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A review on control charts based on different sampling schemes

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Abstract

This paper offers a critical assessment of the book review authored by Woodall [1]. Woodall [1] conducted a review of the book titled "Introduction to Statistical Process Control." His critique primarily focused on the literature that had been published and the sampling schemes employed for designing control charts, all without delving into a comprehensive analysis of the underlying assumptions, the operational processes involved, or the benefits associated with these sampling schemes. Woodall et al. [2] demonstrated the equivalence between control charts employing multiple dependent-state sampling and control charts utilizing runs rules, disregarding the underlying assumptions of both methods. It's important to note that this equivalence does not necessarily imply that the control chart using runs

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rules is more efficient than the run-rule control charts employing multiple dependent-state sampling. Additionally, Saleh et al. [3] observed that when dealing with count data, the control chart using repetitive sampling outperforms the Shewhart control chart. However, they did not provide evidence of the Shewhart control chart's efficiency compared to the control chart using repetitive sampling when both are configured with identical control chart parameters. It should be noted that although there are some typographical errors present in the book, and there is room for additional topics to be incorporated, these issues are expected to be addressed in the upcoming second edition of the book.

Subject Classification: *97K80.*

Keywords: *Control chart, Sampling plans, Average run length.*

1. Introduction

Research is a continuous process of improvements and updating. The criticism for the improvement is always welcome by the researchers. The author beliefs a positive and healthy criticism open the door for the improvement of scientific content. On the other hand, criticism without deep knowledge may discourage the researchers. Aslam, Saghir, and Ahmad [4] wrote a book "introduction to statistical process control" and introduced some sampling schemes in designing the control chart. Their main aim was to introduce control charts that are more efficient than Shewhart control charts. They supplied chapters discussing the design of control charts through repetitive sampling and multiple dependent state sampling. Woodall [1] published a review on the book by Aslam, Saghir, and Ahmad [4] where among other objections, he argued that the control chart using the sequential probability ratio test (SPRT) is efficient proposed by Reynolds and Stoumbos [5] and Stoumbos, and Reynolds Jr [6]. The contents of the book are mainly based on the articles published in well reputed journals. Although some typo errors exist but this is not a sufficient justification to not recommend this book.

2. Repetitive Sampling vs. SPRT

Presently, we conduct a comparative analysis between the repetitive sampling scheme and SPRT. In practical scenarios, decisions regarding the state of the process are typically based on information gathered from the initial sample. There is less chance of in-decision on the basis of the first sample information. In the case of in-decision, the process is repeated until the decision is made. The SPRT was found to be efficient in terms of average sample number (ASN). According to Sherman [7] and Schilling, and Neubauer [8], SPRT is somewhat harder to administer than a repetitive sampling scheme. Additional details can be found in Maji, Singh, and Bandyopadhyay [9] and Khan et al. [10]. A control chart using repetitive sampling scheme can be applied easily like the control chart using a single sampling scheme. In nutshell, although, SPRT provides the smaller ASN as compared to repetitive sampling, but control charts using repetitive sampling are easy in implication. The following Table 1 presents the comparisons between both sampling schemes.

Referring to Table 1, it can be deduced that the operational process of SPRT control charts closely resembles that of control charts using repetitive sampling. Furthermore, it is evident that implementing the SPRT control chart is a complex task, demanding more computational effort to reach a process decision. It is well-established that when employing identical control chart parameter values, the control chart utilizing repetitive sampling consistently surpasses the Shewhart control chart in terms of average run length, as indicated in Aslam, Azam, and Jun [11]. Saleh et al. [3] presented arguments that were deemed illogical in their attempt to demonstrate the efficiency of the Shewhart control chart over the control chart using repetitive sampling. According to Saleh et al. [3], the efficiency of Shewhart control charts could be improved by increasing the sample size, despite the industry's prevailing efforts to minimize sample size to save costs and time. Furthermore, Saleh et al. [3] noted that when dealing with count data, repetitive sampling outperforms the Shewhart control chart. Contrary to this, Muñoz et al. [16] presented evidence demonstrating that the repetitive sampling plan is more effective in terms of average run length (ARL) than the Shewhart control chart. They illustrated that this efficiency is achieved when determining control limits coefficients for an in-control process by equating the average sample number (ASN) to the sample size. Khan et al. [18] proved the efficiency of the control chart using repetitive sampling over the EWMA control in terms of conditional expected delays. In summary, the control chart employing repetitive sampling is both straightforward to implement and more efficient than Shewhart control charts. The suggestion to enhance the efficiency of the control chart using single sampling by increasing the sample size seems to lack sound reasoning and practicality.

