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Process parameters settings and quality investment selection for modified traditional inventory model

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Abstract

In this article, we propose the process parameters settings in the first phase. Then the extended problem with production quantity, quality investment, and parameters of acceptance sampling in the second phase. Taguchi's loss cost is used for evaluating quality.

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Keywords: *Production quantity, Acceptance sampling.*

1. Introduction

The general production problem considers perfect product. Some works have considered the imperfect product, Porteus [1] and Rosenblatt & Lee [2] [3].

The modern production system addresses the control of product quality, production quantity, process promotion and improvement. For the inventory model, one considers the optimization of total cost. For the machine equipment, one considers the trade-off problem of the reliability and maintenance, i.e., maximum availability.

The process parameters affect the bias and variability of product. Taguchi [4] considered the loss cost for evaluating quality. This function connects with the on-line and off-line quality control methods. Chen [5], [6] have addressed the process parameters and production quantity

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settings. Shin et al. [7] also considered the joint setting about process parameters and product tolerance. Quality investment is a selection for promoting quality. Some works have addressed this topic, e.g., Hong et al. [8], Ganeshan, Kulkarni, and Boone [9] Chen & Tsou [10], and Tsou [11].

In this paper, the authors propose the extended problem with quality investment and process parameters setting Firstly, the optimal process parameters are determined by adopting Shin et al.'s [7] first phase. Then the decision variables are to obtain the production quantity, quality investment, and parameters of acceptance sampling.

2. Extended Problem

2.1 *Phase I--- Process Parameters Settings*

According to Shin et al. [7], the process parameters setting model is as follows: One needs to determine the mean and variance through the following model:

$$\begin{aligned}\text{Minimize } E[TC_1(\mu,\ \sigma)] &= \int_{-\infty}^{\infty} k_0(y-T_1)^2 f(y)dy + C_0 + C_1\mu + C_2\sigma + C_3\mu\sigma \\ &= k_0[(\mu-T_1)^2+\sigma^2] + C_0 + C_1\mu + C_2\sigma + C_3\mu\sigma \qquad (1)\end{aligned}$$

where k_0 is quality loss coefficient; μ is unknown mean; σ is unknown standard deviation; Y is normal r.v.; is p.d.f. of Y; T_1 is target value of mean; C_0 is constant in the process cost; C_1 is coefficient of μ in the process cost; C_2 is coefficient of σ in the process cost; C_3 is coefficient of $\mu\sigma$ in the process cost; $E[TC_1(\mu,\ \sigma)]$ is process expected cost.

Let $\frac{\partial E[TC_1(\mu,\ \sigma)]}{\partial \mu}=0$ and $\frac{\partial E[TC_1(\mu,\ \sigma)]}{\partial \sigma}=0$. We have $\mu_0=\frac{4k_1{}^2T_1-2k_1C_1+C_2C_3}{4k_1{}^2-C_3^2}$ and $\sigma_0=\frac{2k_1C_2+2k_1C_3T_1-C_1C_3}{-4k_1{}^2+C_3^2}$. The specified values of μ_0 and σ_0 are the input variables of phase II.

2.3 *Phase II--- Selection of economic manufacturing quantity, quality investment, and parameters of inspection plan*

One needs to determine some decision variables, i.e., production quantity, quality investment, and parameters of acceptance sampling. The mathematical model is as follows:

Maximize

$$ETP(Q,Inv,n,d_0) = M\cdot\left(W_2-\frac{Inv}{Q}\right) - S_t\cdot\frac{M}{Q} - 0.5\cdot Q\cdot\left(1-\frac{Q}{I}\right)h \qquad (2)$$

subject to $$\max_{0\le p\le 1} AOQ = p_L \tag{3}$$

where

$$W_2 = \left\{(A_2 - A_1) + R_I \cdot p + \left(1-\frac{n}{Q}\right) I_c + \left(1-\frac{n}{Q}\right)(LF_1 - LF_0) - \frac{npR_I}{Q}\right\} P(X \le d_0) + (A_1 - R_I \cdot p - I_c - C\mu_I - LF_1) \tag{4}$$

$$\begin{aligned} LF_0 &= \int_{-\infty}^{T_1} k_1 (y-T_1)^2 f(y, Inv)dy + \int_{T}^{\infty} k_2 (y-T_1)^2 f(y, Inv)dy \\ &= k_1 \left\{[(\mu_I - T_1)^2 + \sigma_I^2]\Phi\left(\frac{T_1-\mu_I}{\sigma_I}\right) - \sigma_I(\mu_I - T_1)\phi\left(\frac{T_1-\mu_I}{\sigma_I}\right)\right\} + \\ &\quad k_2 \left\{[(\mu_I - T_1)^2 + \sigma_I^2]\left[1-\Phi\left(\frac{T_1-\mu_I}{\sigma_I}\right)\right] + \sigma_I(\mu_I - T_1)\phi\left(\frac{T_1-\mu_I}{\sigma_I}\right)\right\} \end{aligned} \tag{5}$$

