}}])}{\sqrt{\frac{2}{\theta^2}+\frac{n}{\sigma^2}}\sqrt{\frac{n}{\sigma^2}}(n\theta^2+2\sigma^2)}}{n\theta^2+\sigma^2} \quad (6)$$

When, $\sigma_1 \neq \sigma_2$, then,

$$R_{SYSTEM} = \frac{1}{(n\theta^2+\sigma_1^2)(n\theta^2+\sigma_2^2)(\sigma_1^2+\sigma_2^2)(n\theta^2+\sigma_1^2+\sigma_2^2)^2}((n\theta^2+\sigma_1^2)\sigma_2^4(n\theta^2+\sigma_1^2+$$
$$\sigma_2^2)^2 + \sigma_1^4(n\theta^2+\sigma_2^2)(n\theta^2+\sigma_1^2+\sigma_2^2)^2 + \theta\sigma_1^2(n\theta^2+$$
$$\sigma_2^2)\sqrt{\frac{n\sigma_2^2}{\sigma_1^2(\sigma_1^2+\sigma_2^2)}}\sqrt{\sigma_1^2+\sigma_2^2}(\mathrm{ArcTan}[\frac{\sqrt{n}\theta\sigma_2}{\sigma_1\sqrt{n\theta^2+\sigma_1^2+\sigma_2^2}}](n\theta^2+\sigma_1^2)(\sigma_1^2+$$
$$\sigma_2^2)\sqrt{n\theta^2+\sigma_1^2+\sigma_2^2} + \sqrt{n}\theta\sigma_1\sigma_2(n\theta^2+\sigma_1^2+\sigma_2^2)) + \theta(n\theta^2+$$
$$\sigma_1^2)\sigma_2^2\sqrt{\frac{n\sigma_1^2}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}}\sqrt{\sigma_1^2+\sigma_2^2}(n\theta^2\mathrm{ArcTan}[\frac{\sqrt{n}\theta\sigma_1}{\sigma_2\sqrt{n\theta^2+\sigma_1^2+\sigma_2^2}}](\sigma_1^2+$$
$$\sigma_2^2)\sqrt{n\theta^2+\sigma_1^2+\sigma_2^2} + \sigma_2(\mathrm{ArcTan}[\frac{\sqrt{n}\theta\sigma_1}{\sigma_2\sqrt{n\theta^2+\sigma_1^2+\sigma_2^2}}]\sigma_2(\sigma_1^2+$$
$$\sigma_2^2)\sqrt{n\theta^2+\sigma_1^2+\sigma_2^2} + \sqrt{n}\theta\sigma_1(n\theta^2+\sigma_1^2+\sigma_2^2)))) \quad (7)$$

Definition 8: (Stress-Strength Model of n-components series system without standby components)

Suppose that $X_1 \ldots X_n$ be the lifetime of strength components from Rayleigh distribution with parameter σ and the common stress variable U which is

also Rayleigh distribution with parameter θ. Then the stress-strength reliability of the series system in this case is

$$R = P(X_{(1)} > U)$$
$$= \frac{\sigma^2}{n\theta^2+\sigma^2} \quad (8)$$

The main aim of adding standby components is to increase the reliability system. To show the effect of stand by components we graph the stress strength model using equations (3), (6) and (8). Figure (1) and Figure (2) show the effect of standby redundancy for different values of n and theta respectively.

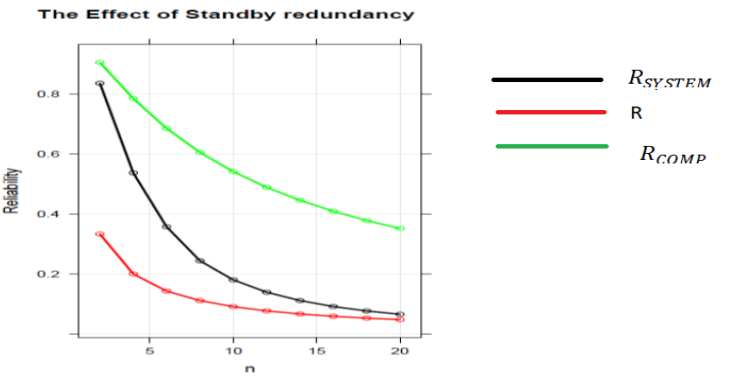


Figure 1
R_{COMP}, R and R_{SYSTEM} for different values of n and $\sigma_1 = 0.22, \sigma_2 = 0.44$, $\sigma = 0.2, \theta = 0.2$.

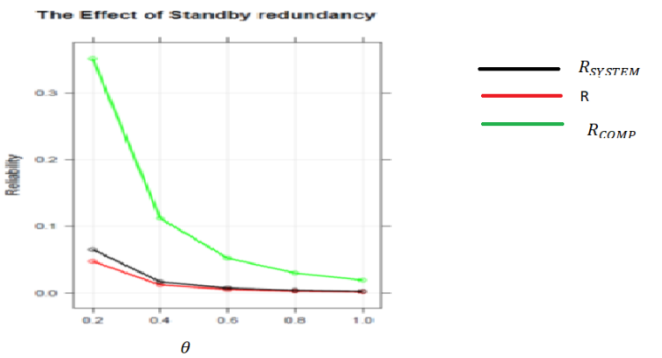


Figure 2
R_{COMP}, R and R_{SYSTEM} for different values of θ and $\sigma_1 = 0.22, \sigma_2 = 0.44$, $\sigma = 0.2, = n = 20$

3. The Maximum Likelihood Estimators of R_{COMP} and R_{SYSTEM}

In this section, we introduce the maximum likelihood estimator for R_{COMP}, and R_{SYSTEM}.

3.1 *Maximum Likelihood estimator of* R_{COMP}

Let the strength sample is $W_{i1} \dots W_{in}$, $i = 1 \dots m$ and the stress U_i, $i = 1 \dots m$, where m represent the number of systems which effect on n −components and n cold standby components in the series system. Then the likelihood function in this case is,

$$L(\theta, \sigma_1, \sigma_2 | \underline{w}, \underline{u}) = \prod_{i=1}^{m} \left(\prod_{j=1}^{n} f_W(w_{ij}) \right) f_U(u_i).$$

Where, $f_W(w_{ij})$ is defined in equation (1) and $f_U(u_i)$ is a Rayleigh distribution with parameter θ. Now, to get the maximum likelihood estimator of $\theta, \sigma_1, \sigma_2$. Taking the logarithm of the likelihood function.

$$Log\ L(\theta, \sigma_1, \sigma_2 | \underline{w}, \underline{u}) = \sum_{i=1}^{m} \sum_{j=1}^{n} Log(f_W(w_{ij})) + \sum_{i=1}^{m} Log(f_U(u_i)).$$

The partial derivatives are given in Appendix 1. From solving the first equation. We get the maximum likelihood estimator of θ which donated by $\hat{\theta}$ as $\hat{\theta} = \left(\frac{\sum_{i=1}^{m} u_i^2}{2\,m}\right)^{0.5}$, but we can't solve other equations analytic then we use the we use the numerical nonlinear global optimization random search method using Mathematica 13.1to get maximum likelihood estimators of σ_1 and σ_2 which are donated by $\hat{\sigma}_1$ and $\hat{\sigma}_2$. Also, using the invariance property of maximum likelihood method to get the maximum likelihood estimator of R_{COMP} which dented by R_{COMP}^{MLE} by replacing $\theta, \sigma_1, \sigma_2$ in equation (3) by its maximum likelihood estimators $\hat{\theta}$, $\hat{\sigma}_1$ and $\hat{\sigma}_2$.

3.2 *Maximum Likelihood estimator of* R_{SYSTEM}

Let $W_{(1),i}$, $i = 1 \dots m$ be the lifetime of i[th] system and U_i, $i = 1 \dots m$ be the stress. Then the likelihood function in this case is defined as

$$L(\theta, \sigma_1, \sigma_2 | \underline{w}, \underline{u}) = \prod_{i=1}^{m} \left(f_{W_{(1),i}}(w_{(1),i}) \right) f_U(u_i).$$

Where, $f_{W_{(1),i}}(W_{(1),i})$ is defined in equation (5) and $f_U(u_i)$ is a Rayleigh distribution with parameter θ. Now, to get the maximum likelihood estimator of $\theta, \sigma_1, \sigma_2$. Taking the logarithm of the likelihood function.

$$Log\ L(\theta, \sigma_1, \sigma_2 | \underline{\boldsymbol{w}}, \underline{\boldsymbol{u}}) = \sum_{i=1}^{m} Log(f_{W_{(1),i}}(w_{(1),i})) + \sum_{i=1}^{m} Log(f_U(u_i)).$$

The partial derivatives are given in Appendix 2. From solving First equation. We get the maximum likelihood estimator of θ which donated by $\hat{\theta}$ as $\hat{\theta} = \left(\frac{\sum_{i=1}^{m} u_i^2}{2\,m}\right)^{0.5}$, but we can't solve others equations analytic then we use the numerical nonlinear global optimization random search method using Mathematica 13.1 to get maximum likelihood estimators of σ_1 and σ_2 which are donated by $\hat{\sigma}_1$ and $\hat{\sigma}_2$. Also, using the invariance property of maximum likelihood method to get the maximum likelihood estimator of R_{SYSTEM} which dented by R_{SYSTEM}^{MLE} by replacing $\theta, \sigma_1, \sigma_2$ in equation (8) by its maximum likelihood estimators $\hat{\theta}$, $\hat{\sigma}_1$ and $\hat{\sigma}_2$.

