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Testing lifetime of the product using new distribution in group acceptance sampling plan

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Abstract

In this article, we investigate three modern probability models for lifetime random variables: elicitation inverse Rayleigh (EIR), generalized inverse weibull (GIW) and Weighted generalized inverse Weibull (WGIW). A group of acceptance sampling plans for EIR, GIW and WGIW distributions are developed. This is when the life test is truncated at a predetermined time. When the acceptance limit and group size are specified, as well as the consumer's risk and test termination time, the minimal number of groups required to claim that the median life exceeds a particular value α_0 with the consumer's confidence level is obtained. In addition, the probabilities of acceptance values of the provided plan for the specified three modern distributions are presented. Also obtain the minimal ratios of the veritable median life α to the required median life α_0 for the lot acceptance with producer's risk of 0.05. Some

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comparisons are made between the results for the ordinary sampling and group sampling plans.

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1. Introduction

In the prevalent sampling plan, only one item is inspected. However, when time and cost need to be saved; multiple items can be inspected simultaneously. In such cases, we deal with what is known as group acceptance sampling plan. This type of sampling plans is proposed by many authors, such as, [1], [2], [8], [11], [14], [15], [20], [22]. Sometimes, a group acceptance sampling plan is preferred to the prevalent sampling plans because it can test a larger number g items in a given test time. For example, this type is widely used in sudden-death tests. In the group sampling plan of action, the number of products to be inspected and the number of groups are determined. The parameters here are the determined acceptance limit and the group size. In accordance with [7], in a group acceptance sampling plan, the selected sample size n is divided into x (number of groups) groups, and a (group size) items are tested in each group, resulting in $n = a.x$. The x items in a group are tested concurrently on each different tester for a predetermined amount of time. If more than the acceptance limit c of failures occurs in any group during the experiment time, the experiment is terminated. The group acceptance sampling plan based on truncated life test under various lifetime distributions are studied by several authors such as, [3, 4], [5], [6], [10], [11], [17-19], considered Burr type X distribution in designing acceptance sampling plan. [21] proposed repetitive sampling using generalized inverted exponential distribution. More details an sampling plans can be seen in [12-13].

There are also few research on elicitation inverse Rayleigh$\left(EIR(\alpha,\beta)\right)$, generalized inverse Weibull $\left(GIW(\alpha,\beta,\gamma)\right)$ and Weighted generalized inverse Weibull $\left(WGIW(\alpha,\beta,\gamma,\lambda)\right)$ in group acceptance sampling plan based on truncated life test. Thus, the purpose of this article is to design a group sampling plan based on truncated life tests when the lifetime of an item follows GIW, SGIW or CIR distributions.

The remainder of the article is structured as follows. Section 2 contains definitions for some modern probability distributions based on life tests. The group acceptance sampling plan is designed in Section 3. Section 4 develops operating characteristic functions and the producer's risk. The minimal ratios of actual median life to specified median life are given in section 5. The numerical case has been discussed in section 6.

2. Life Test–Based Modern Distributions

Let $\mathbb{Z}$ be a random variable distributed according to elicitation inverse Rayleigh distribution with α and β density functions (PDF) as follows

$$f_{\mathbb{Z}}(z) = \frac{2\beta\alpha^{\beta}}{z^3(z^{-2}+\alpha)^{\beta+1}};\ z > 0;\ \alpha, \beta \in \Re^{+}.$$

Where , β are scale and shape parameters respectively. The distribution and the median functions of $\mathbb{Z}|\{\alpha, \beta\}$ are respectively as

$$F_{\mathbb{Z}}(z) = \left(1 + \frac{z^{-2}}{\alpha}\right)^{-\beta},\ z > 0;\ \alpha, \beta \in \Re^{+}. \quad (1)$$

and

$$\mu_{\mathbb{Z}|\{\alpha,\beta\}}(z) = \left\{\alpha\left[(0.5)^{-\frac{1}{\beta}} - 1\right]\right\}. \quad (2)$$

For the EIR distribution, the scale parameter $\alpha_{\mathbb{Z}|\{\alpha,\beta\}}(z)$ from equation (2) can be written as

$$\alpha_{\mathbb{Z}|\{\alpha,\beta\}}(z) = \frac{\mu_{\mathbb{Z}|\{\alpha,\beta\}}(z)}{\left[(0.5)^{-\frac{1}{\beta}}-1\right]}. \quad (3)$$

For more properties of $EIR(\alpha, \beta)$ distribution see [14].

