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Quality investment and tolerance setting under the product inspection

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Abstract

This study considers the modified Shin et. al.'s [14] model with process control and tolerance. In the first phase, the process parameters setting model considers Taguchi's quadratic quality loss and process cost of product for obtaining the original values of mean and standard deviation. Then the second phase, i.e., the tolerance setting model considers the process promotion with quality investment. The optimal tolerance can be also determined under the 100% inspection under the specified C_{pkm} value and sampling inspection with specified C_{pkm} value and consumer's risk, respectively. The numerical result are described for explanation.

Subject Classification: *60E05 Probability distributions: general theory.*

Keywords: *Tolerance, Quality investment, Process capability index, Consumer's risk, Taguchi's quadratic quality loss.*

1. Introduction

Taguchi [15] presented the optimization quality measurement by quadratic quality loss function. This function has been applied in statistical quality control, production, and inventory management. Eleftheriou and Farmakis [3] considered the defective item with expected quality loss for selecting the specification limit coefficient and parameters of inspection plan.

The joint design of process parameters and tolerance settings is an important decision problem for product design. Previous researchers, i.e., Li and Wu [12], Jeang and Chang [10], Jeang [7, 8], Shin et. al. [14], Shen et.

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al. [13], Jeang and Lin [11], Jeang [9], and Hazrati-Marngaloo and Shahriari [5], etc., have addressed the integrated model for process and product parameters settings.

Shin et. al.'s [14] model is a two-phase process for obtaining the mean, standard deviation, and tolerance. Shin et. al.'s [14] process parameters setting model addresses quality loss and process cost. The tolerance design considers the trade-off problem for decreasing the probability of non-conforming product and increasing the manufacturing cost. Their tolerance model, the product with 100% inspection considers the expected total cost including quality loss, rejection cost, and tolerance cost.

Process capability index can be applied in measuring the probability with non-defective product. There are some applicable indices, e.g., C_{pm} and C_{pkm}. The quality investment is a tool for optimization product. Some researchers addresses this topic, e.g., Hong et. al. [6], Ganeshan et. al. [4], Chen and Tsou [2], and Tsou [16]. Chen and Chou [1] further addressed the decision for asymmetric product specifications, quality investment, and rectifying plan's parameters with quality assurance.

In this paper, the authors propose the modified Shin et. al.'s [14] process parameters and tolerance setting models with 100% and sampling inspection under the quality investment. The quality loss of non-defective and defective products are considered in the modified models. The specified C_{pkm} value is addressed for quality protection in the 100% inspection. The specified C_{pkm} value and consumer's risk are addressed for quality protection in the sampling inspection.

2. Modified Model

2.1 *Notations*

k quality loss coefficient of non-defective product

k_1 quality loss coefficient of defective product

μ unknown parameter of mean

σ unknown parameter of standard deviation

m target value of mean

C_0 constant in the process cost

C_1 coefficient of μ in the process cost

C_2 coefficient of σ in the process cost

C_3 coefficient of $\mu\sigma$ in the process cost

$E[TC_1(\mu,\ \sigma)]$	process expected total cost without inspection
$E[TC_2(Inv,\ \delta)]$	expected total cost with 100% inspection
$E[TC_2(Inv,\ \delta,\ n,\ d_0)]$	expected total cost with sampling inspection
$E[TC_2(\delta)]$	expected total cost in Shin et. al.'s [14] model
$\phi(\cdot)$	standard normal p.d.f.
$\Phi(\cdot)$	standard normal c.d.f.
C_R	rejection unit cost
a_0	constant in the manufacturing cost
a_1	coefficient of the tolerance range in the manufacturing cost
LSL	lower specification limit, $LSL = \mu_0 - \delta\sigma_0$
USL	upper specification limit, $USL = \mu_0 + \delta\sigma_0$
μ_0	known value of mean in the phase I
σ_0	known value of standard deviation in the phase I
Inv	quality investment
μ_1	improved mean through quality investment
σ_1	improved standard deviation through quality investment
C_{pkm}	process capability index
K_m	specified C_{pkm}
β_1	exponential declined coefficient for mean through quality investment
α_1	exponential declined coefficient for standard deviation through quality investment
Y	normal quality characteristic
$f(y)$	p.d.f. of Y without quality investment, $f(y) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right),\ -\infty < y < \infty$
$f(y, Inv)$	p.d.f. of Y with quality investment, $f(y, Inv) = \frac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{1}{2}\left(\frac{y-\mu_1}{\sigma_1}\right)^2\right),\ -\infty < y < \infty$