4. Bayesian Estimators of R_{COMP} and R_{SYSTEM}

In this section we introduce the Bayesian estimators of R_{COMP} and R_{SYSTEM}.

4.1 *Bayesian Estimators of* R_{COMP}

To get the Bayes estimator of R_{COMP}, let informative priors of σ_1, σ_2 and θ be independent gamma distribution with parameters (α_i, β_i), $i = 1,2,3$ respectively. The probability densities functions of σ_1, σ_2 and θ are

$$f(\sigma_1) = \frac{{\beta_1}^{\alpha_1}}{\Gamma(\alpha_1)} \, {\sigma_1}^{\alpha_1 - 1} \, e^{-\sigma_1 \beta_1} \qquad \sigma_1 > 0, \alpha_1, \beta_1 > 0, \tag{9}$$

$$f(\sigma_2) = \frac{{\beta_2}^{\alpha_2}}{\Gamma(\alpha_2)} \, {\sigma_2}^{\alpha_2 - 1} \, e^{-\sigma_2 \beta_2} \qquad \sigma_2 > 0, \alpha_2, \beta_2 > 0, \tag{10}$$

$$f(\theta) = \frac{\theta^{\alpha_3}}{\Gamma(\alpha_3)} \, \theta^{\alpha_3 - 1} \, e^{-\theta \beta_3} \qquad \theta > 0, \alpha_3, \beta_3 > 0. \tag{11}$$

Then, the joint prior distribution is $f(\sigma_1, \sigma_2, \theta) = f(\sigma_1) f(\sigma_2) f(\theta)$ and the joint posterior distribution of σ_1, σ_2 and θ is $g\left(\sigma_1, \sigma_2, \theta \middle| \underline{w}, \underline{u}\right) = L\left(\theta, \sigma_1, \sigma_2 \middle| \underline{w}, \underline{u}\right) f(\sigma_1, \sigma_2, \theta)$. Hence, the Bayes estimator of R_{COMP} based on the square error loss function is

$$R_{COMP}^{Bayes} = \int_0^\infty \int_0^\infty \int_0^\infty R_{COMP} \; g\left(\sigma_1, \sigma_2, \theta \middle| \underline{w}, \underline{u}\right) d\sigma_1 d\sigma_2 d\theta$$

Also, we can use the noninformative priors when we replace functions defined in (9), (10), (11) by $f(\sigma_1) \propto \frac{1}{\sigma_1}$, $f(\sigma_2) \propto \frac{1}{\sigma_2}$ and $f(\theta) \propto \frac{1}{\theta}$. For more details about the noninformative prior see [29], [30]. But in both cases of informative or noninformative priors, we can't get the closed form of Bayes estimator of R_{COMP}. So, we use the Monte-Carlo Markov Chain method (MCMC) as an approximation of Bayes estimator. The MCMC method which named Gibbs sampler and Metropolis Hasting (MH) algorithm see [31] and [32]. To apply MH algorithm, we use the following steps.

1. Find the conditional posterior of $\sigma_1, \sigma_2,$ and θ which denoted by $g(\sigma_1 | \sigma_2, \underline{w})$, $g(\sigma_2 | \sigma_1, \underline{w})$, and $g(\theta | \underline{t})$.
2. Guess the start value of $\sigma_1^{(0)}$, $\sigma_2^{(0)}$ and $\theta^{(0)}$.
3. Set i=1
4. Generate $\theta^{(k+1)}$, $\sigma_1^{(k+1)}$, $\sigma_2^{(k+1)}$ using $\pi(\theta | \sigma_1^{(k)}, \sigma_2^{(k)}, \underline{w})$, $\pi(\sigma_1 | \theta^{(k)}, \sigma_2^{(k)}, \underline{w})$ and $\pi(\sigma_2 | \theta^{(k)}, \sigma_1^{(k)}, \underline{w})$ respectively with normal proposal distribution.
5. Repeat step 4 $N = 20000$ times.
6. Get the Bayes estimator of R_{COMP} at every value of $\sigma_1, \sigma_2,$ and θ and calculate the Bayes estimator from $R_{COMP}^{Bayes} = \frac{1}{N - N_0} \sum_{j=N_0}^{N} R_{COMP}^{(j)}$, where, $N_0 = 500$ is the burn -in period of generate Markov chain.

4.2 *Bayesian Estimators of* R_{SYSTEM}

In this subsection, we get the Bayes estimator of R_{SYSTEM} which denoted by R_{SYSTEM}^{Bayes} using the same technique in the previous subsection by replacing R_{COMP} by R_{SYSTEM}.

5. Simulation Study

We construct a Monte-Carlo simulation study to compare the different point estimators for our proposed model R_{COMP} and R_{SYSTEM}. The comparison between different estimators using different statistical criteria such as bias, mean square error (MSE) and relative efficiency (RE). The simulation study results are obtained by Mathematica 13.1 and R- program. To perform the simulation study using the following algorithm

1. Calculate the true values of R_{COMP} and R_{SYSTEM} at different values of different parameters $(\sigma_1, \sigma_2, \theta) = (0.5,2,2), (2.5,1,3), (2,2,5)$.
2. Generate the sample sizes of $m = 4,12,20$ and $n = 4, 8, 12, 16, 20, 50, 75, 100$.
3. For Bayesian estimation, we select the hyperparameters of $\sigma_1, \sigma_2, and\ \theta$ as $(a_i, b_i) = (0.1,2)$ for $i = 1,2,3$ for informative prior case and $(a_i, b_i) = (0,0)$ for noninformative case.
4. We get the maximum likelihood estimators of R_{COMP} and R_{SYSTEM} as method was explained in section 3.
5. We get the Bayesian estimators of R_{COMP} and R_{SYSTEM} as method was explained in section 4.
6. Calculate bias, mean square error (MSE) and relative efficiency (RE) where,

$Bias = E(\hat{R} - R)$, $MSE = E(\hat{R} - R)^2$ where $\hat{R}$ the estimator of R and

$RE1 = \frac{MSE(MLE)}{MSE(Bayes\ informative)}$, $RE2 = \frac{MSE(MLE)}{MSE(Bayes\ non-informative)}$, and

$RE3 = \frac{MSE(Bayes\ non-informative)}{MSE(Bayes\ informative)}$.

Tables 1-3 show the results of maximum likelihood and Bayesian point estimates of R_{COMP}. According to RE, we can decide that the maximum likelihood estimator of R_{COMP} is more efficient than the Bayesian estimators in both cases informative and non-informative prior because the values of $RE1$ and R$E2$ less than 1. Also, for Bayesian estimates we can decide that the informative case is more efficient than non-informative case because the vales of $RE3$ is more than 1. Also, from Tables 1-3 and Figures 3-5, we can see the values of mean square error are decrease when the sample sizes are increase. Tables 4-6 show the results of maximum likelihood and Bayesian point estimates of R_{SYSTEM}. According to RE, we can decide that the maximum likelihood estimator of R_{SYSTEM} is more efficient than the Bayesian estimators in both cases informative and non-informative prior because the values of $RE1$ and R$E2$ less than 1. Also, for Bayesian estimates we can decide that the informative case is more efficient

than non-informative case because the vales of $RE3$ is more than 1. Also, from Tables 4-6 and Figures 6-8, we can see the values of mean square error are decrease when the sample sizes are increase.

Table 1
Maximum Likelihood Estimator and Bayes Estimator of R_{COMP} for $(\sigma_1, \sigma_2, \theta) = (0.5, 2, 2)$