Let $\mathbb{Z}$ be a random variable distributed according to a generalized inverse Weibull distribution with parameters α, γ and β density function (PDF) as follows

$$f_{\mathbb{Z}|\{\alpha,\gamma,\beta\}}(z) = \alpha^{\beta}\gamma\beta z^{-\beta-1}\exp\left[-\gamma\left(\frac{\alpha}{z}\right)^{\beta}\right],\ z > 0;\ \alpha, \gamma, \beta \in \Re^{+}$$

Where γ and β are a shape parameters, α is a scale parameter. The distribution and the median functions of $\mathbb{Z}|\{\alpha, \gamma, \beta\}$ are respectively as

$$F_{\mathbb{Z}|\{\alpha,\gamma,\beta\}}(z) = \exp\left[-\gamma\left(\frac{\alpha}{z}\right)^{\beta}\right],\ z > 0; \quad (4)$$

and

$$\mu_{\mathbb{Z}|\{\alpha,\gamma,\beta\}}(z) = \left[\frac{\ln 2}{(\gamma\alpha^{\beta})}\right]^{-\frac{1}{\beta}}. \quad (5)$$

For more details about properties of GIW distribution see de [9].

Let $\mathbb{Z}$ be a random variable distributed according to Weighted generalized inverse Weibull distribution with parameters α, γ, β and λ density function (PDF) as follows

$$f_{\mathbb{Z}^{w}|\{\alpha,\gamma,\beta,\lambda\}}(z) = \alpha^{\beta}(1+\lambda^{-\beta})\gamma\beta z^{-\beta-1}\exp\left[-\gamma\left(\frac{\alpha}{z}\right)^{\beta}(1+\lambda^{-\beta})\right];\ \text{for } z > 0,$$

Where γ, β and λ are shape parameters and α is a scale parameter. The distribution and the median functions of $\mathbb{Z}^{w}|\{\alpha, \gamma, \beta, \lambda\}$ are respectively as

$$F_{\mathbb{Z}^{w}|\{\alpha,\gamma,\beta,\lambda\}}(z) = \exp\left[-\gamma\left(\frac{\alpha}{z}\right)^{\beta}(1+\lambda^{-\beta})\right];\ z > 0;\ \alpha, \gamma, \beta, \lambda \in \Re^{+} \quad (6)$$

and

$$\mu_{\mathbb{Z}^{w}|\{\alpha,\gamma,\beta,\lambda\}}(z) = \left[\frac{\ln 2}{\left(\gamma\alpha^{\beta}(1+\lambda^{-\beta})\right)}\right]^{-\frac{1}{\beta}}. \tag{7}$$

For more details about the WGIW distribution see [16]. The scale parameter $\alpha_{\mathbb{Z}|\{\alpha,\gamma,\beta,\lambda\}}(z)$ of WGIW distribution can be obtained from the equation (7) as follows

$$\alpha_{\mathbb{Z}^{w}|\{\alpha,\gamma,\beta,\lambda\}}(z) = \frac{\mu_{\mathbb{Z}^{w}|\{\alpha,\gamma,\beta,\lambda\}}(z)\left(\gamma(1+\lambda^{-\beta})\right)^{-\frac{1}{\beta}}}{(\ln 2)^{-\frac{1}{\beta}}} \tag{8}$$

The equations (1) and (6) would be appropriate for determining the experiment's termination time z_0 as a multiple of the stated median life α_0 that $z_0 = \Lambda\alpha_0$ where Λ is a constant (termination time ratio) and α_0 is the specified median life. It is worth noting that the failure probability is independent of the scale parameter and only depends on the shape parameters when transformation $z_0 = \Lambda\alpha_0$. As a result, equations (1) and (6) can be recast as functions of Λ and the ratio $\frac{\alpha}{\alpha_0}$ as follows

$$\Pi_{EIR} = \left\{1 + \left(\frac{\alpha}{\Lambda\alpha_0}\right)^2 \left[(0.5)^{-\frac{1}{\beta_0}} - 1\right]\right\}^{-\beta_0}. \tag{9}$$

And

$$\Pi_{WGIW} = \exp\left[-\left(\frac{\alpha}{\Lambda\alpha_0}\right)^{\beta_0} \ln 2\right]. \tag{10}$$

Where $\frac{\alpha}{\alpha_0}$ is a ratio of veritable median life to the stated median life for the EIR and WGIW distributions. The probability that an item will not work out before the termination time z_0 for the EIR and WGIW distributions in equations (9) and (10) has been acquired using the hypothesis $z_0 = \Lambda.\alpha_0$ and the equations (3) and (7) for the scale parameter of the EIR and WGIW distributions. It's worth noting that the probability of failure for WGIW (the study's emphasis) and GIW takes the same mathematical formula at the hypothesis $z_0 = \Lambda\alpha_0$; resulting in the identical result for both distributions.

