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Process variance setting and quality investment selection for modified traditional inventory model

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Abstract

This work discusses inventory model based on EMQ, process variance, and parameters of inspection plan in the first step. Then the optimal quality investment for reducing process variance is determined in the second step.

In this article, we consider the modified inventory model with EMQ, process variance, and parameters of inspection plan in the first step. Then the optimal quality investment for reducing process bias is determined in the second step.

Subject Classification: *60E05 Probability distributions: general theory.*

Keywords: *EMQ, Acceptance sampling, Mean, Investment of quality.*

1. Introduction

Traditional EMQ model addresses that the perfect production. Porteus [8] & Rosenblatt & Lee [9, 10] have presented a imperfect EMQ model.

Taguchi [12] proposed the quadratic loss function for evaluating the quality. Shin et al.'s [11] process parameters problem including loss and process cost. Chen [2] addressed the design of mean and quantity for modified EMQ model. Chen [3] further proposed the design of mean and variance with uniform characteristic with quality loss.

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Improvement activity is a long-term tool for increasing product quality. Tsou [13], Chen & Tsou [4], Ganeshan et al. [5], and Hong et al. [6] have applied this method. Chase [1] and Jeang [7] addressed the problem of product tolerance design.

In this work, we address a modified model including quality investment and variance. Firstly, we need to have the EMQ, variance, and parameters of inspection plan. Then the quality investment is applied in the decreasing bias and increasing the expected total profit.

2. Model

Step 1. One needs to select the EMQ, process variance setting, and parameters of inspection plan for the modified model.

Maximize

$$ETP(Q_1,\ \sigma_0,\ n_{,1}\ d_0) = M_1 \cdot W_{22} - S_t \cdot \frac{M_1}{Q_1} - 0.5 \cdot Q_1 \cdot \left(1 - \frac{O_1}{I_1}\right) h_1 \tag{1}$$

subject to

$$\max_{0 \le p \le 1} AOQ = p_L \tag{2}$$

where

$$\begin{aligned} W_{22} = &\left\{ (A_{22} - A_{11}) + RI \cdot p_1 + \left(1 - \frac{n_1}{Q_1}\right) IC + \left(1 - \frac{n_1}{Q_1}\right)(LF1 - LF0) \right. \\ &\left. - \frac{n_1 p_1 RI}{Q_1} \right\} P(X_1 \le d_0) + (A_1 - RI \cdot p_1 - IC - C_1 m_0 - LF1) \end{aligned} \tag{3}$$

$$\begin{aligned} LF0 = &k_1 \left\{ [(m_0 - T_1)^2 + \sigma_0^2] \Phi\left(\frac{T_1 - m_0}{\sigma_0}\right) - \sigma_0 (m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) \right\} \\ &+ k_2 \left\{ [(m_0 - T_1)^2 + \sigma_0^2] \left[1 - \Phi\left(\frac{T_1 - m_0}{\sigma_0}\right)\right] + \sigma_0 (m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) \right\} \end{aligned} \tag{4}$$

$$\begin{aligned} LF1 = &k_1 \left\{ [(m_0 - T_1)^2 + \sigma_0^2] \left[\Phi\left(\frac{T_1 - m_0}{\sigma_0}\right) - \Phi\left(\frac{LSL - m_0}{\sigma_0}\right) \right] \right. \\ &\left. - \sigma_0 \left[(m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) - (m_0 - 2T_1 + LSL) \phi\left(\frac{LSL - m_0}{\sigma_0}\right) \right] \right\} \\ &+ k_2 \left\{ [(m_0 - T_1)^2 + \sigma_0^2] \left[\Phi\left(\frac{USL - m_0}{\sigma_0}\right) - \Phi\left(\frac{T_1 - m_0}{\sigma_0}\right) \right] \right. \\ &\left. - \sigma_0 \left[-(m_0 - T_1) \phi\left(\frac{T_1 - m_0}{\sigma_0}\right) + (m_0 - 2T_1 + USL) \phi\left(\frac{USL - m_0}{\sigma_0}\right) \right] \right\} \end{aligned} \tag{5}$$

$$p_1 = 1 - \Phi\left(\frac{USL - m_0}{\sigma_0}\right) + \Phi\left(\frac{LSL - m_0}{\sigma_0}\right) \tag{6}$$

$$P(X_1 \le d_0) = \sum_{x_1=0}^{d_0} \frac{e^{-n_1 p_1}(n_1 p_1)^{x_1}}{x_1!} \tag{7}$$

