Journal of Statistics & Management Systems
ISSN 0972-0510 (Print), ISSN 2169-0014 (Online)
Vol. 28 (2025), No. 1, pp. 133–138
DOI : 10.47974/JSMS-1289

Selection of EMQ, quality investment, and parameters of acceptance sampling

C. H. Chen *
T. H. Lee †
Department of Industrial Management and Technology
Southern Taiwan University of Science and Technology
1 Nan-Tai Street, Yong-Kang Dist.
Tainan 71005
Taiwan, R.O.C.

Abstract

This work discusses the inventory model under EMQ, quality investment, parameters of acceptance sampling, PCI and AOQL.

Subject Classification: *60E05 Probability distributions: general theory.*

Keywords: *EMQ, AOQL, PCI.*

1. Introduction

Traditional EMQ model exists perfect assumption for product. Porteus [1] & Rosenblatt & Lee [2, 3] have considered the bad product for EMQ one. Chen [4] addressed the selection of mean and EMQ with Taguchi's [5] criteria.

The PCI is the available tool to improve quality. Kane [6], Chan et al. [7], Boyles [8], and Pearn et al. [9] studied hard about PCI.

There are some reports for investment of quality, e.g., Tsou [10], Chen & Tsou [11], Ganeshan et al. [12], and Hong et al. [13].

This work proposes the EMQ model based on specified Cpm and AOQL in order to have EMQ, investment of quality, and parameters of acceptance sampling.

* *E-mail:* **chench@stust.edu.tw (Corresponding Author)**
† *E-mail:* **thlee@stust.edu.tw**

2. Model

Consider to maximize

$$ETP(Q_1, Inv, n_1, d_0) = M_1 \cdot \left(W_{22} - \frac{Inv}{Q_1}\right) - S_t \cdot \frac{M_1}{Q_1} - 0.5 \cdot Q_1 \cdot \left(1 - \frac{Q_1}{I_1}\right) h_1 \tag{1}$$

$s.t.$

$$AOQL = p_L \tag{2}$$

$$C_{pm} = K_m \tag{3}$$

where

$$W_{22} = \left\{(A_{22} - A_{11}) + RI \cdot p_1 + \left(1 - \frac{n_1}{Q_1}\right) IC + \left(1 - \frac{n_1}{Q_1}\right)(LF1 - LF0) - \frac{n_1 p_1 RI}{Q_1}\right\}$$
$$P(X_1 \le d_0) + (A_{11} - RI \cdot p_1 - IC - C_1 m_{\mathrm{I}} - LF1) \tag{4}$$

$$LF0 = k_1 \left\{[(m_{\mathrm{I}} - T_1)^2 + \sigma_1^2]\Phi\left(\frac{T_1 - m_{\mathrm{I}}}{\sigma_1}\right) - \sigma_1 (m_{\mathrm{I}} - T_1)\phi\left(\frac{T_1 - m_{\mathrm{I}}}{\sigma_1}\right)\right\}$$
$$+ k_2 \left\{[(m_{\mathrm{I}} - T_1)^2 + \sigma_1^2]\left[1 - \Phi\left(\frac{T_1 - m_{\mathrm{I}}}{\sigma_1}\right)\right] + \sigma_1 (m_{\mathrm{I}} - T_1)\phi\left(\frac{T_1 - m_{\mathrm{I}}}{\sigma_1}\right)\right\} \tag{5}$$

$$LF1 = k_1 \{[(m_I - T_1)^2 + \sigma_1^2]\left[\Phi\left(\frac{T_1 - m_I}{\sigma_1}\right) - \Phi\left(\frac{LSL - m_I}{\sigma_1}\right)\right]$$
$$-\sigma_1 \left[(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_1}\right) - (m_I - 2T_1 + LSL)\phi\left(\frac{LSL - m_I}{\sigma_1}\right)\right]\}$$
$$+k_2 \left\{[(m_I - T_1)^2 + \sigma_1^2]\left[\Phi\left(\frac{USL - m_I}{\sigma_1}\right) - \Phi\left(\frac{T_1 - m_I}{\sigma_1}\right)\right] -\right.$$
$$\left. -\sigma_1 \left[-(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_1}\right) + (m_I - 2T_1 + USL)\phi\left(\frac{USL - m_I}{\sigma_1}\right)\right]\right\} \tag{6}$$