m	n	R_{COMP}	Maximum Likelihood Estimator			Bayes Estimator (Informative prior)			Bayes Estimator (Non-Informative prior)			$RE1$	$RE2$	$RE3$
			R_{COMP}^{MLE}	Bias	MSE	R_{COMP}^{Bayes}	Bias	MSE	R_{COMP}^{Bayes}	Bias	MSE			
4	4	0.1098	0.0699	-0.0997	0.0995	0.0480	-0.0154	0.2387	0.3714	0.0654	0.4277	0.4168	0.2326	1.7917
	8		0.0365	-0.0916	0.0839	0.0156	-0.0117	0.1386	0.1600	0.0062	0.3937	0.6051	0.2130	2.8405
	12		0.0264	-0.0695	0.0483	0.0157	-0.0078	0.0614	0.4211	0.0259	0.6729	0.7854	0.0717	1.0959
	16		0.0184	-0.0512	0.0326	0.0056	-0.0065	0.0424	0.0485	-0.0038	0.1467	0.7686	0.2220	3.4845
	20		0.0150	-0.0474	0.0224	0.0092	-0.0050	0.0253	0.0427	-0.0033	0.0025	0.8853	0.1990	4.4446
	50		0.0874	-0.0056	0.0003	0.0105	-0.0019	0.0039	0.1112	$2.3*10^{-5}$	0.0008	0.0760	0.3826	0.2010
	75		0.0389	-0.0009	0.0002	0.0432	-0.0008	0.0007	0.0687	-0.0005	0.0003	0.2536	0.6659	0.42900
	100		0.0094	-0.0010	0.0001	0.0301	-0.0007	0.0006	0.0618	-0.0004	0.0002	0.1574	0.4340	0.3840
12	4	0.0182	0.1641	0.0364	0.1330	0.0670	0.0122	0.1488	0.4056	0.0968	0.9379	0.8935	0.1417	6.3030
	8		0.0155	0.0003	0.0011	0.0089	0.0011	0.0013	0.1585	0.0175	0.3075	0.8139	0.0035	2.3653
	12		0.0154	0.0002	0.0005	0.0036	-0.0012	0.0014	0.0845	0.0055	0.3052	0.3377	0.0016	2.1800
	16		0.0126	-0.0003	0.0001	0.0047	-0.0008	0.0007	0.0521	0.0021	0.0448	0.1404	0.002	6.4000
	20		0.0089	-0.0004	0.0001	0.0044	-0.0006	0.0004	0.0022	-0.0008	0.0064	0.2100	0.0156	0.1450
	50		0.1463	0.0025	0.0001	0.0275	0.0002	0.0003	0.0186	$8*10^{-6}$	$6*10^{-4}$	0.2890	1.5625	4.000
	75		0.0462	0.0004	$1.4*10^{-6}$	0.0152	$-4*10^{-4}$	$0.16*10^{-4}$	0.0154	$-0.3*10^{-4}$	$0.1*10^{-4}$	0.0875	0.1004	0.8711
	100		0.0116	-0.0006	$4.4*10^{-8}$	0.0142	$-4*10^{-4}$	$0.16*10^{-4}$	0.0225	$0.4*10^{-4}$	$0.1*10^{-4}$	0.0027	0.0023	1.1556
20	4	0.0079	0.1146	0.0266	0.7115	0.0400	0.0080	0.0644	0.4038	0.0989	0.9796	0.1104	0.7236	2.4490
	8		0.0277	0.0024	0.0061	0.0214	0.0016	0.0028	0.1879	0.0013	0.0185	0.2142	0.3291	0.8644
	12		0.0154	0.0006	0.0004	0.0078	-0.0008	0.0069	0.0870	0.0065	0.4345	0.5760	0.0009	5.5705
	16		0.0087	0.0001	0.0002	0.0028	-0.0003	0.0010	0.04944	0.0025	0.0674	0.1968	0.0029	2.4071
	20		0.0047	-0.0001	0.0002	0.0031	-0.0002	0.0005	0.0026	-0.0002	0.0007	0.3472	0.2847	2.2580
	50		0.1156	0.0021	$0.4*10^{-4}$	0.1054	0.0019	0.0003	0.1040	$0.5*10^{-4}$	0.0003	0.1051	0.1600	0.8333
	75		0.0190	0.0001	$2.1*10^{-7}$	0.0083	$5.3*10^{-6}$	$2.8*10^{-7}$	0.0052	$-0.3*10^{-4}$	$1.29*10^{-7}$	0.7382	1.6203	0.4628
	100		0.0071	-0.0001	$6.4*10^{-10}$	0.0064	$-0.1*10^{-4}$	$2.2*10^{-9}$	0.0078	$-1*10^{-6}$	$1*10^{-9}$	0.2844	0.6400	0.4545

Table 2
Maximum Likelihood Estimator and Bayes Estimator of R_{COMP} for $(\sigma_1, \sigma_2, \theta) = (2.5, 1, 3)$

m	n	R_{COMP}	Maximum Likelihood Estimator			Bayes Estimator (Informative prior)			Bayes Estimator (Non-Informative prior)			$RE1$	$RE2$	$RE3$
			R_{COMP}^{MLE}	Bias	MSE	R_{COMP}^{Bayes}	Bias	MSE	R_{COMP}^{Bayes}	Bias	MSE			
4	4	0.1032	0.1827	0.0198	0.3950	0.0751	-0.0070	0.4921	0.3823	0.0697	0.4868	0.8026	0.8113	0.9892
	8		0.0279	-0.0094	0.0885	0.0239	-0.0099	0.9825	0.1581	0.0068	0.4709	0.9000	0.1879	0.4792
	12		0.0180	-0.0071	0.0504	0.0114	-0.0076	0.5852	0.0890	-0.0011	0.1400	0.8612	0.3599	0.2393
	16		0.0055	-0.0061	0.0372	0.0071	-0.0060	0.3607	0.0555	-0.0029	0.8887	0.1031	0.0418	2.4638
	20		0.0046	-0.0049	0.0243	0.0059	-0.0048	0.2366	0.0381	-0.0032	0.1059	0.1026	0.2293	0.4475
	50		0.1014	-0.0003	$1.3*10^{-5}$	0.0971	-0.0001	$1.5*10^{-5}$	0.0906	-0.0002	$0.6*10^{-4}$	0.8734	0.2047	4.2336
	75		0.0233	-0.0011	$1*10^{-5}$	0.1074	$0.6*10^{-5}$	$3.1*10^{-5}$	0.0848	-0.0002	$0.6*10^{-4}$	0.3188	0.1661	4.0125
	100		0.0030	-0.0010	$1*10^{-5}$	0.1357	0.0003	$1.1*10^{-5}$	0.0890	-0.0001	$0.2*10^{-4}$	0.9467	0.4959	1.8330
12	4	0.0181	0.0934	0.0188	0.3543	0.0598	0.0104	0.1086	0.4139	0.0840	0.7064	0.3260	0.0343	6.5046

	8		0.0239	0.0007	0.0005	0.0163	-0.0002	0.0005	0.1658	0.0184	0.0340	0.9876	0.0146	6.8000
	12		0.0174	-0.0001	0.0003	0.0149	-0.0002	0.0007	0.0926	0.0062	0.0038	0.4218	0.0778	5.4285
	16		0.0082	0.0006	0.0003	0.0070	-0.0006	0.0004	0.0593	0.0025	0.0006	0.6233	0.4524	1.5000
	20		0.0067	-0.0005	0.0003	0.0060	-0.0006	0.0003	0.0441	0.0013	0.0016	0.8196	0.1775	5.3333
	50		0.1005	0.0016	$0.21*10^{-4}$	0.0223	$0.8*10^{-4}$	$0.7*10^{-4}$	0.0212	$6*10^{-4}$	$3.8*10^{-5}$	0.2976	0.5463	0.5491
	75		0.0262	0.0001	$1.21*10^{-7}$	0.0137	$-0.5*10^{-4}$	$3.4*10^{-7}$	0.0200	$0.2*10^{-4}$	$6.4*10^{-7}$	0.3515	0.1885	1.8875
	100		0.0089	-0.0001	$8.51*10^{-8}$	0.0211	$0.3*10^{-4}$	$9*10^{-8}$	0.0249	$0.7*10^{-4}$	$4.62*10^{-7}$	0.9455	0.1840	5.1377
20	4	0.0080	0.1212	0.0283	0.8008	0.0781	0.0175	0.3071	0.3410	0.0832	0.6930	0.2607	1.1551	2.2565
	8		0.0371	0.0036	0.0132	0.0410	0.0041	0.0170	0.1363	0.0160	0.0257	0.7757	0.5132	1.5117
	12		0.0262	0.0015	0.0023	0.0088	0.0001	0.0044	0.0736	0.0054	0.0029	0.5175	0.7696	0.6590
	16		0.0100	0.0001	0.0001	0.0086	0.0001	0.0023	0.0455	0.0023	0.0005	0.0400	0.1820	0.2000
	20		0.0083	0.0001	0.0001	0.0109	0.0001	0.0210	0.0324	0.0012	0.0001	0.0047	0.6718	0.0047
	50		0.0787	0.0014	$2*10^{-4}$	0.0050	$-0.6*10^{-4}$	0.0004	0.0096	$0.3*10^{-4}$	0.0010	0.5555	0.1953	2.56
	75		0.0137	0.0001	$5.8*10^{-8}$	0.0076	$-5.3*10^{-6}$	$2.8*10^{-7}$	0.0106	$0.3*10^{-4}$	$1.2*10^{-7}$	0.2039	0.4826	0.4292
	100		0.0071	-0.0001	$0.8*10^{-9}$	0.0086	$6*10^{-6}$	$3.6*10^{-9}$	0.0072	$-8*10^{-6}$	$6.4*10^{-9}$	0.2222	0.1250	1.7777

Table 3
Maximum Likelihood Estimator and Bayes Estimator of R_{COMP} for $(\sigma_1, \sigma_2, \theta) = (2, 2, 5)$.