$ETP(Q_1,\ \sigma_0, n_1,\ d_0)]$ is expected total profit; W_{22} is ETP; A_{11} is sold price with rejected product; A_{22} is sold price for accepted product; Q_1 is production size; X is no. of bad unit; n is sampling no.; d_0 is available no. of bad unit; IC_c is inspection expense; RI is replacement expense; p_1 is bad prob.; C is expense of material; $LF0$ is expected loss for no inspection; $LF1$ is expected loss for inspection; $\phi(\cdot)$ is p.d.f. of SND; $\Phi(\cdot)$ is c.d.f. of SND; LSL is min level; USL is max level; m_0 is process expectation; σ_0 is SD; M_1 is sold product (time); O_1 is sold product (day); I_1 is production product per day; S_t is preparing expense; h is store expense; k_1 & k_2 are coefficients of loss.

The decision variables $(Q_1,\ \sigma_0, n_1,\ d_0)$ need to solve under maximizing $ETP(Q_1, \sigma_0, n_1,\ d_0)]$. Consider following steps:

Step 1. Let maximum Q_1 be $Q_{\max}$ and initial $Q_1 = Q_{\max}$.

Step 2. From Dodge-Romig, we have $(n_1,\ d_0)$ satisfying Eq. (2).

Step 3. Searching the interval $0.01 < \sigma_0 < (USL - LSL)/8$, one can obtain the corresponding $ETP(Q_1,\ \sigma_0, n_1,\ d_0)$ from Eq. (1).

Step 4. Set $Q_1 = Q_1 + 1$ and continue Steps 2-3 with $Q_1 = Q_{\max}$. The $(Q_1, \sigma_0, n_1,\ d_0)$ with maximum $ETP(Q_1, \sigma_0, n_1,\ d_0)$ is the optimal solution.

The optimal combination of $(Q_1, \sigma_0, n_1,\ d_0)$ is the input variables in the next step.

Step 2. One needs to select the quality investment. The model is
Maximize

$$ETP(Inv) = M_1 \cdot \left(W_{22} - \frac{Inv}{Q_1}\right) - S_t \cdot \frac{M_1}{Q_1} - 0.5 \cdot Q_1 \cdot \left(1 - \frac{O_1}{I_1}\right) h_1 \tag{8}$$

where

$$W_{22} = \left\{(A_{22} - A_{11}) + RI \cdot p_1 + \left(1 - \frac{n_1}{Q_1}\right) IC + \left(1 - \frac{n_1}{Q_1}\right)(LF1 - LF0) - \frac{n_1 p_1 RI}{Q_1}\right\} P(X_1 \le d_0) + (A_{11} - RI \cdot p_1 - IC - C_1 m_I - LF1) \tag{9}$$

$$LF0 = k_1\left\{[(m_I - T_1)^2 + \sigma_0^2]\Phi\left(\frac{T_1 - m_I}{\sigma_0}\right) - \sigma_0(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right\} + \\ + k_2\left\{[(m_I - T_1)^2 + \sigma_0^2]\left[1 - \Phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right] + \sigma_0(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right\} \quad (10)$$

$$LF1 = k_1\left\{[(m_I - T_1)^2 + \sigma_0^2]\left[\Phi\left(\frac{T_1 - m_I}{\sigma_0}\right) - \Phi\left(\frac{LSL - m_I}{\sigma_0}\right)\right] \\ - \sigma_0\left[(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right) - (m_I - 2T_1 + LSL)\phi\left(\frac{LSL - m_I}{\sigma_0}\right)\right]\right\} \\ + k_2\left\{[(m_I - T_1)^2 + \sigma_0^2]\left[\Phi\left(\frac{USL - m_I}{\sigma_0}\right) - \Phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right] \\ - \sigma_0\left[-(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right) + (m_I - 2T_1 + USL)\phi\left(\frac{USL - m_I}{\sigma_0}\right)\right]\right\} \quad (11)$$

$$m_I^2 = T_1^2 + (m_0^2 - T_1^2)\exp(-\beta Inv) \quad (12)$$

$$p_1 = 1 - \Phi\left(\frac{USL - m_I}{\sigma_0}\right) + \Phi\left(\frac{LSL - m_I}{\sigma_0}\right) \quad (13)$$

$$P(X_1 \le d_0) = \sum_{x_1=0}^{d_0} \frac{e^{-n_1 p_1}(n_1 p_1)^{x_1}}{x_1!} \quad (14)$$

$ETP(Inv)$ is ETP; Inv is quality investment; m_I is improved mean; β is declined coefficient.