σ_T	target value of standard deviation
δ	coefficient of tolerance with symmetric range
I_c	inspection unit cost
β	customer's risk
n	sample size
X	number of non-conformance
d_0	clearance number
N	lot size
$P(X \le d_0)$	accepted probability of inspected lot, $P(X \le d_0) = 1 - P(X > d_0)$
$LTPD$	lot tolerance percent defective
$P_a(p = LTPD)$	accepted probability of inspected lot when p = $LTPD$.

2.2 *Assumptions*

1. The normal distribution is applied for describing quality.
2. The symmetric quadratic quality loss function is addressed. Both the non-defective and defective items have the quality loss.
3. The quality investment functions of mean and standard deviation consider the declining exponential reduction.
4. Consider the symmetric tolerance.
5. The specified C_{pkm} is considered as the quality protection when one adopts the 100% inspection in the quality investment and specification limits settings model. The specified C_{pkm} and consumer's risk are considered as the quality protection when one adopts the sampling inspection.

2.3 *Phase I-Process Parameters Setting*

According to Shin et. al. [14], the process parameters setting model is as follows:

One needs to solve the following model:

Minimize

$$E[TC_1(\mu,\ \sigma)] = \int_{-\infty}^{\infty} k(y-m)^2 f(y)dy + C_0 + C_1\mu + C_2\sigma + C_3\mu\sigma$$
$$= k[(\mu-m)^2 + \sigma^2] + C_0 + C_1\mu + C_2\sigma + C_3\mu\sigma \qquad (1)$$

Let the first derivative of mean and standard deviation be zero. We have the optimal settings $\mu_0 = \frac{4k^2m-2kC_1+C_2C_3}{4k^2-C_3^2}$ and $\sigma_0 = \frac{2kC_2+2kC_3m-C_1C_3}{-4k^2+C_3^2}$. The specified values of μ_0 and σ_0 are the input variables of phase II.

2.4 *Phase II--- Quality Investment Setting Models*

2.4.1 100% Inspection

The expected total cost considers quality loss for non-defective and defective products, tolerance cost, inspection cost, and quality investment. According to Shin et. al. [14], the modified tolerance setting model is as follows:

Minimize

$$\begin{aligned} &E[TC_2(Inv,\ \delta)] \\ &= \int_{LSL}^{USL} k(y-m)^2 f(y,Inv)dy + \left[\int_{-\infty}^{LSL} k_1(y-LSL)^2 f(y,Inv)dy \right. \\ &\quad \left. + \int_{USL}^{\infty} k_1(y-USL)^2 f(y,Inv)dy\right] + a_0 + 2a_1\delta\sigma_1 + I_c + Inv \\ &= k[(\mu_I - m)^2][2\Phi(\delta)-1] - k\sigma_1^2[2\delta\phi(\delta)+1-2\Phi(\delta)] \\ &\quad + k_1\left\{\{\delta^2\sigma_1^2 + \frac{\sigma_1^2}{[1-\Phi(\delta)]\sqrt{\pi}}\left[\frac{\delta}{\sqrt{2}}\exp(-0.5\delta^2) + \sqrt{\pi}(1-\Phi\left(\frac{\delta}{\sqrt{2}}\right)\right]\right\} \\ &\quad + a_0 + 2a_1\delta\sigma_1 + I_c + Inv \end{aligned} \qquad (2)$$

subject to

$$LSL = \mu_1 - \delta\sigma_1 \qquad (3)$$
$$USL = \mu_1 + \delta\sigma_1 \qquad (4)$$
$$f(y,Inv) = \frac{1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{1}{2}\left(\frac{y-\mu_1}{\sigma_1}\right)^2\right), -\infty < y < \infty \qquad (5)$$
$$\mu_1^2 = m^2 + (\mu_0^2 - m^2)\exp(-\beta_1 Inv) \qquad (6)$$
$$\sigma_1^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha_1 Inv) \qquad (7)$$
$$C_{pkm} = \frac{(\delta\sigma_1 - |\mu_1 - m|)}{3\sqrt{(\mu_1-m)^2+\sigma_1^2}} = K_m \qquad (8)$$