$$\begin{aligned} LF_1 &= \int_{LSL}^{T_1} k_1 (y-T_1)^2 f(y,\ Inv)dy + \int_{T_1}^{USL} k_2 (y-T_1)^2 f(y,\ Inv)dy \\ &= k_1 \left\{[(\mu_I - T_1)^2 + \sigma_I^2]\left[\Phi\left(\frac{T_1-\mu_I}{\sigma_I}\right) - \Phi\left(\frac{LSL-\mu_I}{\sigma_I}\right)\right] - \right. \\ &\quad \left. \sigma_I\left[(\mu_I - T_1)\phi\left(\frac{T_1-\mu_I}{\sigma_I}\right) - (\mu_I - 2T_1 + LSL)\phi\left(\frac{LSL-\mu_I}{\sigma_I}\right)\right]\right\} + \\ &\quad k_2 \left\{[(\mu_I - T_1)^2 + \sigma_I^2]\left[\Phi\left(\frac{USL-\mu_I}{\sigma_I}\right) - \Phi\left(\frac{T_1-\mu_I}{\sigma_I}\right)\right] - \right. \\ &\quad \left. \sigma_I\left[-(\mu_I - T_1)\phi\left(\frac{T_1-\mu_I}{\sigma_I}\right) + (\mu_I - 2T_1 + USL)\phi\left(\frac{USL-\mu_I}{\sigma_I}\right)\right]\right\} \end{aligned} \tag{6}$$

$$\mu_I^2 = T_1^{\ 2} + (\mu_0^2 - T_1^{\ 2})\exp(-\beta Inv) \tag{7}$$

$$\sigma_I^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha Inv) \tag{8}$$

$$p = 1 - \Phi\left(\frac{USL-\mu_I}{\sigma_I}\right) + \Phi\left(\frac{LSL-\mu_I}{\sigma_I}\right) \tag{9}$$

$$P(X \le d_0) = \sum_{x=0}^{d_0} \frac{e^{-np}(np)^x}{x!} \tag{10}$$

$ETP(Q,\ Inv, n,\ d_0)]$ is expected profit/unit time; W_2 is expected profit/unit; A_1 is sold price for bad lot; A_2 is sold price for good lot; Q is lot quantity; X is defective number; n is sample quantity; d_0 is largest number of defective item; I_c is inspection cost; R_I is replacement cost; p is defective probability; C is material cost; LF_0 is expected loss cost in the good lot; LF_1 is expected loss cost in the bad lot; $\phi(\cdot)$ is standard normal p.d.f.; $\Phi(\cdot)$ is standard normal c.d.f.; *LSL* is smallest value of conforming unit; *USL* is largest value of conforming unit; μ_0 is setting value of mean; σ_0 is known standard deviation; *M* is demand rate in the lot; *O* is demand rate per day; *I* is production rate per day; S_t is set-up cost; *h* is handing cost; k_1 and k_2 are loss coefficients; AOQ is average outgoing quality; p_L is specified AOQL; *Inv* is quality investment; μ_I is improved mean; β is exponential declined coefficient for mean; σ_I is improved standard deviation; α is exponential declined coefficient for standard deviation; *Y* is normal r.v.; $f(y, Inv)$ is the p.d.f. of *Y* with quality investment.

The decision variable *Inv* needs to be obtained by maximizing $ETP(Q, Inv, n, d_0)$. To solve Eqs. (2)-(10) is to do:

Step 1. According to Dodge-Romig [12], one has the combination of $(n,\ d_0)$ satisfying Eq. (3).

Step 2. Let the maximum *Q* be $Q_{\max}$. Set the initial $Q = n$.

Step 3. Let the maximum *Inv* be $Inv_{\max}$. Set the initial $Inv = 0$.

Step 4. Compute the corresponding $ETP(Q, Inv, n, d_0)$ from Eq. (2).

Step 5. For $Inv = Inv + 1$, repeat Step 4 until $Inv = Inv_{\max}$.

Step 6. For $Q = Q + 1$, repeat the above Steps when $Q < Q_{\max}$. The parameters $(Q, Inv, n,\ d_0)$ with largest $ETP(Q, Inv, n,\ d_0)$ is our solution.

3. Numerical Example

Some parameters are considered: $k_0 = 200$, $k_1 = 200$, $k_2 = 300$, $T_1 = 50$, $C_0 = 100$, $C_1 = 100$, $C_2 = -2$, $C_3 = -3$, $\alpha_1 = 0.5$, $\beta = 0.1$, $\sigma_T = 0.3$, $A_1 = 67.5$, $A_2 = 80$, $R_I = 30.5$, $M = 800$, $O = 80$, $I = 100$, $h = 10$, $c = 1$, $S_t = 20$, $LSL = 48$, $USL = 52$, $p_L = 2\%$, and $I_c = 0.5$.