The Inv needs to solve with maximizing $ETP(Inv)$. Its solution procedure is

Step 1. Let maximum Inv be $Inv_{\max}$ Let initial $Inv = 0$.

Step 2. Compute the $ETP(Inv)$ from Eq. (8).

Step 3. For $Inv = Inv + 1$, repeat the 2nd Step with $Inv = Inv_{\max}$.

Step 4. The solution is that Inv with maximum $ETP(Inv)$.

3. Numerical Example

Table 1 presents the change of $\pm 20\% \sim \pm 30\%$ for the basic numerical data. From Eqs. (1)-(7), we have $Q_1 = 152$, $\sigma_0 = 0.01$, $n_1 = 18$, $d_0 = 0$ and $ETP(Q_1, \sigma_0, n_1, d_0) = -135520.6$. From Eqs. (8)-(14), we have $Inv = 42$ with $ETP(Inv) = 23434.75$.

Table 1 presents some results:

1. The setting value of variance approaches to zero, i.e., $\sigma_0 \to 0,$ in the first step
2. The M_1, O_1, I_1, S_t, h_1, m_0, and IC have a large contribution for EMQ.
3. The β has a large contribution for investment of quality.
4. The *Inv* has a large contribution for ETP.

4. Conclusions

This work addresses the EMQ one considering quality investment and variance. Numerical results show that (1) the setting value of process variance approaches to zero, i.e., $\sigma_0 \to 0,$ in the first step; (2) the *Inv* can reduce bias and increase expected total profit. Further work should include the integrated design of unknown parameters and tolerance in the model.

Table 1

Some results

C_1	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
0.8	(152, 0.01, 18, 0)	-127680.6	43	31432.48
1.2	(152, 0.01, 18, 0)	-143360.6	42	15435.65
A_{22}	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
64	(152, 0.01, 18, 0)	-148320.6	42	10634.75
96	(152, 0.01, 18, 0)	-122720.6	42	36234.75
A_{11}	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
54	(152, 0.01, 18, 0)	-135520.6	42	23434.75
81	(152, 0.01, 18, 0)	-135520.6	42	23434.75
M_1	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
640	(136, 0.01, 18, 0)	-108445.3	42	18698.23
960	(166, 0.01, 18, 0)	-162592.9	43	28176.31
Q_1	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
64	(114, 0.01, 18, 0)	-135624.7	41	23257.49
96	(341, 0.01, 18, 0)	-135352.2	46	23731.35
I_1	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
90	(202, 0.01, 18, 0)	-135443.1	44	23568.34
120	(117, 0.01, 18, 0)	-135609.3	41	23280.28

Contd...

S_t	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
16	(141, 0.01, 18, 0)	-135498.9	42	23439.29
24	(161, 0.01, 18, 0)	-135541.0	43	23426.94
h_1	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
8	(170, 0.01, 18, 0)	-135488.5	43	23490.76
12	(138, 0.01, 18, 0)	-135549.7	42	23383.24
RI	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
24.4	(152, 0.01, 18, 0)	-135520.6	42	23434.75
36.6	(152, 0.01, 18, 0)	-135520.6	42	23434.75
k_1	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
160	(152, 0.01, 18, 0)	-103517.4	41	23445.26
240	(151, 0.01, 18, 0)	-167523.8	43	23424.25
k_2	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
240	(152, 0.01, 18, 0)	-135520.6	42	23434.87
360	(152, 0.01, 18, 0)	-135520.6	42	23434.64
β	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
0.08	(152, 0.01, 18, 0)	-135520.6	52	23379.66
0.12	(152, 0.01, 18, 0)	-135520.6	36	23472.32
m_0	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
48	(122, 0.01, 18, 0)	-209637.6	48	23328.23
51	(152, 0.01, 18, 0)	-217128.6	46	23397.15
IC	$(Q_1, \sigma_0, n_1, d_0)$	ETP $(Q_1, \sigma_0, n_1, d_0)$	*Inv*	*ETP*(*Inv*)
0.4	(171, 0.01, 18, 0)	-135511.0	43	23466.16
0.6	(157, 0.01, 18, 0)	-135529.9	43	23432.53

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