We need to find the combination (Inv, δ) with minimum $E[TC_2(Inv, \delta)]$ under the specified C_{pkm}. One considers the following solution procedure:

Step 1. Set maximum *Inv* be $Inv_{\max}$.

Step 2. Set initial *Inv* = 0. One can compute the corresponding δ satisfying Eq. (8) and $E[TC_2(Inv, \delta)]$ from Eq. (2).

Step 3. Set *Inv* = *Inv* +0.01 and repeat Step 2 until $Inv = Inv_{\max}$. The optimum solution is (Inv, δ) with minimum $E[TC_2(Inv, \delta)]$.

2.4.2 Sampling Inspection

All units are inspected when the lot is rejected. If the inspected unit is the defective, then it will be rejected and replaced by conforming one. The expected total cost includes quality loss for accepted lot, quality loss for rejected lot, quality loss of defective product, inspection cost, and quality investment. The specified C_{pkm} and consumer's risk are applied for assuring the shipment quality of product. According to Shin et. al. [14], the modified tolerance setting model is as follows:

Minimize

$$
\begin{aligned}
&E[TC_2(Inv,\ \delta, n, d_0)] \\
&= \{[nI_c + npC_R + (N-n)LF_0 + nLF_1]P(X \le d_0) \\
&\quad + [NI_c + NpC_R + NLF_1]P(X > d_0) + (a_0 + 2a_1\delta\sigma_1)N + Inv\} / N \\
&= \{[(N-n)LF_0 + nLF_1]P(X \le d_0) + NLF_1 P(X > d_0) \\
&\quad + NC_R\left[1 - \Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) + \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\cdot[N - (N-n)P(X \le d_0)] \\
&\quad + (a_0 + 2a_1\delta\sigma_1)N + Inv + [NI_c - I_c(N-n)P(X \le d_0)]\} / N \\
&= \left[\left(1-\frac{n}{N}\right)LF_0 + \frac{n}{N}LF_1\right]P(X \le d_0) + LF_1 \cdot P(X > d_0) \\
&\quad + C_R\left[1 - \Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) + \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\left[1 - \left(1-\frac{n}{N}\right)P(X \le d_0)\right. \\
&\quad + (a_0 + 2a_1\delta\sigma_1) + \frac{Inv}{N} + \left[I_c - I_c\left(1-\frac{n}{N}\right)P(X \le d_0)\right]
\end{aligned} \tag{9}
$$

subject to

$$LSL = \mu_1 - \delta\sigma_1 \tag{10}$$

$$USL = \mu_1 + \delta\sigma_1 \tag{11}$$

$$f(y,Inv)=\frac{1}{\sqrt{2\pi}\sigma_1}\exp\left[-\frac{1}{2}\left(\frac{y-\mu_1}{\sigma_1}\right)^2\right],\ LSL<y<USL \tag{12}$$