$$m_{\mathrm{I}}^2 = T_1^2 + (m_0^2 - T_1^2)\exp(-\beta Inv) \tag{7}$$

$$\sigma_1^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha Inv) \tag{8}$$

$$p_1 = 1 - \Phi\left(\frac{\mathrm{USL} - m_{\mathrm{I}}}{\sigma_1}\right) + \Phi\left(\frac{\mathrm{LSL} - m_{\mathrm{I}}}{\sigma_1}\right) \tag{9}$$

$$P(X_1 \le d_0) = \sum_{x_1=0}^{d_0} \frac{e^{-n_1 p_1}(n_1 p_1)^{x_1}}{x_1!} \tag{10}$$

$ETP(Q_1,\ Inv, n_1,\ d_0)]$ is expected total profit; W_{22} is ETP; A_{11} is sold price with rejected product; A_{22} is sold price for accepted product; Q_1 is production size; X is no. of bad unit; n is sampling no.; d_0 is available no. of bad unit; IC is inspection expense; $RI$is replacement expense; p_1 is bad prob.; C_1 is expense of material; $LF0$ is expected loss for no inspection; $LF1$ is expected loss for inspection; $\phi(\cdot)$ is p.d.f. of SND; $\Phi(\cdot)$ is c.d.f. of SND; LSL is min level; USL is max level; m_0 is process expectation; σ_0 is SD; M_1 is sold product (time); O_1 is sold product (day); I_1 is production product per day; S_t is preparing expense; h_1 is store expense; k_1 & k_2 are coefficients of loss; Inv is quality investment; m_I is improved mean; σ_1 is improved SD; K_m is specified PCI; β is declined coefficient for mean; α is declined coefficient for variance.

Consider following steps:

Step 1. From Dodge-Romig, we have $(n_1,\ d_0)$.

Step 2. Set limit Q_1 as $Q_{\max}$. Start $Q_1 = n_1$.

Step 3. Set limit Inv as $Inv_{\max}$. Start $Inv = 0$.

Step 4. We have C_{pm} and $ETP(Q_1, Inv, n_1, d_0)$.

Step 5. Set Inv = Inv +0.01, repeat Step 4 until $Inv_{\max}$.

Step 6. Set $Q_1 = Q_1 + 1$, repeat Steps 3-4 until $Q_{\max}$. We have $(Q_1, Inv, n_1,\ d_0)$ with maximum $ETP(Q_1, Inv, n_1,\ d_0)$.

3. Numerical Example

Table 1 presents the change of $\pm 20\% \sim \pm 30\%$ for the basic numerical data. From

Eqs. (1)-(10), we have $Q_1 = 320$, $Inv = 25.48$, $n_1 = 18$, $d_0 = 0$, $m_I = 50$, & $\sigma_1 = 0.556$ with $ETP(Q_1,\ Inv,\ n_1,\ d_0)$ = 17125.57.

Table 1 presents some results.

1. The α, σ_0, LSL, USL, K_m, & σ_T have a major effect for Inv.
2. The M_1, O_1, I_1, and h_1 have a large contribution for EMQ.
3. The $C_1, A_2, M_1, O_1, I_1, k_1, k_2, T_1, K_m$, LSL, USL, and σ_T have a large contribution for ETP.

4. Conclusions

We address a extended EMQ one under specified PCI and AOQL. Further study can apply this method for extended SCM.

Table 1

Numerical results

C_1	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.8	(320, 25.48, 0, 18)	25125.57
1.2	(320, 25.48, 0, 18)	9125.69
A_{22}	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
64	(320, 25.48, 0, 18)	4399.27
96	(321, 25.48, 0, 18)	29851.87
A_{11}	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
54	(319, 25.48, 0, 18)	17063.38
81	(320, 25.48, 0, 18)	17187.75
M_1	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
640	(286, 25.48, 0, 18)	13639.91
960	(350, 25.48, 0, 18)	20617.72
O_1	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
64	(239, 25.48, 0, 18)	16906.52
88	(412, 25.48, 0, 18)	17270.09
I_1	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
90	(428, 25.48, 0, 18)	17288.84
120	(248, 25.48, 0, 18)	16938.99
S_t	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
16	(315, 25.48, 0, 18)	17135.63
24	(324, 25.48, 0, 18)	17115.66
h_1	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
8	(358, 25.48, 0, 18)	17193.26
12	(292, 25.48, 0, 18)	17064.37
RI	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
24.4	(319, 25.48, 0, 18)	17125.66
36.6	(320, 25.48, 0, 18)	17125.47
k_1	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
160	(320, 25.48, 0, 18)	17619.31
240	(320, 25.48, 0, 18)	16631.83