m	n	R_{COMP}	Maximum Likelihood Estimator			Bayes Estimator (Informative prior)			Bayes Estimator (Non-Informative prior)			$RE1$	$RE2$	$RE3$
			R_{COMP}^{MLE}	Bias	MSE	R_{COMP}^{Bayes}	Bias	MSE	R_{COMP}^{Bayes}	Bias	MSE			
4	4	0.0543	0.0275	-0.0067	0.0448	0.0378	0.006	0.0369	0.8287	0.2011	0.4044	1.2140	0.1107	0.1213
	8		0.0360	-0.0022	0.0052	0.0118	-0.0053	0.0282	0.7526	0.0872	0.0761	0.1843	0.0682	0.1842
	12		0.0138	-0.0033	0.0113	0.0025	-0.0043	0.0186	0.2392	0.0154	0.0237	0.6075	0.4759	0.6064
	16		0.0137	-0.0025	0.0064	0.0050	0.0030	0.0094	0.2741	0.0137	0.0188	0.6808	0.3391	0.6741
	20		0.0029	-0.0025	0.0060	0.0018	-0.0026	0.0068	0.3797	0.0162	0.0264	0.8823	0.2266	0.8707
	50		0.0621	0.0002	$2.4*10^{-7}$	0.0529	$-0.2*10^{-4}$	$7.84*10^{-7}$	0.0425	-0.0002	$5.56*10^{-7}$	0.3061	0.4309	0.7104
	75		0.0118	-0.0006	$0.3*10^{-7}$	0.0554	$0.1*10^{-4}$	$2.15*10^{-7}$	0.1028	0.0006	$4.18*10^{-7}$	0.1394	0.0717	1.9450
	100		0.0033	-0.0005	$0.2*10^{-7}$	0.0501	$-0.4*10^{-4}$	$1.76*10^{-6}$	0.0652	0.0001	$1.18*10^{-7}$	0.0113	0.1683	0.0675
12	4	0.0096	0.0447	0.0087	0.0770	0.0432	-0.0027	0.0077	0.9689	0.2398	0.5751	10	0.1338	0.9999
	8		0.0137	0.0005	0.0004	0.0108	-0.0054	0.0295	0.4053	0.0494	0.0244	0.0135	0.0163	0.0135
	12		0.0089	0.0005	0.0003	0.0074	-0.0039	0.0152	0.4746	0.0387	0.0150	0.0197	0.0199	0.0196
	16		0.0052	-0.0027	0.0075	0.0053	-0.0030	0.0093	0.3482	0.0211	0.0044	0.8064	1.6746	0.7996
	20		0.0045	-0.0025	0.0065	0.0056	-0.0024	0.0059	0.2338	0.0112	0.0125	1.1016	0.5172	0.1096
	50		0.0688	0.0012	$0.1*10^{-4}$	0.0082	$-0.2*10^{-4}$	$0.8*10^{-4}$	0.0068	$-0.5*10^{-4}$	$0.3*10^{-}$	0.1275	0.3188	0.3920
	75		0.0050	-0.0001	$3.7*10^{-8}$	0.0077	$-0.3*10^{-4}$	$6.4*10^{-8}$	0.0079	$-0.2*10^{-4}$	$5.14*10^{-8}$	0.5765	0.7201	0.8027
	100		0.0049	-0.0004	$2.2*10^{-8}$	0.0078	$-0.2*10^{-4}$	$3.24*10^{-8}$	0.0105	$9*10^{-6}$	$8.1*10^{-8}$	0.6790	0.2716	2.5000
20	4	0.0043	0.0731	0.1720	0.2958	0.0521	0.0119	0.1928	0.2813	0.0692	0.4795	1.5342	0.6168	0.2071
	8		0.0154	0.0138	0.0019	0.0131	0.0011	0.0012	0.0219	0.0022	0.04840	0.0833	0.0392	0.1570
	12		0.0088	0.0037	0.0001	0.00061	0.0001	0.0022	0.0092	0.0004	0.0016	0.0454	0.0599	0.1333
	16		0.0063	0.0012	0.0001	0.0057	0.0001	0.0012	0.0067	0.0001	0.0002	0.5000	0.4444	0.0790
	20		0.0039	0.0002	0.0004	0.0049	0.0001	0.0001	0.0062	0.0001	0.0009	0.4444	0.4432	0.1111
	50		0.0525	0.0001	$0.9*10^{-7}$	0.0035	$-0.2*10^{-4}$	$2.56*10^{-7}$	0.0057	$0.2*10^{-4}$	$7.84*10^{-7}$	0.3515	0.1147	3.0625
	75		0.0100	0.0001	$5.7*10^{-8}$	0.0046	$4*10^{-6}$	$1.6*10^{-7}$	0.0038	$6*10^{-6}$	$4.44*10^{-7}$	0.3562	0.1282	2.7777
	100		0.0046	$0.3*10^{-5}$	$0.9*10^{-11}$	0.0056	$0.13*10^{-4}$	$1.69*10^{-10}$	0.0036	$-7*10^{-6}$	$4.9*10^{-10}$	0.0532	0.0183	2.8994

Table 4
Maximum Likelihood Estimator and Bayes Estimator of R_{SYSTEM} for $(\sigma_1, \sigma_2, \theta) = (0.5, 2, 2)$

m	n	R_{SYSTEM}	Maximum Likelihood Estimator			Bayes Estimator (Informative prior)			Bayes Estimator (Non-Informative prior)			$RE1$	$RE2$	$RE3$
			R_{SYSTEM}^{MLE}	Bias	MSE	R_{SYSTEM}^{Bayes}	Bias	MSE	R_{SYSTEM}^{Bayes}	Bias	MSE			
4	4	0.2763	0.3820	0.0264	0.0698	0.1554	-0.0651	0.4249	0.1799	-0.0291	0.5808	0.1642	0.1201	1.3669
	8		0.1080	-0.0210	0.0442	0.1918	-0.0211	0.0446	0.0766	-0.0249	0.6231	0.9964	0.0709	1.3970
	12		0.0814	-0.0162	0.0263	0.0734	-0.0169	0.0285	0.1499	-0.0105	0.1109	0.9194	0.7336	3.8912
	16		0.1741	-0.0063	0.0040	0.0821	-0.0121	0.0147	0.2096	-0.0041	0.0173	0.2743	0.3201	1.1768
	20		0.0432	-0.0116	0.0013	0.0518	-0.0112	0.0126	0.0342	-0.0111	0.1233	0.1031	0.0105	1.7857
	50		0.0200	-0.0051	$2.6*10^{-5}$	0.0312	-0.0049	0.0024	0.0223	-0.0050	0.0025	0.0108	0.0100	1.07527
	75		0.0140	-0.0034	$1.2*10^{-5}$	0.0121	-0.0035	0.0012	0.0042	-0.0036	0.0013	0.0101	0.0092	1.0968
	100		0.0230	-0.0025	$6.4*10^{-6}$	0.0170	-0.0025	0.0007	0.0157	-0.0026	0.0007	0.0091	0.0094	0.9701
12	4	0.1097	0.3296	0.0549	0.0302	0.1516	0.0140	0.1974	0.1767	0.0167	0.2805	0.1582	0.1076	1.4209
	8		0.1314	0.0027	0.0007	0.0639	-0.0039	0.0155	0.1013	-0.0010	0.0011	0.4192	0.6349	0.0709
	12		0.2250	0.0096	0.0092	0.1668	0.0059	0.0354	0.0844	-0.0021	0.0044	0.0028	0.2069	1.4242
	16		0.0766	-0.0020	0.0004	0.0773	-0.0011	0.0127	0.1292	0.0012	0.0014	0.5047	0.2692	0.1102
	20		0.0916	-0.0009	0.0001	0.0534	-0.0021	0.0441	0.1075	-0.0001	0.0001	0.0022	0.8264	0.0022
	50		0.0266	-0.0016	$2.7*10^{-6}$	0.0342	-0.0015	0.0002	0.0416	-0.0018	$3.5*10^{-6}$	0.0130	0.7356	0.0176
	75		0.0153	-0.0012	$1.58*10^{-6}$	0.0358	-0.0009	0.0001	0.0385	-0.0009	$9.01*10^{-7}$	0.0158	0.0175	0.9010
	100		0.0183	-0.0009	$8.3*10^{-7}$	0.0189	-0.0009	0.0001	0.0275	0.0008	$0.6*10^{-5}$	0.0083	0.0122	0.6756
20	4	0.0684	0.2693	0.0502	0.2522	0.1795	0.0301	0.0906	0.2678	0.0498	0.2485	0.5816	0.1014	2.7428
	8		0.1321	0.0079	0.0063	0.0777	0.0023	0.0054	0.2842	0.0269	0.0727	0.9064	0.0865	1.3463
	12		0.1086	0.0033	0.0011	0.0884	0.0024	0.0059	0.1144	0.0038	0.0014	0.1845	0.7485	0.2372
	16		0.0771	0.0005	0.0001	0.0400	0.0011	0.0014	0.0498	-0.0011	0.0001	0.3508	0.7399	0.0714
	20		0.0777	0.0004	0.0001	0.0598	0.0003	0.0001	0.0522	-0.0008	0.0001	0.1632	0.1524	1
	50		0.0291	-0.0007	$6.17*10^{-7}$	0.0366	-0.0006	$0.4*10^{-4}$	0.0414	-0.0013	0.0001	0.0167	0.0035	4.6648