$$\mu_1^2=m^2+(\mu_0^2-m^2)\exp(-\beta_1 Inv) \tag{13}$$

$$\sigma_1^2=\sigma_T^2+(\sigma_0^2-\sigma_T^2)\exp(-\alpha_1 Inv) \tag{14}$$

$$p=2[1-\Phi(\delta)] \tag{15}$$

$$\begin{aligned}LF_0&=p(\int_{-\infty}^{LSL}k_1(y-LSL)^2 f(y,Inv)dy+\int_{USL}^{\infty}k_1(y-USL)^2 f(y,Inv)dy)\\&\quad+(1-p)\int_{LSL}^{USL}k(y-m)^2 f(y,\ Inv)dy\\&=pk_1\left\{\delta^2\sigma_1^2+\frac{\sigma_1^2}{[1-\Phi(\delta)]\sqrt{\pi}}\left[\frac{\delta}{\sqrt{2}}\exp(-0.5\delta^2)+\sqrt{\pi}(1-\Phi\left(\frac{\delta}{\sqrt{2})}\right)\right]\right\}\\&\quad+(1-p)k\{(\mu_1-m)^2[2\Phi(\delta)-1]-\sigma_1^2[2\delta\varphi(\delta)+1-2\Phi(\delta)]\}\end{aligned} \tag{16}$$

$$\begin{aligned}LF_1&=\frac{\int_{LSL}^{USL}k(y-m)^2 f(y,\ Inv)dy}{\int_{LSL}^{USL}f(y,\ Inv)dy}\\&=\{k[(\mu_1-m)^2][2\Phi(\delta)-1]\\&\quad-k\sigma_1^2[2\delta\phi(\delta)+1-2\Phi(\delta)]\}/[2(1-\Phi(\delta)]\end{aligned} \tag{17}$$

$$C_{pkm}=\frac{(\delta\sigma_1-|\mu_1-m|)}{3\sqrt{(\mu_1-m)^2+\sigma_1^2}}=K_m \tag{18}$$

$$P_a(p=LTPD)=\beta \tag{19}$$

We need to find $(Inv,\ \delta,\ n,\ d_0)$ with minimum $E[TC_2(Inv,\delta,\ n,\ d_0)]$ under the specified C_{pkm} and consumer's risk. One can adopt the following solution procedure:

Step 1. Set maximum d_0 be $d_{\max}$ and maximum *Inv* be $Inv_{\max}$. Set initial $Inv=0$ and $d_0=0$.

Step 2. For known *LTPD*, β, and d_0, we have corresponding *n* satisfying Eq. (19).

Step 3. We can compute the corresponding δ satisfying Eq. (18) and $E[TC_2(Inv,\delta,\ n,\ d_0)]$.

Step 4. Set $Inv = Inv + 1$ and repeat Step 3 until $Inv=Inv_{\max}$.

Step 5. Set $d_0=d_0+1$ and repeat Step 2 until $d_0=d_{\max}$. The optimum solution is $(Inv,\ \delta,\ n,\ d_0)$ with minimum $E[TC_2(Inv,\delta,\ n,\ d_0)]$.

3. Numerical Examples and Sensitivity Analysis

3.1 *Example 1*

Consider the following parameters: $k = 100$, $T = 50$, $C_0 = 100$, $C_1 = 6.1$, $C_2 = -3.1$, $C_3 = -3.85$, $a_0 = 100$, $a_1 = -0.2$, $k_1 = 10$, $\alpha_1 = 0.5$, $\beta_1 = 0.1$, $\sigma_T = 0.5$, and $K_m = 1$.

The optimal solution of Eq. (1) is $\mu_0 = 49.988$ and $\sigma_0 = 0.9778$ with $E[TC_1(\mu, \sigma)] = 309.338$. By solving Eqs. (2)-(8), the optimal solution for 100% inspection is $\mu_I = 49.996$, $\sigma_I = 0.505$, $Inv = 10.05$, $\delta = 3.01$, $LSL = 47.047$, $USL = 52.930$, and $E[TC_2(Inv, \delta)] = 215.635$.

Table 1 presents some sensitivity results. We have some conclusions:

1. The parameters, C_3, k, α_1, and K_m, have a significant effect on the quality investment.
2. The parameter, K_m, has a significant effect on the coefficient of tolerance.
3. The parameters, k, σ_T, and K_m, have a significant effect on the expected total cost.

3.2 *Example 2*

Consider the following parameters: $k = 100$, $T = 50$, $C_0 = 100$, $C_1 = 6.1$, $C_2 = -3.1$, $C_3 = -3.85$, $a_0 = 100$, $a_1 = -0.2$, $C_R = 100$, $\alpha_1 = 0.5$, $k_1 = 10$, $\beta_1 = 0.1$, $\sigma_T = 0.5$, $K_m = 1$, $LTPD = 5\%$, $\beta = 10\%$, $I_c = 1$, and $N = 500$. The optimal solution of Eq. (1) is $\mu_0 = 49.988$ and $\sigma_0 = 0.9778$ with $E[TC_1(\mu, \sigma)] = 309.338$.