By solving Eq. (1), one obtains the optimal $\mu_0 = 49.753$ and $\sigma_0 = 0.379$. By solving Eqs. (2)-(10), the optimal solution is $Q = 190$, $Inv = 16$, $n = 18$, $d_0 = 0$, $\mu_I = 49.950$, and $\sigma_I = 0.3$ with $ETP(Q,\ Inv, n, d_0)$ $= 6118.92$.

Table 1 shows the sensitivity results. Some results are as follows:

1. The $k_1, k_2, \beta, T_1,$ and σ_T have a significant contribution for quality investment.
2. The $M, O, I, S_t, k_1, k_2, \beta, T_1,$ and σ_T have a significant contribution for production quantity.
3. The $C,\ A_2,\ M, O, I, k_1, k_2, T_1,$ and σ_T have a significant contribution for expected profit.

4. Conclusions

This work have addressed the extended production quantity problem with quality investment and parameters of acceptance sampling. Further work should extend this model applied in the buyer-seller system with supplier, producer, wholesaler, retailer, and end-user for formulating the modified traditional inventory model.

Table 1

Sensitivity results

c	$(\mu_I,\ \sigma_I,\ Inv,\ Q, n, d_0)$	$ETP(Q, Inv, n, d_0)$
0.8	(49.950, 0.3, 16, 190, 0, 18)	14110.95
1.2	(49.950, 0.3, 16, 190, 0, 18)	-1873.12
A_2	$(\mu_I,\ \sigma_I,\ Inv,\ Q, n, d_0)$	$ETP(Q, Inv, n, d_0)$
64	(49.950, 0.3, 16, 190, 0, 18)	-6681.08
96	(49.950, 0.3, 16, 190, 0, 18)	18918.92
A_1	$(\mu_I,\ \sigma_I,\ Inv,\ Q, n, d_0)$	$ETP(Q, Inv, n, d_0)$
54	(49.950, 0.3, 16, 190, 0, 18)	6118.92
81	(49.950, 0.3, 16, 190, 0, 18)	6118.92
M	$(\mu_I,\ \sigma_I,\ Inv,\ Q, n, d_0)$	$ETP(Q, Inv, n, d_0)$
640	(49.950, 0.3, 16, 170, 0, 18)	4859.30
960	(49.950, 0.3, 16, 208, 0, 18)	7382.38
O	$(\mu_I,\ \sigma_I,\ Inv,\ Q, n, d_0)$	$ETP(Q, Inv, n, d_0)$
64	(49.950, 0.3, 16, 141, 0, 18)	5989.27
96	(49.950, 0.3, 16, 425, 0, 18)	6328.69
I	$(\mu_I,\ \sigma_I,\ Inv,\ Q, n, d_0)$	$ETP(Q, Inv, n, d_0)$
90	(49.950, 0.3, 16, 254, 0, 18)	6215.55
120	(49.950, 0.3, 16, 493, 0, 18)	6008.49 *Contd...*

S_t	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
16	(49.950, 0.3, 16, 181, 0, 18)	6136.18
24	(49.950, 0.3, 16, 198, 0, 18)	6102.41
h	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
8	(49.950, 0.3, 16, 212, 0, 18)	6158.98
12	(49.950, 0.3, 16, 173, 0, 18)	6082.70
R_I	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
24.4	(49.950, 0.3, 16, 190, 0, 18)	6118.92
36.6	(49.950, 0.3, 16, 190, 0, 18)	6118.92
k_1	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
160	(49.926, 0.3, 12, 181, 0, 18)	8110.64
240	(49.970, 0.3, 21, 200, 0, 18)	4344.46
k_2	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
240	(49.973, 0.3, 22, 202, 0, 18)	7855.95
360	(49.933, 0.3, 13, 183, 0, 18)	4574.57
α	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
0.4	(49.950, 0.3, 16, 190, 0, 18)	6105.06
0.6	(49.950, 0.3, 16, 190, 0, 18)	6121.71
β	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
0.08	(49.946, 0.3, 19, 196, 0, 18)	6107.10
0.12	(49.948, 0.3, 13, 183, 0, 18)	6119.41
σ_T	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
0.24	(49.959, 0.24, 18, 194, 0, 18)	12417.90
0.36	(49.939, 0.36, 14, 185, 0, 18)	-1580.28
T	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
49.5	(49.502, 0.3, 50, 251, 0, 18)	5862.79
50.5	(50.450, 0.3, 27, 212, 0, 18)	5678.57
I_c	$(\mu_I,\ \sigma_I,\ Inv,\ Q,n,d_0)$	$ETP(Q,Inv,n,d_0)$
0.4	(49.950, 0.3, 16, 186, 0, 18)	6126.58
0.6	(49.950, 0.3, 16, 193, 0, 18)	6111.40

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