3. Multiple Dependent State Sampling vs. Run Rule Chart

Woodall [1] pointed out that multiple dependent state sampling (MDSS) is the same as the run rule and referred to the work of [2]. Woodall

Table 1

Comparisons between three sampling schemes

Sampling schemes	Sample selection	Operational Process	Efficiencies
Single sampling	In single sampling a single sample is taken to make decision.	A single sample is selected and information is collected.	While less efficient compared to repetitive and sequential sampling methods, simple and straightforward to implement, these charts are not suitable for use in indecisive situations.
Repetitive Sampling	According to Aslam, Azam, and Jun [11] Step 1: Take a sample of size n. Plot the number of non-conforming items (denoted by D). Step2: Declare the process as out-of-control if $D \geq UCL_1$ or $D \leq LCL_1$ (UCL_1 and LCL_1 are called the outer control limits). Declare the process as in-control if $LCL_2 \leq D \leq UCL_2$ (UCL_2 and LCL_2 are called the inner control limits). Otherwise, go to Step1 and repeat the process.	A sample of size n is initially chosen, and in the event of indecision, the process is iterated until a conclusive decision is reached.	Outperforming Shewhart control charts in terms of average run length efficiency and presenting a simpler implementation compared to SPRT charts.
SPRT	According to Reynolds and Stoumbos [5] "SPRT chart can be applied in situations in which items are inspected one by one, and the results of each inspection can be conveniently recorded before the next item is inspected"	Stoumbos, and Reynolds Jr [12] pointed out that "As stated by Stoumbos and Reynolds [8], the SPRT charts are particularly effective in the situations, namely, the items can be inspected one by one and the time required to inspect	While more effective than repetitive sampling, this approach demands additional time, effort, resources, and energy.

Contd...

		a single item is so short that it can be neglected, the testing is expensive and/or destructive, and it is expected to detect a shift of any size as effectively as possible”.	

[1] also referred the papers by Page [13] and Champ, and Woodall [14]. Based on the literature review, we present the difference between the control chart with run rules and MDSS.

i. *Operational process of the control chart with supplementary run rules*

As per Page [13], the operational process of the control chart with run rules, based on the actual limits and two warning limits, is stated as follows:

Rule 1. “Choose k, n and N. Take samples of size N. Take action if any point falls outside the action lines or if any k out of the last n points fall outside the warning lines”

Rule 2. “Take action after the nth sample if any of the inequalities”

Rule 3. “Choose n, N. Take samples of size N. Take action if two points out of any sequence of n fall between the warning and action lines or if any point falls outside the action lines”

Rule 4. “Choose n, N. Take samples of size N. Take action if n consecutive points fall between the warning and action lines or if any point falls outside the action lines”

Rule 5. “Choose n, N. Plot the means of samples of size N on a chart on which are drawn two warning and two action lines. Take action if: (i) any point falls outside the action lines, (ii) n consecutive points fall outside the warning lines, (iii) two out of any set of n consecutive points fall outside opposite warning lines”

Champ, and Woodall [14] mentioned some runs role and one of them is stated as follows

“Rule 3 signals if four of the last five standardized sample means are between -3 and -1 or if four of the last five are between 1 and 3”.

ii. *Operational process of control chart using MDSS*

The operational process of control chart using MDSS from [15] is stated as follows

"**Step-0**: Establish the outer control limits of UCL_1 and LCL_1 as well as the inner control limits of UCL_2 and LCL_2 using the data for the in-control process.

Step-1: Select a random sample of size n from the production process at each subgroup and count the number of defectives D.

Step-2: Declare the process as in-control if $LCL_2 \le D \le UCL_2$. Declare the process to be out-of-control if $D \ge UCL_1$ or $D \le LCL_1$. Otherwise, go to Step-3

Step-3: Declare the process is in control if I proceeding subgroups declared the process as in-control. Otherwise, declare the process to be out-of-control".

It is clear that the operational process of the MDSS chart is not different from run rule chart but they can be different based on the applicability, assumption, efficiency and extensions. If one subgroup falls within warning limits, the MDSS approach suggests looking at m preceding subgroups and if they fall within inner limits then the subgroup will be accepted. If p_0 denotes the probability of falling a subgroup in the inner limits, the probability of acceptance in the MDSS approach is p_0^m in this case. On the other hand, in 2 out of m+1 run rule approach, if one subgroup falls within the warning limit, the probability of acceptance in run rule is still p_0^m. Thus, these two methods are theoretically the same for one subgroup case. When $m+1$ subgroups are given in practical situations, the probability of accepting the process in MDSS is equal to $p_0 + p_1 p_0^m$ based on the situation of the last subgroup (p_1 denotes the probability of falling the last subgroup in warning limits) and the probability of accepting the process in 2 out of $m+1$ run rule is equal to (1-rejection probability) that is equal to $1-\sum_{j=2}^{m+1}\binom{j-1}{1}p_1^2 p_0^{j-2} = 1-(p_1^2+2p_1^2p_0^1 + 3p_1^2p_0^2 + \ldots + (m)p_1^2p_0^{m-1})$ (assuming that the number of inspected subgroups follows negative binomial distribution because when two subgroups fall within warning limit then we classify the process as out of control). This means that these two methods are different in decision making based on m initial subgroups because in MDSS method, the decision-making starts after gathering at least m subgroups but run rule method can be applied from the start of the process. For example at any point of restarting and adjusting the process, we can apply run rule method for decision making that this is not possible for MDSS because of the need for inspecting at least m subgroups. The original form of MDSS and run rule will be the same but the MDSS method can be extended in different direction while these extensions are not possible in the run rule method. If current subgroup falls within