By solving Eqs. (9)-(19), the optimal solution for sampling inspection is $\mu_I = 49.988$, $\sigma_I = 0.500$, $Inv = 19.88$, $\delta = 3.00$, $LSL = 47.052$, $USL = 52.925$, $d_0 = 1$, $n = 78$, and $E[TC_2(Inv, \delta, d_0, n)] = 124.067$.

Table 2 presents some sensitivity results. We have some conclusions:

1. The parameters, C_3 and α_1, have a significant effect on the quality investment.
2. The parameter, K_m, has a significant effect on the coefficient of tolerance.
3. The parameters, k and σ_T, have a significant effect on the expected total cost.

4. The parameters, K_m, LTP), and β, have a significant effect on the parameters of inspection plan.

3.3 *Discussions*

The phase II of the original Shin et. al.'s [14] model only considers the tolerance setting. According to Shin et. al. [14], the optimal tolerance can be obtained by solving the following Eq. (20).

$$
\begin{aligned}
&E[TC_2(\delta)] \\
&= \int_{LSL}^{USL} k(y-m)^2 f(y)dy + C_R[\int_{-\infty}^{LSL} f(y)dy + \int_{USL}^{\infty} f(y)dy] + a_0 + a_1(USL-LSL) \\
&= k[(\mu_0 - m)^2 + \sigma_0^2][2\Phi(\delta)-1] - 2k\sigma_0^2\delta\phi(\delta) + a_0 + 2a_1\delta\sigma_0 \qquad (20)
\end{aligned}
$$

Similar to numerical examples 1 and 2, Eq. (20) has the optimal $\delta = 0.323$ with the corresponding LSL = 49.673, USL = 50.304, and $E[TC_2(\ \delta)] = 108.300$. The original Shin et. al.'s [14] model shows that it has smaller expected total cost than those of examples 1 and 2. That reason is because the original Shin et. al.'s [14] model only considers the linear cost of non-conforming unit. They don't consider both quadratic quality loss of defective unit and specified C_{pkm} for the quality protection. Hence, they underestimate the expected total cost. The modified model addresses the infinite cost for defective unit and quality protection under the high process capability index. Thus, the modified model can obtain customer's confidence.

4. Conclusions

The authors have presented the modified Shin et. al.'s [14] model with quality investment, specified C_{pkm}, and consumer's risk for quality protection in the tolerance setting model. Numerical results show that quality loss coefficient for the non-defective unit, exponential declined coefficient for the standard deviation through quality investment, target value of standard deviation, and specified C_{pkm} are the significant effect on the solution. Further study should consider the integrated model applied in the system with supplier, manufacturer, retailer, and customer.

Table 1

The sensitivity results for numerical example 1

C_0	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
120	49.998	0.978	49.996	0.505	10.05	3.01	215.635
80	49.998	0.978	49.996	0.505	10.05	3.01	215.635
C_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
7.32	49.982	0.978	49.994	0.504	10.13	3.01	216.007
4.88	49.994	0.978	49.998	0.505	9.98	3.00	215.268
C_2	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
-2.48	49.988	0.975	49.996	0.504	10.11	3.01	215.622
-3.72	49.988	0.981	49.996	0.504	10.12	3.01	215.647
C_3	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
-3.08	49.982	0.785	49.992	0.504	8.88	3.02	214.895
-4.62	49.997	1.170	49.999	0.505	10.88	3.00	216.045
k	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
120	49.988	0.815	49.995	0.504	9.14	3.01	219.634
80	49.991	1.222	49.997	0.505	11.05	3.01	211.583
T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
51	50.989	0.997	50.996	0.505	10.17	3.01	215.711
49	48.988	0.959	48.996	0.505	9.97	3.01	215.557
k_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
12	49.988	0.978	49.996	0.505	10.05	3.01	215.635
8	49.988	0.978	49.996	0.505	10.05	3.01	215.635
α	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
0.6	49.988	0.978	49.995	0.504	8.79	3.01	214.050
0.4	49.988	0.978	49.997	0.506	12.10	3.01	217.940
β	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
0.12	49.988	0.978	49.997	0.504	10.13	3.01	215.504
0.08	49.988	0.978	49.995	0.504	10.12	3.01	215.795
σ_T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
0.60	49.988	0.978	49.996	0.604	9.78	3.01	260.479
0.40	49.988	0.978	49.996	0.406	10.36	3.01	178.915
a_0	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
120	49.988	0.978	49.996	0.505	10.05	3.01	235.635
80	49.988	0.978	49.996	0.505	10.05	3.01	195.635