3. Design of Group Acceptance Sampling Plan

Consider to be the veritable median α and α_0 is the stated median life of a product, assumption that the product's lifetime follows the EIR, GIW, or WGIW distributions. The lot is approved if $H_0 = \alpha \geq \alpha_0$; else, the lot is refused. This hypothesis is evaluated in acceptance sampling techniques based on the number of failures in a sample within a predetermined period. [3] gave the following suggestion for a group acceptability sampling plan according to a truncated life test:

i. The No. of groups x is determined and allocate a items to each group, so that $n = a.x$

ii. The acceptance limit c for every group is selected and the termination time z_0 is specified.

iii. The experiment for the group is performed simultaneously and the number of failure during this experiment for each group is observed. The lot is accepted, if c or fewer failures occur in each and all group. Otherwise, the lot is rejected, and the experiment is terminated. A full inspection for the remaining quantity $(N - n)$ is done.

The main focus of this section is determining the No. of groups x in the sampling plan so that the consumer's risk does not exceed $1 - \mathfrak{Z}^*$ as for the different values of the No. of an item in a group, acceptance limit c and termination time z_0 are assumed to be given. The minimum No. of group can be found which satisfy the following inequality:

$$\left[\sum_{d=0}^{c} \binom{a}{d} \Pi^d (1 - \Pi)^{a-d}\right] \leq 1 - \mathfrak{Z}^*, \tag{11}$$

Where Π is given in equations (4), (9) and (10). The group plan becomes the single sampling plan when $a = 1$. The minimum No. of a group can be determined by taking the normal logarithm for two side inequality (11) then, we can rewrite as follows

$$x \geq \left[\frac{\ln(1-\mathfrak{Z}^*)}{\ln\left(\sum_{d=0}^{c}\binom{a}{d}\Pi^d(1-\Pi)^{a-d}\right)}\right], \tag{12}$$

When $c = 0$, the equation (12) becomes as follows

$$x \geq \left[\frac{\ln(1-\mathfrak{Z}^*)}{a.\ln(1-\Pi)}\right]$$

The minimum No. of groups x satisfying the inequality (12) according to different values of confidence level $\mathfrak{Z}^*$, termination time ratio Λ, the No. of tester (group size) a, acceptance limits c are obtained and given in Tables (1-3). The minimum No. of groups is evaluated at the shape parameters $\beta_0 = 2$ for EIR, GIW, and WGIW distributions and $\gamma_0 = 0.25$ for only GIW distribution

Table 1

Minimum number of group for EIR distribution $\beta_0 = 2$

3^*	a	c	$\Lambda = \frac{z}{\alpha_0}$							
			0.5	**0.75**	**1.0**	**1.5**	**2.0**	**2.75**	**3.0**	**3.55**
0.75	2	0	5	2	1	1	1	1	1	1
	3	1	25	5	2	1	1	1	1	1
	4	2	136	12	4	1	1	1	1	1
	5	3	776	53	9	2	1	1	1	1
	6	4	4591	104	19	2	1	1	1	1
	7	5	*	207	23	3	1	1	1	1
0.90	2	0	8	3	2	1	1	1	1	1
	3	1	41	8	3	1	1	1	1	1
	4	2	225	20	6	2	1	1	1	1
	5	3	1288	53	19	3	2	1	1	1
	6	4	7625	207	37	4	2	1	1	1
	7	5	*	344	47	5	2	1	1	1
0.95	2	0	10	4	2	1	1	1	1	1
	3	1	53	10	4	2	1	1	1	1
	4	2	293	26	5	3	2	1	1	1
	5	3	1676	104	19	4	2	1	1	1
	6	4	9920	207	37	5	2	1	1	1
	7	5	*	448	47	7	3	2	1	1
0.99	2	0	15	6	3	2	1	1	1	1
	3	1	82	15	7	3	2	1	1	1
	4	2	451	40	12	4	2	2	1	1
	5	3	2577	104	37	6	3	2	2	1
	6	4	*	412	74	8	4	2	2	2
	7	5	*	688	93	10	5	2	2	2

Table 2

Minimum number of group for WGIW distribution $\beta_0 = 2$

3^*	a	c	$\Lambda = \frac{z}{\alpha_0}$							
			0.5	**0.75**	**1.0**	**1.5**	**2.0**	**2.75**	**3.0**	**3.55**
0.75	2	0	11	2	1	1	1	1	1	1
	3	1	123	6	2	1	1	1	1	1
	4	2	1489	17	4	1	1	1	1	1
	5	3	*	83	7	2	1	1	1	1
	6	4	*	144	12	2	1	1	1	1
	7	5	*	429	21	3	1	1	1	1
0.90	2	0	18	3	2	1	1	1	1	1
	3	1	204	10	3	1	1	1	1	1

Contd...

