Journal of Statistics & Management Systems
ISSN 0972-0510 (Print), ISSN 2169-0014 (Online)
Vol. 28 (2025), No. 1, pp. 89–93
DOI : 10.47974/JSMS-1209

Optimal process parameters setting based on time value of money

Chung-Ho Chen *
Tzu-Hsien Lee †
Department of Industrial Management and Technology
Southern Taiwan University of Science and Technology
1 Nan-Tai Street, Yong-Kang Dist.
Tainan 71005
Taiwan, R.O.C.

Abstract

In this research, we consider the process parameters setting by addressing Taguchi's cost for product and process adjustment with time value of money. The process parameters with the constant standard deviation and linear mean drift are considered. Some parameters with optimal process parameters and production run length are determined.

Subject Classification (2010): *60E05 Probability distributions: general theory.*

Keywords: *Process parameters, Taguchi's cost, Production run length, Time value of money.*

1. Introduction

In 1986, Taguchi [1] considered the quality cost for measuring the product. His loss concept can be applied in production process. Teran et al. [2] transferred this quality performance measurement into the present worth of expected quality loss per unit. Hence, Teran et al.'s [2] study also considered the time value of money when the customer holds it. Chou and Liang [3] and Chou and Chen [4] have presented the bivariate/multivariable quality loss with the time value of money. Recently, there are many works about this research by integrating Taguchi's [1] loss function. Chen [5, 6] presented the optimal manufacturing target. The decision criteria needs to obtain the minimum expected total cost. Chen

* *E-mail:* **chench@stust.edu.tw (Corresponding Author)**
† *E-mail:* **thlee@stust.edu.tw**

and Chou [7] addressed the optimum supply chain model with process and product parameters.

The joint design of product and tolerance is an significant decision for product design. Huang [8] considered that the adjustment cost problem. The tolerance design usually considers the trade-off balance for decreasing the probability of defective product and increasing the production cost. Shin et al. [9] considered the model with optimal process and product parameters. Hence, Shin et al.'s [9] process parameters and tolerance setting models addressed the application with product design and statistical process control.

In this research, we considered Shin et al.'s [9] model with time value of money. It The model is to get the optimal manufacturing target, standard deviation, and process cycle with minimum expected total cost.

2. Modified Shin et al.'s [9] Process Parameters Setting Model with Time Value of Money

Consider that the initial production process for the manufacturer needs to do the work about process parameters settings. The model considers the normal quality under the unknown mean and variability. The process mean occurs the linear mean drift. From Shin et al. [9] and Huang [8], the modified process parameters setting model with quality loss cost under the time value of money is as follows:

Minimize

$$E[TC(\mu,\ \sigma,\ T)] = \{\int_0^T k_1[\sigma_1{}^2(t) + (\mu_1(t) - M_1)^2]e^{-rt}dt + \int_0^T [\beta_1(\mu_1(t)^2) + \beta_2 \frac{1}{(\sigma_1(t))^2}]e^{-rt}dt\} / T \quad (1)$$

where

a	coefficient of linear drift of process mean
r	interest rate
k_1	quality loss coefficient
T	production run length
μ	unknown parameter of process mean
σ	unknown parameter of standard deviation
M_1	target value of product
β_1	coefficient of μ^2 in the process cost

β_2	coefficient of $1/\sigma^2$ in the process cost
$E[TC(\mu,\ \sigma,T)]$	expected total cost
$\mu_1(t)=\mu+at$	process mean during the time period t
$\sigma_1(t)=\sigma$	process standard deviation during the time period t

$$\int_0^T k_1[\sigma_1{}^2(t)+(\mu_1(t)-M_1)^2]e^{-rt}dt$$
$$=k_1\sigma^2 I(0,r,T)+k[(\mu_1(t)-M_1)^2 I(0,r,T)+2a(\mu_1(t)-M_1)I(1,r,T)+a^2 I(2,r,T)]$$
$$\int_0^T[\beta_1(\mu_1(t)^2)+\beta_2\frac{1}{(\sigma_1(t))^2}]e^{-rt}dt$$
$$=\beta_1[\mu^2 I(0,r,T)+2a\mu I(1,r,T)+a^2 I(2,r,T)]+\beta_2\frac{1}{\sigma^2}I(0,r,T)$$
$$I(0,r,T)=\int_0^T e^{-rt}dt=\frac{1}{r}(1-e^{-rT})$$
$$I(1,r,T)=\int_0^T te^{-rt}dt=\frac{1}{r^2}(1-e^{-rT})-\frac{Te^{-rT}}{r}$$
$$I(2,r,T)=\int_0^T t^2e^{-rt}dt=\frac{2}{r}[\frac{1}{r^2}(1-e^{-rT})-\frac{Te^{-rT}}{r}]-\frac{T^2e^{-rT}}{r}$$