Contd...

a_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
-0.16	49.988	0.978	49.996	0.505	10.05	3.01	215.757
-0.24	49.988	0.978	49.996	0.505	10.05	3.01	215.514
K_m	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
1.2	49.988	0.978	49.996	0.502	11.51	3.61	291.650
0.8	49.988	0.978	49.995	0.507	9.17	2.41	178.381
I_c	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv,\delta)]$
1.2	49.988	0.978	49.996	0.505	10.05	3.01	215.835
0.8	49.988	0.978	49.996	0.505	10.05	3.01	215.435

Table 2

The sensitivity results for numerical example 2

C_0	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
120	49.988	0.978	49.998	0.500	19.88	3.00	124.067	(1, 78)
80	49.988	0.978	49.998	0.500	19.88	3.00	124.067	(1, 78)
C_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
7.32	49.982	0.978	49.998	0.500	20.14	3.00	124.069	(1, 78)
4.88	49.994	0.978	49.999	0.500	19.76	3.00	124.064	(1, 78)
C_2	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
-2.48	49.988	0.975	49.998	0.500	19.96	3.00	124.067	(1, 78)
-3.72	49.988	0.981	49.998	0.500	19.83	3.00	124.067	(1, 78)
C_3	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
-3.08	49.982	0.785	49.997	0.500	19.24	3.01	124.068	(1, 78)
-4.62	49.997	1.170	50.000	0.500	20.58	3.00	124.065	(1, 78)
k	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
120	49.988	0.815	49.998	0.500	19.46	3.01	128.912	(1, 78)
80	49.991	1.222	49.998	0.500	20.24	3.00	119.221	(1, 78)
T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)

Contd...

51	50.989	0.997	50.998	0.500	20.00	3.00	124.067	(1, 78)
49	48.988	0.959	48.998	0.500	19.93	3.00	124.067	(1, 78)
k_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
12	49.988	0.978	49.998	0.500	19.88	3.00	124.101	(1, 78)
8	49.988	0.978	49.998	0.500	19.88	3.00	124.032	(1, 78)
α	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
0.6	49.988	0.978	49.998	0.500	17.18	3.00	124.062	(1, 78)
0.4	49.988	0.978	49.999	0.500	24.06	3.00	124.075	(1, 78)
β	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
0.12	49.988	0.978	49.999	0.500	19.92	3.00	124.065	(1, 78)
0.08	49.988	0.978	49.998	0.500	19.91	3.00	124.069	(1, 78)
σ_T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
0.60	49.988	0.978	49.998	0.60	19.68	3.00	134.683	(1, 78)
0.40	49.988	0.978	49.998	0.40	20.03	3.00	115.402	(1, 78)
a_0	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
120	49.988	0.978	49.998	0.500	19.88	3.00	144.067	(1, 78)
80	49.988	0.978	49.998	0.500	19.88	3.00	104.067	(1, 78)
a_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
-0.16	49.988	0.978	49.998	0.500	19.88	3.00	124.187	(1, 78)
-0.24	49.988	0.978	49.998	0.500	19.90	3.00	123.946	(1, 78)
K_m	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
1.2	49.988	0.978	49.998	0.500	19.54	3.60	124.351	(0, 46)
0.8	49.988	0.978	49.998	0.500	19.91	2.40	122.896	(3, 134)
I_c	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
1.2	49.988	0.978	49.998	0.500	19.88	3.00	124.101	(1, 78)
0.8	49.988	0.978	49.998	0.500	19.88	3.00	124.032	(1, 78)

Contd...