Table 5
Maximum Likelihood Estimator and Bayes Estimator of R_{SYSTEM} for $(\sigma_1, \sigma_2, \theta) = (2.5, 1, 3)$

m	n	R_{SYSTEM}	Maximum Likelihood Estimator			Bayes Estimator (Informative prior)			Bayes Estimator (Non-Informative prior)			$RE1$	$RE2$	$RE3$
			R_{SYSTEM}^{MLE}	Bias	MSE	R_{SYSTEM}^{Bayes}	Bias	MSE	R_{SYSTEM}^{Bayes}	Bias	MSE			
4	4	0.2471	0.3190	0.0179	0.0323	0.1439	-0.0258	0.0665	0.2086	-0.0096	0.0092	0.4852	0.3486	0.13834
	8		0.1279	-0.0149	0.0220	0.1369	-0.0137	0.0189	0.1209	-0.0157	0.0248	0.1159	0.8840	1.3121
	12		0.0736	-0.0144	0.0209	0.1033	-0.0119	0.0143	0.0951	-0.0124	0.0154	0.1455	0.1355	1.0769
	16		0.0837	-0.0102	0.0104	0.0988	-0.0092	0.0085	0.0981	0.0093	0.0086	0.1210	0.1199	1.0117
	20		0.1048	-0.0071	0.0050	0.0276	-0.0109	0.0120	0.0485	-0.0099	0.0098	0.4151	0.5070	0.8166
	50		0.0540	-0.0038	$0.1*10^{-5}$	0.0490	-0.0039	0.0015	0.0282	-00043	0.0019	0.0006	0.0005	1.2777
	75		0.0152	-0.0031	$9.5*10^{-6}$	0.0372	-0.0027	0.0007	0.0121	-0.0047	0.0022	0.0135	0.0043	3.1557
12	4	0.0954	0.3190	0.0559	0.3124	0.1334	0.0095	0.0090	0.2524	0.0392	0.1540	0.3461	0.2086	1.7111

	8		0.1603	0.0081	0.0065	0.0987	0.0004	0.0017	0.1423	0.0058	0.0034	0.3820	0.1891	2.0000
	12		0.1025	0.0005	0.0001	0.0408	-0.0045	0.0020	0.1057	0.0008	0.0007	0.0483	0.1357	0.3500
	16		0.0521	0.0254	0.0646	0.0484	-0.0029	0.0862	0.1329	0.0023	0.0054	0.7486	0.1176	0.0323
	20		0.0678	-0.0013	0.0001	0.0933	-0.0001	0.0001	0.0389	-0.0028	0.0079	0.9070	0.0125	0.7900
	50		0.0328	-0.0013	$1.56*10^{-6}$	0.0317	-0.0012	0.0001	0.0248	-0.0014	0.0001	0.0156	0.0078	1.9937
	75		0.0224	-0.0009	$9.4*10^{-7}$	0.0216	-0.0009	$0.9*10^{-4}$	0.0172	-0.0010	0.0001	0.0005	0.0086	0.0063
	100		0.0129	-0.0008	$9.8*10^{-7}$	0.0114	-0.0008	$0.7*10^{-4}$	0.0115	-0.0008	$0.7*10^{-4}$	0.014	0.0139	1.0056
20	4	0.0591	0.1510	0.0229	0.0527	0.0600	0.0002	0.0005	0.2029	0.0359	0.1292	0.1040	0.4077	2.5800
	8		0.1152	0.0070	0.0049	0.2947	-0.0294	0.0867	0.0869	0.0034	0.0012	0.0564	0.4057	0.0138
	12		0.1005	0.0034	0.0011	0.1075	0.0040	0.0016	0.0666	0.0006	0.0039	0.6761	0.2816	2.4375
	16		0.0552	-0.0002	0.0005	0.0653	0.0003	0.0015	0.0900	0.0019	0.0372	0.3329	0.0134	2.4800
	20		0.0561	-0.0015	0.0002	0.0574	-0.0001	0.0007	0.0353	-0.0011	0.0141	0.2768	0.0141	2.0142
	50		0.0360	-0.0004	$2.1*10^{-7}$	0.0390	-0.0004	$0.1*10^{-4}$	0.0284	-0.0006	$0.3*10^{-4}$	0.021	0.0055	3.769
	75		0.0162	-0.0006	$3.3*10^{-8}$	0.0260	-0.0004	$0.1*10^{-4}$	0.0165	-0.0005	$0.3*10^{-4}$	0.0033	0.0010	3.2262
	100		0.0123	-0.0005	$2.1*10^{-8}$	0.0214	-0.0003	$0.1*10^{-4}$	0.0118	-0.0004	$0.2*10^{-4}$	0.0021	0.0009	2.2372

Table 6
Maximum Likelihood Estimator and Bayes Estimator of R_{SYSTEM} for $(\sigma_1, \sigma_2, \theta) = (2, 2, 5)$

m	n	R_{SYSTEM}	Maximum Likelihood Estimator			Bayes Estimator (Informative prior)			Bayes Estimator (Non-Informative prior)			$RE1$	$RE2$	$RE3$
			R_{SYSTEM}^{MLE}	Bias	MSE	R_{SYSTEM}^{Bayes}	Bias	MSE	R_{SYSTEM}^{Bayes}	Bias	MSE			
4	4	0.1286	0.0063	0.1194	0.1426	0.1106	-0.0045	0.2025	0.0872	-0.0103	0.1071	0.7041	0.1331	0.5288
	8		0.0088	-0.0149	0.0022	0.0395	-0.0111	0.1240	0.0625	-0.0082	0.6826	0.0177	0.0032	5.5048
	12		0.0022	-0.0105	0.0011	0.0348	-0.0078	0.0611	0.0414	-0.0072	0.5280	0.0180	0.0020	8.6415
	16		0.0102	-0.0074	0.0005	0.0329	-0.0059	0.0357	0.0344	-0.0058	0.3466	0.0139	0.0014	9.7086
	20		0.0064	-0.0061	0.0003	0.0193	-0.0054	0.0029	0.0255	-0.0051	0.2657	0.1004	0.0011	9.1620
	50		0.0070	-0.0024	$5.9*10^{-6}$	0.0130	-0.0023	0.0005	0.0182	-0.0022	0.0005	0.0118	0.0121	0.9750
	75		0.0097	-0.0015	$2.6*10^{-6}$	0.0118	-0.0015	0.0002	0.0062	-0.0016	0.0002	0.013	0.0097	1.3317
	100		0.0120	-0.0011	$1.4*10^{-6}$	0.0052	-0.0012	0.0001	0.0071	-0.0012	0.0001	0.014	0.0094	1.4762
12	4	0.0459	0.0025	-0.0108	0.0011	0.1504	0.0261	0.0682	0.1003	0.0136	0.1849	0.0161	0.0059	2.7111
	8		0.0006	-0.0056	0.0003	0.0321	-0.0034	0.0011	0.0549	0.0011	0.0012	0.2520	0.2370	1.0909
	12		0.0880	0.0035	0.0001	0.0266	-00016	0.0002	0.0976	0.0043	0.0185	0.3865	0.0053	9.2500
	16		0.0460	0.0006	0.0001	0.0254	-0.0012	0.0001	0.0374	-0.0005	0.0002	0.6091	0.3543	2.0000
	20		0.0003	-0.2280	0.0001	0.0380	-0.0003	0.0001	0.0189	-0.0013	0.0018	0.6409	0.0548	1.8000
	50		0.0107	-0.0007	$4.9*10^{-7}$	0.0075	-0.0008	$0.6*10^{-4}$	0.0112	-0.0006	$0.4*10^{-4}$	0.0816	0.0101	8.0272
20	4	0.0279	0.0022	-0.0064	0.0041	0.0905	0.0156	0.0244	0.0875	0.0149	0.2220	0.1614	0.0184	9.0900
	8		0.0009	-0.0033	0.0011	0.0561	0.0035	0.0012	0.709	0.0053	0.0288	0.8852	0.0380	2.4000
	12		0.0005	-0.0022	0.0005	0.0442	0.0013	0.0001	0.0265	-0.0001	0.0013	0.2709	0.3673	1.3000
	16		0.0002	-0.0017	0.0002	0.0354	0.0004	0.0002	0.0427	0.0009	0.0855	0.9102	0.0023	4.2700
	20		0.0003	-0.0013	0.0001	0.0182	-0.0004	0.0002	0.0494	0.0011	0.1155	0.4259	0.0008	5.7000
	50		0.0125	-0.0003	$9.4*10^{-8}$	0.0090	-0.0004	$0.1*10^{-4}$	0.0132	-0.0002	$8.64*10^{-6}$	0.0094	0.0108	0.8643
	75		0.0078	-0.0002	$7.1*10^{-8}$	0.0068	-0.0028	$7.9*10^{-6}$	0.0061	-0.0002	$8.4*10^{-8}$	0.0089	0.0084	1.0694

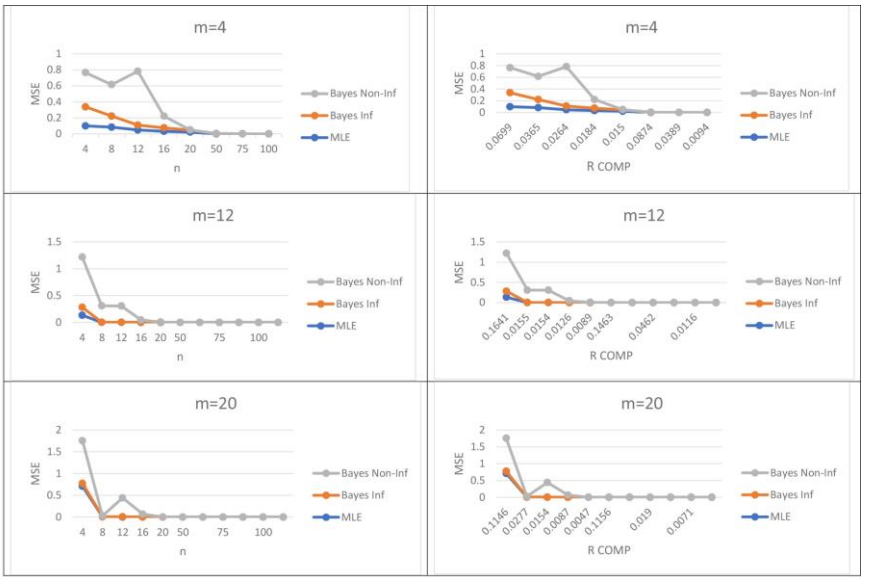


Figure 3
Plots of MSE for R_{COMP} for $(\sigma_1, \sigma_2, \theta) = (0.5, 2, 2)$.