	4	2	2473	29	6	2	1	1	1	1
	5	3	*	82	11	3	1	1	1	1
	6	4	*	239	20	3	2	1	1	1
	7	5	*	711	36	4	2	1	1	1
0.95	2	0	23	4	2	1	1	1	1	1
	3	1	265	13	4	2	1	1	1	1
	4	2	3217	37	8	2	1	1	1	1
	5	3	*	106	14	3	2	1	1	1
	6	4	*	311	26	4	2	1	1	1
	7	5	*	962	46	6	3	1	1	1
0.99	2	0	36	7	3	2	1	1	1	1
	3	1	408	20	7	3	2	1	1	1
	4	2	*	57	12	4	2	1	1	1
	5	3	*	207	37	5	3	2	2	1
	6	4	*	851	74	7	3	2	2	1
	7	5	*	1423	93	9	4	2	2	2

Table 3

Minimum number of group for GIW distribution with $\gamma_0 = 0.25$ and $\beta_0 = 2$

β^*	a	c	$\Lambda = \frac{z}{\alpha_0}$							
			0.5	**0.75**	**1.0**	**1.5**	**2.0**	**2.75**	**3.0**	**3.55**
0.75	2	0	2	1	1	1	1	1	1	1
	3	1	4	2	1	1	1	1	1	1
	4	2	9	2	1	1	1	1	1	1
	5	3	21	3	1	1	1	1	1	1
	6	4	49	5	2	1	1	1	1	1
	7	5	116	10	3	1	1	1	1	1
0.90	2	0	3	1	1	1	1	1	1	1
	3	1	6	2	1	1	1	1	1	1
	4	2	15	3	1	1	1	1	1	1
	5	3	34	5	2	1	1	1	1	1
	6	4	81	8	3	1	1	1	1	1
	7	5	193	9	4	1	1	1	1	1
0.95	2	0	3	1	1	1	1	1	1	1
	3	1	8	2	1	1	1	1	1	1
	4	2	19	4	2	1	1	1	1	1
	5	3	45	8	3	1	1	1	1	1

Contd...

	6	4	105	10	4	1	1	1	1	1
	7	5	251	12	5	2	1	1	1	1
0.99	2	0	5	2	2	1	1	1	1	1
	3	1	13	4	2	1	1	1	1	1
	4	2	30	6	3	2	1	1	1	1
	5	3	69	15	4	2	1	1	1	1
	6	4	162	17	7	2	2	1	1	1
	7	5	385	19	8	3	2	1	1	1

Note. The cells with star (*) indicate that the number of group is found to be very large.

According to Tables (1), (2), and (3), the GIW distribution requires fewer groups than the WGIW and EIR distributions. Although, the No. of groups in the three distributions decreases with the increase of the termination time ratio of the experiment.

4. Operating Characteristic Functions

When calculating the minimal number of groups in a sampling plan so that the consumer's risk does not exceed $1-\beta^*, \beta^*$ is referred to as the consumer's confidence level. The chance of lot acceptance will thereafter be easy to calculate if and only if c or less than the failure items happened in each and every group. Otherwise, the lot should be rejected. The OC function can be described as follows for the group acceptance sampling plan.

$$OC(\Pi) = \left[\sum_{d=0}^{c} \binom{a}{d} \Pi^d (1-\Pi)^{a-d}\right]^x, \tag{13}$$

where Π is the failure probability that an item in each group in the termination time z_0. According to (13), the OC values as a function of $\frac{\alpha}{\alpha_0}$ when the lifetime follows the EIR, WGIW, or GIW distributions are provided as follows:

$$OC_{GIW}(\Pi) = \left\{\sum_{d=0}^{c} \binom{a}{d} \left[\mathrm{e}^{-\gamma_0\left(\frac{z}{\alpha_0}\cdot\frac{\alpha}{\alpha_0}\right)^{-\beta_0}}\right]^d \left[1-\mathrm{e}^{-\left(\frac{z}{\alpha_0}\cdot\frac{\alpha}{\alpha_0}\right)^{-\beta_0}}\right]^{a-d}\right\}^x;$$

$$OC_{WGIW}(\Pi) = \left\{\sum_{d=0}^{c} \binom{a}{d} \left[\mathrm{e}^{-\left[\left(\frac{\alpha}{\Lambda\,\alpha_0}\right)^{\beta_0}\ln 2\right]}\right]^d \left[1-\mathrm{e}^{-\left[\left(\frac{\alpha}{\Lambda\,\alpha_0}\right)^{\beta_0}\ln 2\right]}\right]^{a-d}\right\}^x;$$

or

$$OC_{EIR}(\Pi) = \left\{\sum_{d=0}^{c} \binom{a}{d} \left[\left\{1+\left(\frac{\alpha}{\Lambda\alpha_0}\right)^2\left[(0.5)^{-\frac{1}{\beta_0}}-1\right]\right\}^{-\beta_0}\right]^d \left[1-\left(1+\left(\frac{\alpha}{\Lambda\alpha_0}\right)^2\left[(0.5)^{-\frac{1}{\beta_0}}-1\right]\right)^{-\beta_0}\right]^{a-d}\right\}^x$$

The probabilities of acceptance of the lot according to (13) for a given different values of median ratio $\frac{\alpha}{\alpha_0}$, time ratio Λ, confidence level $\mathfrak{Z}^*$, group size a = 4, and the concluded values of x are shown in Tables (4-6) values for GIW distribution at γ_0 = 0.25 and β_0 = 2 for the three distributions.