Table 2

Modified MDSS with m=3

number	subgroup	Run Rule	MDSS	MDSS with tighter intervals	Modified MDSS
1	Warning limit	In-control	In-control	Warning limit (IC in MDSS)	In-control
2	Warning limit	Out-of-control	Out-of-control	Warning limit (OOC in MDSS)	In-control (First OOC subgroup)
3	Inner limit	In-control	In-control	Inner limit (IC in MDSS)	In-control
4	Warning limit	Out-of-control	Out-of-control	Warning limit (OOC in MDSS)	Out-of-control (Second OOC subgroup)

warning limit then we can employ different policies for decision making about the process condition based on m preceding subgroups denoting the different applicability of these two methods. For example, assume modified MDSS method is designed as following, if m preceding subgroups have been in-control based on MDSS strategy with tighter limits then we can assume that current subgroup is in-control. Table 2 denotes the performance of modified MDSS method and it is obvious that the results will be different. As can be seen in Table 2, both run rule and MDSS method have the same result and classify the process as out-of-control at the second and forth subgroup. Also since MDSS with tighter limits classifies the process as out-of-control at the second and forth subgroup thus modified MDSS classifies the process as out-of-control at the forth subgroup because at the forth subgroup, we reach to two out-of-control subgroups based on applying MDSS policy for each subgroup.

Woodall et al. [2] put forth the argument that MDSS is equivalent to the control chart run rule if there are multiple subgroups in the indecision area. It is essential to consider that MDSS operates under the assumption that there is a relatively low chance of having multiple subgroups in the indecision region. Consequently, the occurrence of two or more subgroups within the indecision region would violate the fundamental assumption of MDSS. The research by Woodall et al. [2] indicates that the ARL values of MDSS are lower compared to the control chart using run rules. In contrast, recent work by Muñoz et al. [17] demonstrated that the control chart using MDSS outperforms the control chart using run rules in terms of ARL. Their findings provide compelling evidence for the superior efficiency of the control chart using MDSS in detecting process mean shifts. In summary, the equivalence claimed between MDSS and the run rule by [2] is contingent on specific conditions that may not hold in practice, making their assertion less applicable. The study by Muñoz et al. [17] highlights the enhanced efficiency of the control chart using MDSS in comparison to the control chart using run rules, supporting the use of MDSS for improved process monitoring. In addtion, Woodall et al. [2] presented the incorrect simulation procedure using the generalized multiple dependent state sampling (GMDS). Similar to MDS, GMDS has demonstrated greater efficiency compared to the run-rule chart.

4. Concluding Remarks

In this paper, we critiqued Woodall's [1] book review, which evaluated the book "Introduction to Statistical Process Control" by Aslam, Saghir, and Ahmad [4]. While Woodall's [1] raised concerns about the book's content, including the sampling schemes used in designing control charts, the review lacked a comprehensive analysis of the assumptions, operational processes, and benefits of these sampling schemes. We acknowledged the presence of some typographical errors in the book, but emphasized that these issues could be rectified in the second edition. Furthermore, we compared the repetitive sampling scheme with the sequential probability ratio test (SPRT), highlighting that SPRT exhibits greater efficiency in terms of average sample number (ASN). However, we noted that control charts using repetitive sampling are easier to apply and more user-friendly, making them a practical choice for process monitoring. Additionally, we discussed the multiple dependent state sampling (MDSS) and its comparison to the run rule chart. While Woodall's [1] claimed the equivalence of MDSS and the run rule but the design of run-rule control chart using MDSS will be more efficient than the exiting run-rule control charts. We supported our argument with evidence from recent research by Muñoz et al. [17] Muñoz et al. [17], which demonstrated the superior efficiency of the control chart using MDSS in detecting process mean shifts. In conclusion, we provided a balanced assessment of the book review by Woodall's [1] and presented valid points regarding the efficiency and applicability of different control charting techniques. We encourage further research and constructive criticism to enhance the field of statistical process control and promote continuous improvement in quality control processes.

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