LTPD	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
0.06	49.988	0.978	49.998	0.500	19.91	2.40	122.846	(1, 65)
0.04	49.988	0.978	49.998	0.500	19.33	2.40	123.351	(1, 97)
β	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
0.05	49.988	0.978	49.998	0.500	19.33	2.40	123.323	(1, 95)
0.01	49.988	0.978	49.998	0.500	19.21	2.40	123.776	(1, 133)
N	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, n, d_0)]$	(d_0, n)
600	49.988	0.978	49.998	0.500	20.32	3.00	124.029	(1, 78)
400	49.988	0.978	49.998	0.500	19.47	3.00	124.122	(1, 78)

References

[1] C. H. Chen and C. Y. Chou, "The asymmetric specification limit and quality investment settings for rectifying inspection plan," *Journal of Information and Optimization Sciences*, vol. 43, no. 2, pp. 323-330 (2022).

[2] J. M. Chen and J. C. Tsou, "An optimal design for process quality improvement: modeling and application," *Production Planning & Control*, vol. 14, pp. 603-612 (2003).

[3] M. Eleftheriou and N. Farmakis, "Expected cost for continuous sampling plans," *Communications in Statistics—Theory and Methods*, vol. 40, pp. 2969-2984 (2011).

[4] R. Ganeshan, S. Kulkarni, and T. Boone, "Production economics and process quality: a Taguchi perspective," *International Journal of Production Economics*, vol. 71, pp. 343-350 (2001).

[5] H. Hazrati-Marangaloo and H. Shahriari, "A novel approach to simultaneous robust design of product parameters and tolerances using quality loss and multivariate ANOVA concepts," *Quality and Reliability Engineering International*, vol. 33, pp. 71-85 (2017).

[6] J. D. Hong, S. H. Xu, and J. C. Hayya, "Process quality improvement and setup reduction in dynamic lot-sizing," *International Journal of Production Research* vol. 31, pp. 2693-2708 (1993).

[7] A. Jeang, "Optimal parameter and tolerance design with a complete inspection plan," *International Journal of Advanced Manufacturing Technology*, vol. 20, pp. 121-127 (2002).

[8] A. Jeang, "Robust computer-aided parameter and tolerance determination for an electronic circuit design," *International Journal of Production Research*, vol. 41, pp. 883-895 (2003).

[9] A. Jeang, "Concurrent product and process parameters determination under deterioration process," *Applied Mathematics and Computation*, vol. 219, pp. 9132-9141 (2013).

[10] A. Jeang and C. L. Chang, "Combined robust parameter and tolerance design using orthogonal arrays," *International Journal of Advanced Manufacturing Technology*, vol. 19, pp. 442-447 (2002).

[11] A. Jeang and Y. Lin, "Product and process parameters determination for quality and cost," *International Journal of Systems Science*, vol. 45, pp. 2042-2054 (2012).

[12] W. Li and C. J. Wu, "An integrated method of parameter design and tolerance design," *Quality Engineering*, vol. 11, pp. 417-425 (1999).

[13] L. Shen, J. Yang, and Y. Zhao, "Simultaneous optimization of robust parameter and tolerance design based on generalized linear models," *Quality and Reliability Engineering International*, vol. 98, pp. 1107-1115 (2012).

[14] S. Shin, P. Kongsuwon, and B. R. Cho, "Development of the parametric tolerance modeling and optimization schemes and cost-effective solutions," *European Journal of Operational Research*, vol. 207, pp. 1728-1741 (2010).

[15] G. Taguchi, *Introduction to Quality Engineering*. Tokyo, Japan: Asian Productivity Organization (1986).

[16] J. C. Tsou, "Decision-making on quality investment in a dynamic lot sizing production system," *International Journal of Systems Science*, vol. 37, pp. 463-475 (2006).

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