Table 4

OC values of group sampling plan (x, c, Λ) for a given $\mathfrak{Z}^*$ with c = 2, a = 4 and β_0= 2 for EIR distribution

$\mathfrak{Z}^*$	x	Λ	α/α_0							
			2	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.75	136	0.5	0.997	1	1	1	1	1	1	1
	12	0.75	0.988	1	1	1	1	1	1	1
	4	1	0.963	0.999	1	1	1	1	1	1
	1	1.5	0.867	0.987	0.999	1	1	1	1	1
	1	2	0.687	0.939	0.99	0.998	1	1	1	1
	1	2.75	0.396	0.76	0.928	0.98	0.995	0.998	1	1
	1	3	0.325	0.687	0.89	0.966	0.99	0.997	0.999	1
	1	3.55	0.21	0.532	0.785	0.915	0.969	0.989	0.996	0.998
0.90	225	0.5	0.995	1	1	1	1	1	1	1
	20	0.75	0.98	1	1	1	1	1	1	1
	6	1	0.939	0.998	1	1	1	1	1	1
	2	1.5	0.788	0.979	0.998	1	1	1	1	1
	1	2	0.635	0.926	0.988	0.998	1	1	1	1
	1	2.75	0.396	0.76	0.928	0.98	0.995	0.998	1	1
	1	3	0.325	0.687	0.89	0.966	0.99	0.997	0.999	1
	1	3.55	0.21	0.532	0.785	0.915	0.969	0.989	0.996	0.998
0.95	293	0.5	0.994	1	1	1	1	1	1	1
	26	0.75	0.974	0.999	1	1	1	1	1	1
	8	1	0.922	0.997	1	1	1	1	1	1
	3	1.5	0.734	0.973	0.997	1	1	1	1	1
	2	2	0.554	0.905	0.984	0.997	0.999	1	1	1
	1	2.75	0.388	0.756	0.926	0.98	0.995	0.998	1	1
	1	3	0.325	0.687	0.89	0.966	0.99	0.997	0.999	1
	1	3.55	0.21	0.532	0.785	0.915	0.969	0.989	0.996	0.998
0.99	451	0.5	0.991	1	1	1	1	1	1	1
	40	0.75	0.961	0.999	1	1	1	1	1	1
	12	1	0.882	0.996	1	1	1	1	1	1
	4	1.5	0.621	0.959	0.996	0.999	1	1	1	1
	2	2	0.404	0.858	0.976	0.996	0.999	1	1	1

Contd...

	2	2.75	0.233	0.65	0.889	0.969	0.992	0.998	0.999	1
	1	3	0.202	0.586	0.847	0.952	0.986	0.996	0.999	1
	1	3.55	0.154	0.469	0.748	0.899	0.963	0.987	0.995	0.998

Table 5

OC values of group sampling plan (x, c, Λ) for a given $\mathfrak{Z}^*$ with c = 2, a = 4 and β_0= 2 for WGIW distribution

$\mathfrak{Z}^*$	x	Λ	α/α_0							
			2	3	4	5	6	7	8	9
0.75	1489	0.5	1	1	1	1	1	1	1	1
	17	0.75	1	1	1	1	1	1	1	1
	4	1	0.997	1	1	1	1	1	1	1
	1	1.5	0.914	0.999	1	1	1	1	1	1
	1	2	0.687	0.969	0.999	1	1	1	1	1
	1	2.75	0.361	0.774	0.959	0.996	1	1	1	1
	1	3	0.287	0.687	0.922	0.989	0.999	1	1	1
	1	3.55	0.177	0.508	0.803	0.948	0.991	0.999	1	1
0.90	2473	0.5	1	1	1	1	1	1	1	1
	37	0.75	1	1	1	1	1	1	1	1
	8	1	0.993	1	1	1	1	1	1	1
	2	1.5	0.824	0.998	1	1	1	1	1	1
	1	2	0.587	0.956	0.999	1	1	1	1	1
	1	2.75	0.361	0.774	0.959	0.996	1	1	1	1
	1	3	0.287	0.687	0.922	0.989	0.999	1	1	1
	1	3.55	0.177	0.508	0.803	0.948	0.991	0.999	1	1
0.95	3217	0.5	1	1	1	1	1	1	1	1
	37	0.75	1	1	1	1	1	1	1	1
	8	1	0.993	1	1	1	1	1	1	1
	2	1.5	0.824	0.998	1	1	1	1	1	1
	1	2	0.587	0.956	0.999	1	1	1	1	1
	1	2.75	0.361	0.774	0.959	0.996	1	1	1	1
	1	3	0.287	0.687	0.922	0.989	0.999	1	1	1
	1	3.55	0.177	0.508	0.803	0.948	0.991	0.999	1	1
0.99	4945	0.5	1	1	1	1	1	1	1	1
	57	0.75	1	1	1	1	1	1	1	1