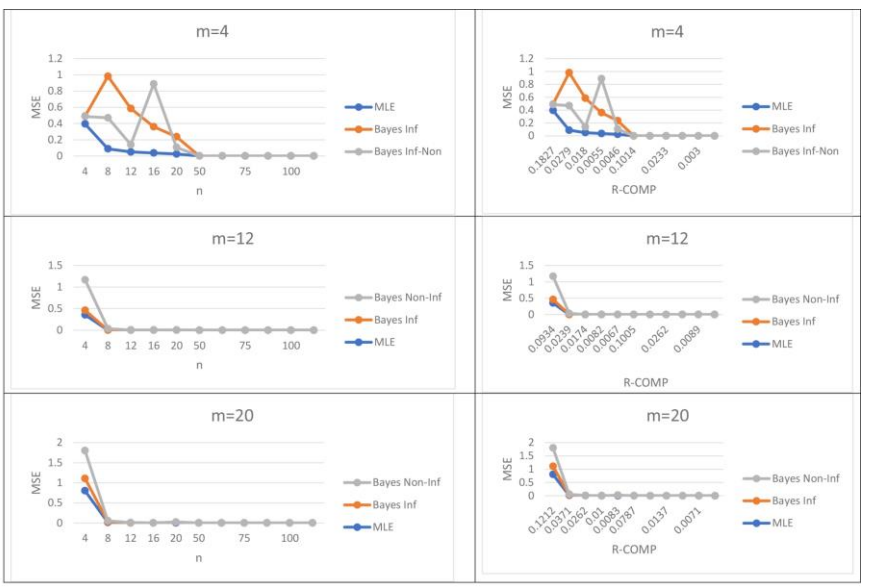


Figure 4
Plots of MSE for R_{COMP} for $(\sigma_1, \sigma_2, \theta) = (2.5, 1, 3)$.

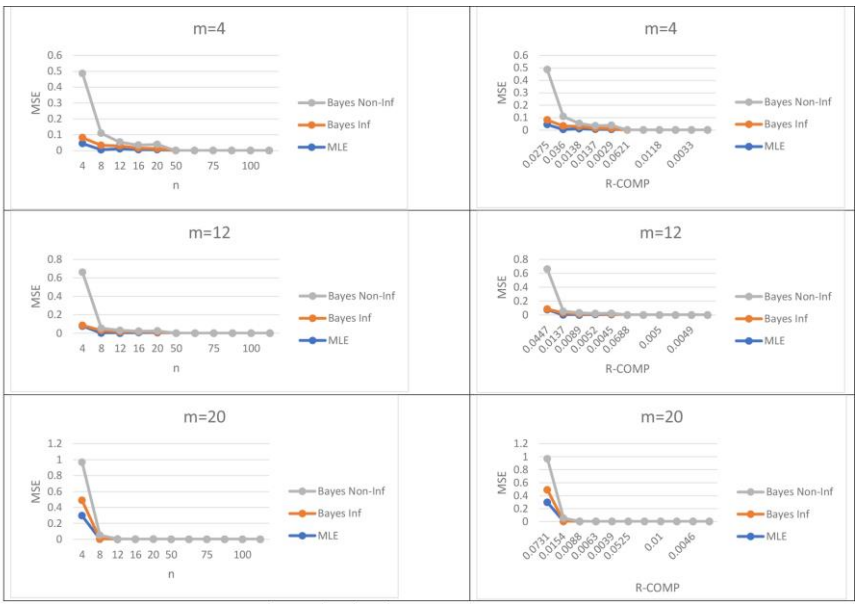


Figure 5
Plots of MSE for R_{COMP} for $(\sigma_1, \sigma_2, \theta) = (2, 2, 5)$.

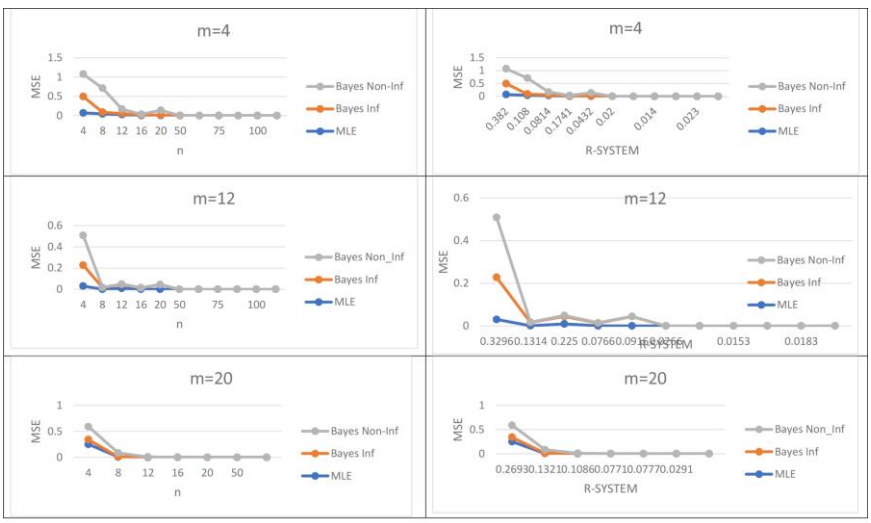


Figure 6
Plots of MSE for R_{SYSTEM} for $(\sigma_1, \sigma_2, \theta) = (0.5, 2, 2)$.

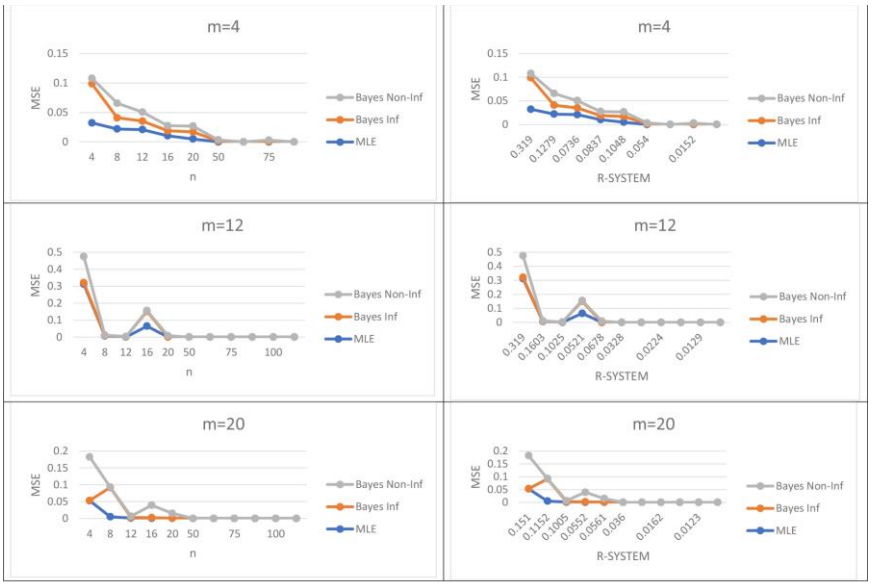


Figure 7
Plots of MSE for R_{SYSTEM} for $(\sigma_1, \sigma_2, \theta) = (2.5, 1, 3)$.

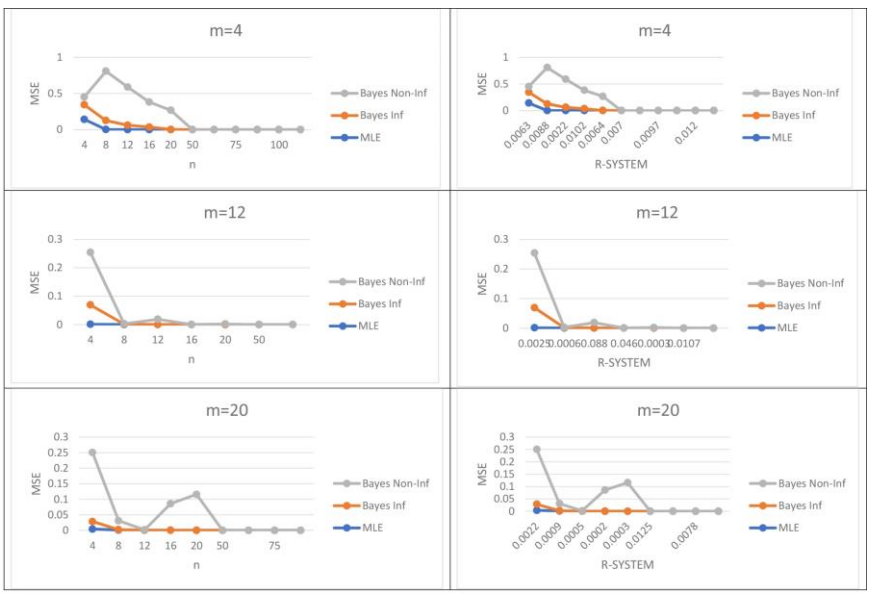


Figure 8
Plots of MSE for R_{SYSTEM} for $(\sigma_1, \sigma_2, \theta) = (2, 2, 5)$.