Contd...

k_2	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
240	(319, 25.48, 0, 18)	17866.16
360	(320, 25.48, 0, 18)	16384.97
α	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.08	(320, 31.85, 0, 18)	17125.57
0.12	(319, 21.23, 0, 18)	17125.26
β	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.4	(319, 25.48, 0, 18)	17125.78
0.6	(320, 25.48, 0, 18)	17125.57
σ_T	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.4	(319, 17.29, 0, 18)	17118.00
0.6	(319, 79.34, 0, 18)	15996.59
T_1	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
49.5	(317, 25.48, 0, 18)	16962.01
50.5	(316, 25.48, 0, 18)	16170.95
m_0	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
48.5	(319, 25.48, 0, 18)	17125.56
49.5	(320, 25.48, 0, 18)	17125.57
σ_0	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.8	(319, 18.94, 0, 18)	17125.76
1.2	(320, 30.09, 0, 18)	17124.78
IC	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.4	(318, 25.48, 0, 18)	17130.51
0.6	(322, 25.48, 0, 18)	17120.66
p_L	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.01	(331, 25.48, 0, 37)	17041.19
0.05	(314, 25.48, 0, 7)	17175.07
LSL	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
47.5	(319, 16.70, 0, 18)	15404.42
48.5	(319, 87.23, 0, 18)	18100.83
USL	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
51.5	(319, 87.23, 0, 18)	18103.09
52.5	(320, 16.70, 0, 18)	15402.44
K_m	(Q_1, Inv, n_1, d_0)	$ETP(Q_1, Inv, n_1, d_0)$
0.96	(312, 11.71, 0, 18)	13036.14
1.44	(320, 87.23, 0, 18)	18343.31

References

[1] E.L. Porteus, Optimal lot sizing, process quality improvement and set-up cost reduction, *Operations Research,* vol. 34, pp. 137-144 (1986).

[2] M.J. Rosenblatt and H.L. Lee, Economic production cycles with imperfect production processes, *IIE Transactions*, vol. 17, pp. 48-54 (1986a).

[3] M.J. Rosenblatt and H.L. Lee, A comparative study of continuous and periodic inspection policies in deteriorating production systems, *IIE Transactions,* vol. 18, pp. 2-9 (1986b).

[4] C.H. Chen, Process mean and production quantity settings by considering modified economic manufacturing quantity model, *Journal of Information and Optimization Sciences*, vol. 39, pp. 1187-1198 (2018).

[5] G. Taguchi, Introduction to Quality Engineering, Asian Productivity Organization, Tokyo (1986).

[6] V.E. Kane, Process capability indices, *Journal of Quality Technology,* vol. 18, pp. 41-52 (1986).

[7] L.K. Chan, S.W. Cheng, and F.A. Spring, A new measure of process capability: Cpm, *Journal of Quality Technology,* vol. 30, pp. 162-175 (1988).

[8] R.A. Boyles, The Taguchi capability index, *Journal of Quality Technology,* vol. 23, pp. 107-126 (1991).

[9] W.L. Pearn, S. Kotz, and N.L. Johnson, Distributional and inferential properties of process capability indices, J*ournal of Quality Technology,* vol. 24, pp. 216-231 (1992).

[10] J.C. Tsou, Decision-making on quality investment in a dynamic lot sizing production system, *International Journal of Systems Science,* vol. 37, pp. 463-475 (2006).

[11] J.M. Chen and J.C. Tsou, An optimal design for process quality improvement: modeling and application, *Production Planning & Control,* vol. 14, pp. 603-612 (2003).

[12] R. Ganeshan, S. Kulkarni, and T. Boone, Production economics and process quality: a Taguchi perspective, *International Journal of Production Economics,* vol. 71, pp. 343-350 (2001).

[13] J.D. Hong, S.H. Xu, and J.C. Hayya, Process quality improvement and setup reduction in dynamic lot-sizing, I*nternational Journal of Production Research*, vol. 31, pp. 2693-2708 (1993).

Received September, 2022