Contd...

	12	1	0.989	1	1	1	1	1	1	1
	4	1.5	0.742	0.997	1	1	1	1	1	1
	2	2	0.441	0.933	0.998	1	1	1	1	1
	1	2.75	0.23	0.691	0.942	0.995	1	1	1	1
	1	3	0.195	0.612	0.9	0.986	0.999	1	1	1
	1	3.55	0.145	0.471	0.784	0.942	0.989	0.999	1	1

Table 6

OC values of group sampling plan $(x, c, z/\alpha_0)$ for a given $\mathfrak{Z}^*$ with c = 2, γ_0 = 0.25 and β_0 = 2 for GIW distribution

$\mathfrak{Z}^*$	x	z/α_0	α/α_0							
			2	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.75	9	0.5	1	1	1	1	1	1	1	1
	2	0.75	0.967	1	1	1	1	1	1	1
	1	1	0.856	0.996	1	1	1	1	1	1
	1	1.5	0.453	0.856	0.983	0.999	1	1	1	1
	1	2	0.214	0.576	0.856	0.969	0.996	1	1	1
	1	2.75	0.078	0.274	0.543	0.775	0.913	0.974	0.994	0.999
	1	3	0.057	0.214	0.453	0.688	0.856	0.946	0.983	0.996
	1	3.55	0.031	0.128	0.299	0.509	0.703	0.845	0.93	0.973
0.90	15	0.5	1	1	1	1	1	1	1	1
	3	0.75	0.952	1	1	1	1	1	1	1
	1	1	0.792	0.994	1	1	1	1	1	1
	1	1.5	0.453	0.856	0.983	0.999	1	1	1	1
	1	2	0.214	0.576	0.856	0.969	0.996	1	1	1
	1	2.75	0.078	0.274	0.543	0.775	0.913	0.974	0.994	0.999
	1	3	0.057	0.214	0.453	0.688	0.856	0.946	0.983	0.996
	1	3.55	0.031	0.128	0.299	0.509	0.703	0.845	0.93	0.973
0.95	19	0.5	1	1	1	1	1	1	1	1
	4	0.75	0.938	1	1	1	1	1	1	1
	2	1	0.739	0.992	1	1	1	1	1	1
	1	1.5	0.436	0.849	0.982	0.999	1	1	1	1
	1	2	0.214	0.576	0.856	0.969	0.996	1	1	1
	1	2.75	0.078	0.274	0.543	0.775	0.913	0.974	0.994	0.999
	1	3	0.057	0.214	0.453	0.688	0.856	0.946	0.983	0.996
	1	3.55	0.031	0.128	0.299	0.509	0.703	0.845	0.93	0.973
0.99	30	0.5	0.999	1	1	1	1	1	1	1
	6	0.75	0.906	1	1	1	1	1	1	1

Contd...

	3	1	0.628	0.987	1	1	1	1	1	1
	2	1.5	0.279	0.778	0.973	0.999	1	1	1	1
	1	2	0.162	0.521	0.832	0.963	0.995	1	1	1
	1	2.75	0.078	0.274	0.543	0.775	0.913	0.974	0.994	0.999
	1	3	0.057	0.214	0.453	0.688	0.856	0.946	0.983	0.996
	1	3.55	0.031	0.128	0.299	0.509	0.703	0.845	0.93	0.973

Although the number of groups at GIW distribution is less than the number of groups at SGIW taken into consideration that the values of the number of groups are equal with the increase of the time ratio of the experiment as has been mentioned earlier- yet the probabilities of acceptance in first distribution (GIW) are less than these found at SGIW distribution. This shows that, the SGIW distribution is a strong competitor to GIW distribution.