6. Application

In this section, we introduce the real application to show the validity of our proposed models in real life. [33] introduced data about the lifetime of steel specimens under the different stress level. Now, let us have the series system which has *n* active components (*X*-variable) which tested at 36 stress level with corresponding *n* standby components (*Y*- variable) which tested at 36.5 stress level. The main aim, an engineer wants to make the comparison between the system which contain the components *X* and *Y* by another component *U* which tested at 33 stress level where,

X:173 -218 -162 -288 -394 -585 -295- 262- 127- 151-181 -209 -141 -186 -309- 192- 117- 203- 198- 255.

Y: 118 -273 -192- 238- 105- 398- 108- 182- 130 -170-181 119- 152- 199- 89- 211- 324- 164- 133- 121.

U: 184- 241- 273- 1842- 371- 830- 683- 1306- 562- 166-981- 1867 -493- 418- 2978- 1463- 2220- 312- 251- 760.

To estimate the reliability of the series system using these data. Now, to verify our aim must perform the following steps:

1. Modify each data set X, Y, U by divided it by 100.
2. Divide the strength X and Y into m parts each one has n unit to construct W and U is getting as the mean value of each part we study two cases for R_{COMP}.

Case 1: For $n = 4, m = 5$,

$$W = \begin{pmatrix} 2.91 & 4.91 & 3.54 & 5.26 \\ 4.99 & 9.83 & 4.03 & 4.44 \\ 2.57 & 3.21 & 3.62 & 3.28 \\ 2.93 & 3.85 & 3.98 & 4.03 \\ 4.41 & 3.67 & 3.31 & 3.76 \end{pmatrix} \quad , \quad U_1 = \begin{pmatrix} 6.35 \\ 7.975 \\ 8.94 \\ 13.38 \\ 8.857 \end{pmatrix}.$$

Case 2: For $n = 5, m = 4$,

$$W = \begin{pmatrix} 2.91 & 4.91 & 3.54 & 5.26 & 4.94 \\ 9.83 & 4.03 & 4.44 & 2.57 & 3.21 \\ 3.62 & 3.28 & 3.93 & 3.85 & 3.98 \\ 4.03 & 4.41 & 3.67 & 3.31 & 3.75 \end{pmatrix} \quad , \quad U_2 = \begin{pmatrix} 5.822 \\ 7.094 \\ 13.474 \\ 10.012 \end{pmatrix}.$$

We study two cases for R_{SYSTEM} ,

Case 1: For $n = 4, m = 5$,

$$W = \begin{pmatrix} 2.91 \\ 4.03 \\ 2.57 \\ 2.93 \\ 3.31 \end{pmatrix} \quad , \quad U_1 = \begin{pmatrix} 6.35 \\ 7.975 \\ 8.94 \\ 13.38 \\ 8.857 \end{pmatrix}.$$

Case 2: For $n = 5, m = 4$,

$$W = \begin{pmatrix} 2.91 \\ 2.75 \\ 2.93 \\ 3.31 \end{pmatrix} \quad , \quad U_2 = \begin{pmatrix} 5.822 \\ 7.094 \\ 13.474 \\ 10.012 \end{pmatrix}.$$

3. Perform the goodness of fit using to approach tests and graphs to show that the Rayleigh distribution is a good fit to X, Y, U_1, U_2 as shown in Table 7 and Figure 9.

Table 7
The Result of Goodness of fit Tests for Data

Data	Goodness of Fit Test			
	Anderson- Darling		Kolmogorov- Smirnov	
	Statistic	P- Value	Statistic	P- Value
$\boldsymbol{X}$	1.0674	0.0980	0.1897	0.2025
$\boldsymbol{Y}$	0.8756	0.1663	0.1995	0.1554
$\boldsymbol{U_1}$	0.7781	0.2126	0.3667	0.1979
$\boldsymbol{U_2}$	0.3594	0.7157	0.3095	0.5522

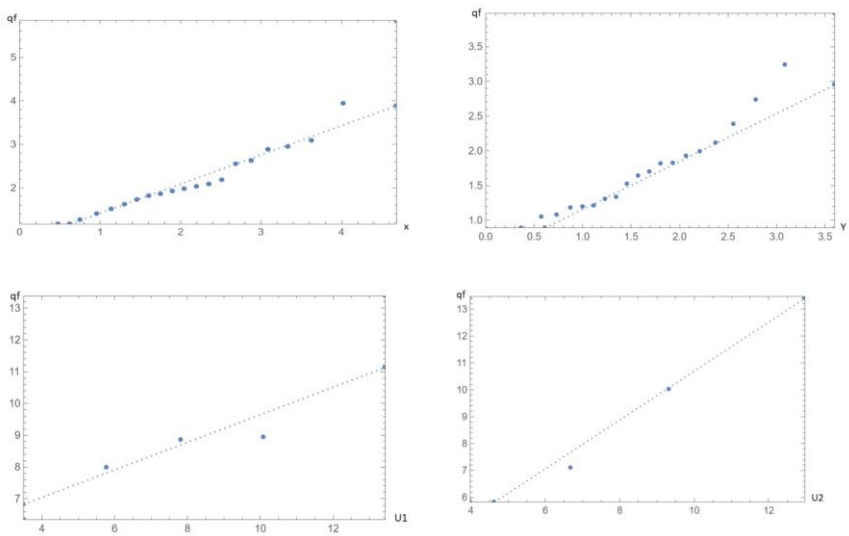


Figure 9
Q-Q Plot For X, Y, U_1, U_2

4. Finally, we calculate different estimates for R_{COMP} and R_{SYSTEM} as shown in Table 8,

Table 8
Different Estimates for R_{COMP} and R_{SYSTEM} for Data

$n = 4, m = 5$			
Estimate	**MLE**	**Bayes Informative Prior**	**Bayes Non-Informative Prior**
R_{COMP}	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ =(1.6397, 1.6397, 6.7639) $R_{COMP}^{MLE} = 0.0017$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (1.8468, 2.0161, 7.8697) $R_{COMP}^{Bayes} = 0.0016$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (1.6719, 2.3818, 7.0072) $R_{COMP}^{Bayes} = 0.0022$
R_{SYSTEM}	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (3.2794, 3.2794, 6.6432) $R_{SYSTEM}^{MLE} = 0.0421$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (3.4046, 4.2490, 8.8887) $R_{SYSTEM}^{Bayes} = 0.0323$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (3.4906, 4.4734, 9.3851) $R_{SYSTEM}^{Bayes} = 0.0314$
$n = 5, m = 4$			
R_{COMP}	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (1.6397, 1.6397, 6.6432) $R_{COMP}^{MLE} = 0.0016$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (3.8598, 6.400, 15.9728) $R_{COMP}^{Bayes} = 0.0026$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (2.2852, 1.6397, 7.7645) $R_{COMP}^{Bayes} = 0.0017$
R_{SYSTEM}	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (3.6665, 3.6665, 6.7639) $R_{SYSTEM}^{MLE} = 0.0504$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (4.5927, 5.2615, 7.7521) $R_{SYSTEM}^{Bayes} = 0.0683$	$(\hat{\sigma}_1, \hat{\sigma}_2, \hat{\theta})$ = (5.3414, 7.7974, 8.6598) $R_{SYSTEM}^{Bayes} = 0.0584$

7. Conclusions

We can state that the results and conclusions of our study as this study is considered Bayesian and non-Bayesian statistical inference of stress- strength for the series of system when adds the cold standby components at component levels and system levels. We are using Rayleigh distribution because it has many applications specially in engineering field. For non-Bayesian estimation we used maximum likelihood estimator also for Bayesian estimates we used MCMC algorithm. Simulation study is performed to compare the behavior of estimates which show that the maximum likelihood estimator is more efficient than Bayesian estimates using RE criterion. Finally, we use real engineering application to show the validity of our model in real life.

Acknowledgements

The authors are deeply thankful to the editor and reviewers for their valuable suggestions to improve the quality and presentation of the paper.