One can obtain the first partial derivative of Eq. (1) with respect to μ and σ as follows:

$$\frac{\partial E[TC(\mu,\ \sigma,T)]}{\partial\mu}=[2k_1(\mu-M_1)I(0,r,T)+2akI(1,r,T)$$
$$+2\mu\beta_1 I(0,r,T)+2a\beta_1 r(1,r,T)]/T$$
$$\frac{\partial E[TC(\mu,\ \sigma,T)]}{\partial\sigma}=[2k\sigma I(0,r,T)-2\beta_2\sigma^{-3}I(0,r,T)]/T$$

Let $\frac{\partial E[TC(\mu,\ \sigma,T)]}{\partial\mu}=0$ and $\frac{\partial E[TC(\mu,\ \sigma,T)]}{\partial\sigma}=0$, one obtains the optimal $\mu_0=\frac{kMI(0,r,T)-akI(1,r,T)-a\beta_1 I(1,r,T)}{kI(0,r,T)+\beta_1 I(0,r,T)}$ and $\sigma_0=\sqrt[4]{\frac{\beta_2}{k}}$. The specified values of μ_0 and σ_0 are the input values of Eq.(1) for determining T. We use the direct search method for solution. T.

3. Numerical Example

Consider some parameters: k = 200, M =50, $\beta_1=2$, $\beta_2=6$, $a=1$, and $r=3\%$.

By solving Eq. (1), the optimal solution is $\mu_0=48.383$, $\sigma_0=0.416$. $t=2.27$ with $E[TC(\mu,\sigma,T)]$ = 4936.245. Following Table resents the result of sensitivity analysis. From Table 1, one has some results:

1. The M, and β_1 have a large effect on the process mean.

2. The k and β_2 have a large effect on the standard deviation.
3. The k , M, β_1, a, and r have a large effect on the production cycle.
4. The M and β_1 have a large effect on the expected total cost.

Table 1
Some results for example

a	μ_0	σ_0	t	$E[TC(\mu,\sigma,T)]$
0.8	48.145	0.416	3.46	4890.342
1.2	48.612	0.416	1.50	4960.825
k	μ_0	σ_0	t	$E[TC(\mu,\sigma,T)]$
160	48.032	0.416	2.74	4897.221
240	48.670	0.398	1.85	4963.835
β_1	μ_0	σ_0	t	$E[TC(\mu,\sigma,T)]$
1.6	48.687	0.416	1.85	3982.872
2.4	48.071	0.416	2.71	5879.386
β_2	μ_0	σ_0	t	$E[TC(\mu,\sigma,T)]$
4.8	48.383	0.394	2.27	4929.174
7.2	48.383	0.436	2.27	4942.637
M	μ_0	σ_0	t	$E[TC(\mu,\sigma,T)]$
40	38.860	0.416	1.50	3202.165
60	57.812	0.416	3.24	7027.434
r	μ_0	σ_0	t	$E[TC(\mu,\sigma,T)]$
2.4%	48.656	0.416	1.71	4965.407
3%	48.182	0.416	2.69	4900.204

4. Conclusions

In this article, the authors address the Shin et al.'s [9] model with time value of money. Numerical results show that M and β_1 have a significant effect on the solution. Further study should consider the model with process capability and time value of money.

References

[1] G. Taguchi, *Introduction to Quality Engineering*, Asian Productivity Organization, Tokyo (1986).

[2] A. Teran, D. B. Pratt, and K. E. Case, "Present worth of external quality losses for symmetric nominal-is-better quality characteristics," *The Engineering Economist*, vol. 42, pp. 39-52 (1996).

[3] C. Y. Chou and W. H. Liang, "A study on the present worth of bivariate quality loss," *Journal of the Chinese Institute of Engineers*, vol. 23, pp. 251-259 (2000).

[4] C. Y. Chou and C. H. Chen, "On the present worth of multivariable quality loss," *International Journal of Production Economics*, vol. 70, pp. 279-288 (2001).

[5] C. H. Chen, "Optimal process mean setting under the quality and cost," *Journal of Statistics and Management Systems*, vol. 21, pp. 807-815 (2018).

[6] C. H. Chen, "Optimal process mean setting based on asymmetric linear quality loss function," *Journal of Information and Optimization Sciences*, vol. 40, pp. 37-41 (2019).

[7] C. H. Chen and C. Y. Chou, "Optimal process mean, specification limits, replenishment cycle time, and credit period for the buyer-seller model with specified capability value," *Journal of Information and Optimization Sciences*, vol. 42, pp. 845-854 (2021).

[8] Y. F. Huang, "Trade-off between quality and cost," *Quality & Quantity*, vol. 35, pp. 265-276 (2001).

[9] S. Shin, P. Kongsuwon, and B. R. Cho, "Development of the parametric tolerance modeling and optimization schemes and cost-effective solutions," *European Journal of Operational Research*, vol. 207, pp. 1728-1741 (2010).

Received February, 2023