5. The Minimum Ratios

The manufacturer may be interested in improving the product's quality so that the acceptance probability exceeds a predetermined level Rao (2009). The smallest ratio of $\frac{\alpha}{\alpha_0}$ for the three distributions can be calculated at the producer's risk ∇ by satisfying the following inequality:

$$PR(\Pi) = \left[\sum_{d=0}^{c} \binom{a}{d} \Pi^d (1-\Pi)^{a-d}\right]^x \geq 1-\nabla, \tag{14}$$

For a given group sampling plans (x, c, Λ) and a given value of producer's risk ∇ = 0.05, the smallest value of (α/α_0) that will ensure that the producer's risk is more than or equal to $(1-\nabla)$ have also been computed in Tables (7-9). The parameters γ_0 and β_0 are studied at value 0.25 and 2 respectively.

Table 7
The ratio of α/α_0 or the EIR distribution with parameter β_0 = 2

$\mathfrak{Z}^*$	a	c	Λ							
			0.5	**0.75**	**1.0**	**1.5**	**2.0**	**2.75**	**3.0**	**3.55**
0.75	2	0	2.799	3.267	3.572	5.358	7.144	9.822	10.715	12.68
	3	1	1.761	2.028	2.328	3.055	4.073	5.601	6.11	7.23
	4	2	1.486	1.682	1.916	2.423	3.116	4.285	4.674	5.531
	5	3	1.356	1.596	1.769	2.131	2.616	3.597	3.924	4.643
	6	4	1.281	1.44	1.638	1.942	2.34	3.158	3.445	4.077
	7	5	1.232	1.337	1.488	1.809	2.169	2.849	3.108	3.677
0.9	2	0	3.123	3.607	4.134	5.358	7.144	9.822	10.715	12.68
	3	1	1.899	2.205	2.55	3.278	4.073	5.601	6.11	7.23
	4	2	1.568	1.791	2.056	2.631	3.221	4.285	4.674	5.531

Contd...

	5	3	1.414	1.596	1.91	2.288	2.792	3.597	3.924	4.643
	6	4	1.326	1.524	1.751	2.071	2.516	3.192	3.445	4.077
	7	5	1.269	1.388	1.581	1.918	2.32	2.934	3.139	3.677
0.95	2	0	3.347	3.875	4.451	5.649	7.144	9.822	10.715	12.68
	3	1	1.974	2.3	2.669	3.446	4.233	5.601	6.11	7.23
	4	2	1.612	1.849	2.13	2.739	3.364	4.302	4.674	5.531
	5	3	1.445	1.703	1.91	2.369	2.902	3.705	3.971	4.643
	6	4	1.35	1.524	1.751	2.136	2.606	3.319	3.556	4.077
	7	5	1.288	1.415	1.581	1.974	2.397	3.044	3.259	3.733
0.99	2	0	3.746	4.351	5.011	6.386	7.773	9.837	10.715	12.68
	3	1	2.1	2.461	2.867	3.725	4.597	5.897	6.328	7.269
	4	2	1.684	1.944	2.251	2.916	3.599	4.624	4.964	5.709
	5	3	1.496	1.703	2.052	2.502	3.08	3.952	4.242	4.878
	6	4	1.389	1.61	1.863	2.243	2.751	3.523	3.78	4.343
	7	5	1.32	1.459	1.674	2.063	2.52	3.219	3.452	3.964

Table 8

The ratio of α/α_0 for the WGIW distribution with parameter $\beta_0 = 2$

3^*	a	c	Λ							
			0.5	**0.75**	**1.0**	**1.5**	**2.0**	**2.75**	**3.0**	**3.55**
0.75	2	0	1.476	1.883	2.303	3.454	4.606	6.333	6.909	8.175
	3	1	1.265	1.538	1.843	2.548	3.397	4.671	5.096	6.03
	4	2	1.183	1.389	1.633	2.154	2.834	3.897	4.251	5.031
	5	3	1.14	1.344	1.508	1.958	2.486	3.419	3.729	4.413
	6	4	1.113	1.248	1.423	1.823	2.243	3.084	3.364	3.981
	7	5	1.095	1.21	1.362	1.721	2.102	2.833	3.09	3.657
0.90	2	0	1.536	1.989	2.455	3.454	4.606	6.333	6.909	8.175
	3	1	1.301	1.605	1.942	2.636	3.397	4.671	5.096	6.03
	4	2	1.209	1.438	1.709	2.284	2.866	3.897	4.251	5.031
	5	3	1.16	1.343	1.57	2.068	2.578	3.419	3.729	4.413
	6	4	1.13	1.282	1.476	1.917	2.377	3.084	3.364	3.981
	7	5	1.109	1.239	1.408	1.805	2.226	2.861	3.09	3.657
0.95	2	0	1.567	2.041	2.531	3.51	4.606	6.333	6.909	8.175
	3	1	1.319	1.638	1.991	2.717	3.444	4.671	5.096	6.03
	4	2	1.222	1.463	1.747	2.348	2.957	3.897	4.251	5.031

Contd...