Appendix 1. Likelihood equations of R_{COMP}

$$\frac{\partial Log\ L}{\partial \theta} = -\frac{2\,m}{\theta} + \sum_{i=1}^{m} \frac{u_i^2}{\theta^3}$$

$$\frac{\partial Log\ L}{\partial \sigma_1}$$

$$= \sum_{i=1}^{m}\sum_{j=1}^{n} (e^{\frac{w_{ij}^2\left(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}\right)}{2\sigma_1^2}} \sqrt{2\pi} (\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}]$$

$$+ \mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}]) w_{ij}^4 \sigma_1^3 \sigma_2 + e^{\frac{w_{ij}^2\left(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}\right)}{2\sigma_1^2}} \sqrt{2\pi} (\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}]$$

$$+ \mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}]) \sigma_1\sigma_2(2\sigma_1^2-\sigma_2^2)(\sigma_1^2+\sigma_2^2)^2$$

$$+ 2e^{\frac{w_{ij}^2}{2(\sigma_1^2+\sigma_2^2)}} w_{ij}^3 \sigma_1^2 \sqrt{\sigma_1^2+\sigma_2^2} (e^{\frac{w_{ij}^2\sigma_1^2}{2\sigma_2^2(\sigma_1^2+\sigma_2^2)}}\sigma_1^2 + e^{\frac{w_{ij}^2\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)}}\sigma_2^2)$$

$$+ e^{\frac{w_{ij}^2\left(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}\right)}{2\sigma_1^2}} \sqrt{2\pi} (\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}]$$

$$+ \mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}]) w_{ij}^2 \sigma_1\sigma_2(-5\sigma_1^4 - 4\sigma_1^2\sigma_2^2 + \sigma_2^4) - 2e^{\frac{w_{ij}^2}{2(\sigma_1^2+\sigma_2^2)}} w_{ij}(\sigma_1^2$$

$$+ \sigma_2^2)^{3/2} (5e^{\frac{w_{ij}^2\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)}}\sigma_1^2\sigma_2^2 + e^{\frac{w_{ij}^2\sigma_1^2}{2\sigma_2^2(\sigma_1^2+\sigma_2^2)}}(2\sigma_1^4 - 4\sigma_1^2\sigma_2^2$$

$$- \sigma_2^4))) / (\sigma_1(\sigma_1^2+\sigma_2^2)^2 (2e^{\frac{w_{ij}^2}{2\sigma_2^2}} w_{ij}\sigma_1^2\sqrt{\sigma_1^2+\sigma_2^2} + 2e^{\frac{w_{ij}^2}{2\sigma_1^2}} w_{ij}\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}$$

$$- e^{\frac{w_{ij}^2\left(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}\right)}{2\sigma_1^2}} \sqrt{2\pi} (\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}]$$

$$+ \mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}]) \sigma_1\sigma_2(-w_{ij}^2 + \sigma_1^2$$

$$+ \sigma_2^2)))$$

$$\frac{\partial Log\ L}{\partial \sigma_2} =$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} (e^{\frac{w_{ij}^2(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)})}{2\sigma_1^2}} \sqrt{2\pi} (\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}] +$$

$$\mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])w_{ij}^4\sigma_1\sigma_2^3 + e^{\frac{w_{ij}^2(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)})}{2\sigma_1^2}}\sqrt{2\pi}(\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}] +$$

$$\mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])w_{ij}^2\sigma_1\sigma_2(\sigma_1^2-5\sigma_2^2)(\sigma_1^2+\sigma_2^2) -$$

$$e^{\frac{w_{ij}^2(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)})}{2\sigma_1^2}}\sqrt{2\pi}(\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}] + \mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])\sigma_1\sigma_2(\sigma_1^2 -$$

$$2\sigma_2^2)(\sigma_1^2+\sigma_2^2)^2 + 2e^{\frac{w_{ij}^2}{2(\sigma_1^2+\sigma_2^2)}}w_{ij}^3\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}(e^{\frac{w_{ij}^2\sigma_1^2}{2\sigma_2^2(\sigma_1^2+\sigma_2^2)}}\sigma_1^2 + e^{\frac{w_{ij}^2\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)}}\sigma_2^2) +$$

$$2e^{\frac{w_{ij}^2}{2(\sigma_1^2+\sigma_2^2)}}w_{ij}(\sigma_1^2 +$$

$$\sigma_2^2)^{3/2}(-5e^{\frac{w_{ij}^2\sigma_1^2}{2\sigma_2^2(\sigma_1^2+\sigma_2^2)}}\sigma_1^2\sigma_2^2 + e^{\frac{w_{ij}^2\sigma_2^2}{2\sigma_1^2(\sigma_1^2+\sigma_2^2)}}(\sigma_1^4+4\sigma_1^2\sigma_2^2-2\sigma_2^4))))/(\sigma_2(\sigma_1^2 +$$

$$\sigma_2^2)^2(2e^{\frac{w_{ij}^2}{2\sigma_2^2}}w_{ij}\sigma_1^2\sqrt{\sigma_1^2+\sigma_2^2} + 2e^{\frac{w_{ij}^2}{2\sigma_1^2}}w_{ij}\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2} -$$

$$e^{\frac{w_{ij}^2(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)})}{2\sigma_1^2}}\sqrt{2\pi}(\mathrm{Erf}[\frac{w_{ij}\sigma_1}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}] + \mathrm{Erf}[\frac{w_{ij}\sigma_2}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])\sigma_1\sigma_2(-w_{ij}^2 + \sigma_1^2 +$$

$$\sigma_2^2))).$$

Appendix 2. Likelihood equations of R_{SYSTEM}

$$\frac{\partial Log\ L}{\partial\theta} = -\frac{2\,m}{\theta} + \sum_{i=1}^{m}\frac{u_i^2}{\theta^3},$$

$$\frac{\partial Log\ L}{\partial\sigma_1} =$$

$$\sum_{i=1}^{m}(\sqrt{n}(-4e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}\sqrt{n}\sigma_1^6\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i} - 10e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sigma_1^4\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i} +$$

$$4e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}\sqrt{n}\sigma_1^4\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i} - 10e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sigma_1^2\sigma_2^4\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i} +$$

$$10e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}\sqrt{n}\sigma_1^2\sigma_2^4\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i} + 2e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}\sqrt{n}\sigma_2^6\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i} +$$

$$2e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}n^{3/2}\sigma_1^4\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}^3 +$$

$$2e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}n^{3/2}\sigma_1^2\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}^3 + e^{\frac{n(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)})w_{1,i}^2}{2\sigma_1^2}}\sqrt{2\pi}(\mathrm{Erf}[\frac{\sqrt{n}\sigma_1 w_{1,i}}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}] +$$

$$\mathrm{Erf}[\frac{\sqrt{n}\sigma_2 w_{1,i}}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])\sigma_1\sigma_2(2\sigma_1^6+3\sigma_1^4\sigma_2^2-\sigma_2^6+n(-5\sigma_1^4-4\sigma_1^2\sigma_2^2+\sigma_2^4)w_{1,i}^2+$$

$$n^2\sigma_1^2 w_{1,i}^4))))/((\sigma_1^2+\sigma_2^2)^2(2e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}n\sigma_1^3\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}+$$

$$2e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}n\sigma_1\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}-e^{\frac{n(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)})w_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sqrt{2\pi}(\mathrm{Erf}[\frac{\sqrt{n}\sigma_1 w_{1,i}}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}]+$$

$$\mathrm{Erf}[\frac{\sqrt{n}\sigma_2 w_{1,i}}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])\sigma_1^2\sigma_2(\sigma_1^2+\sigma_2^2-nw_{1,i}^2))),$$

$$\frac{\partial Log\,L}{\partial\sigma_2}=$$

$$\sum_{i=1}^{m}(\sqrt{n}(2e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sigma_1^6\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}+10e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sigma_1^4\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}-$$

$$10e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}\sqrt{n}\sigma_1^4\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}+4e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sigma_1^2\sigma_2^4\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}-$$

$$10e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}\sqrt{n}\sigma_1^2\sigma_2^4\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}-$$

$$4e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sigma_2^6\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}+2e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}n^{3/2}\sigma_1^2\sigma_2^2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}^3+$$

$$2e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}n^{3/2}\sigma_2^4\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}^3-e^{\frac{n\left(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}\right)w_{1,i}^2}{2\sigma_1^2}}\sqrt{2\pi}(\mathrm{Erf}[\frac{\sqrt{n}\sigma_1 w_{1,i}}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}]+$$

$$\mathrm{Erf}[\frac{\sqrt{n}\sigma_2 w_{1,i}}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])\sigma_1\sigma_2(\sigma_1^6-3\sigma_1^2\sigma_2^4-2\sigma_2^6-n(\sigma_1^2-5\sigma_2^2)(\sigma_1^2+\sigma_2^2)w_{1,i}^2-$$

$$n^2\sigma_2^2 w_{1,i}^4))))/((\sigma_1^2+\sigma_2^2)^2(2e^{\frac{nw_{1,i}^2}{2\sigma_2^2}}n\sigma_1^2\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}+$$

$$2e^{\frac{nw_{1,i}^2}{2\sigma_1^2}}n\sigma_2^3\sqrt{\sigma_1^2+\sigma_2^2}w_{1,i}-e^{\frac{n\left(1+\frac{\sigma_1^4}{\sigma_2^2(\sigma_1^2+\sigma_2^2)}\right)w_{1,i}^2}{2\sigma_1^2}}\sqrt{n}\sqrt{2\pi}(\mathrm{Erf}[\frac{\sqrt{n}\sigma_1 w_{1,i}}{\sqrt{2}\sigma_2\sqrt{\sigma_1^2+\sigma_2^2}}]+$$

$$\mathrm{Erf}[\frac{\sqrt{n}\sigma_2 w_{1,i}}{\sqrt{2}\sigma_1\sqrt{\sigma_1^2+\sigma_2^2}}])\sigma_1\sigma_2^2(\sigma_1^2+\sigma_2^2-nw_{1,i}^2))).$$

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