	5	3	1.17	1.364	1.601	2.122	2.656	3.456	3.729	4.413
	6	4	1.138	1.299	1.503	1.964	2.445	3.168	3.408	3.981
	7	5	1.116	1.253	1.431	1.847	2.287	2.952	3.173	3.657
0.99	2	0	1.616	2.125	2.65	3.702	4.743	6.333	6.909	8.175
	3	1	1.348	1.691	2.068	2.846	3.624	4.78	5.162	6.03
	4	2	1.243	1.503	1.806	2.449	3.1	4.072	4.394	5.098
	5	3	1.187	1.413	1.707	2.207	2.777	3.633	3.917	4.538
	6	4	1.152	1.359	1.604	2.038	2.552	3.325	3.582	4.145
	7	5	1.128	1.277	1.491	1.913	2.383	3.093	3.33	3.848

Table 9

The ratio of α/α_0 for the GIW distribution with parameters γ_0= 0.25 and β_0 = 2.

ℨ*	a	c	z/α_0							
			0.5	**0.75**	**1.0**	**1.5**	**2.0**	**2.75**	**3.0**	**3.55**
0.75	2	0	2.021	3.133	3.835	5.752	7.669	10.545	11.504	13.613
	3	1	1.637	2.302	2.828	4.243	5.657	7.778	8.485	10.041
	4	2	1.467	1.918	2.36	3.539	4.719	6.489	7.079	8.376
	5	3	1.368	1.74	2.126	3.105	4.14	5.692	6.21	7.348
	6	4	1.303	1.662	2.06	2.801	3.735	5.135	5.602	6.629
	7	5	1.257	1.609	1.954	2.573	3.43	4.717	5.145	6.089
0.90	2	0	2.142	2.921	3.835	5.752	7.669	10.545	11.504	13.613
	3	1	1.714	2.286	2.865	4.243	5.657	7.778	8.485	10.041
	4	2	1.525	1.993	2.476	3.539	4.719	6.489	7.079	8.376
	5	3	1.415	1.843	2.284	3.105	4.14	5.692	6.21	7.348
	6	4	1.342	1.719	2.077	2.836	3.735	5.135	5.602	6.629
	7	5	1.291	1.6	1.968	2.648	3.43	4.717	5.145	6.089
0.95	2	0	2.202	3.019	3.835	5.752	7.669	10.545	11.504	13.613
	3	1	1.753	2.351	2.958	4.243	5.657	7.778	8.485	10.041
	4	2	1.554	2.044	2.549	3.56	4.719	6.489	7.079	8.376
	5	3	1.438	1.918	2.301	3.187	4.14	5.692	6.21	7.348
	6	4	1.362	1.758	2.174	2.925	3.735	5.135	5.602	6.629
	7	5	1.308	1.633	2.034	2.728	3.464	4.717	5.145	6.089

Contd...

0.99	2	0	2.298	3.174	4.047	5.769	7.669	10.545	11.504	13.613
	3	1	1.814	2.454	3.103	4.392	5.665	7.778	8.485	10.041
	4	2	1.6	2.124	2.664	3.746	4.817	6.489	7.079	8.376
	5	3	1.476	2.003	2.409	3.346	4.29	5.692	6.21	7.348
	6	4	1.394	1.822	2.282	3.065	3.92	5.189	5.609	6.629
	7	5	1.336	1.684	2.116	2.854	3.641	4.811	5.198	6.089

Values of Tables (7), (8) and (9) refer to the need for higher level quality products at GIW distribution compared to the other distributions: SGIW and CIR. For example, the tester demands to increase the veritable median life to (2.124) times the determined median life to accept the lot with producer's risk at 5% while he wants to increase the true median life at 1.503 and 1.994 times for SGIW and CIR distribution respectively.

6. Table Explanation and Explicative Case

The EIR (α, β), WGIW $(\alpha, \beta, \gamma, \lambda)$ and GIW $(\alpha, \beta, \gamma, \lambda)$ distributions in GASP are discussed in this section. The parameters β_0 = 2 for the three distributions and γ_0 = 0.25 for GIW distribution are determined and shown in Tables 1-9.

Tables 1, 2 and 3 exhibit the number of groups with various values of time ratio $\Lambda = \frac{z}{\alpha_0}$ = 0.5, 0.75, 1.0, 1.5, 2.0, 2.75, 3.0 and 3.55, confidence level $\mathfrak{Z}^* =$ 0.75, 0.90, 0.95 and 0.99, number of tester a = 4 and acceptance limit c = 0 (1)5. Tables 4, 5 and 6 illustrate the operating characteristic values for the plan $\left(x, a, c, \frac{z}{\alpha_0}\right)$ with given various values of $\mathfrak{Z}^*$, c = 2 and a = 4. Finally, Tables 7, 8 and 9 determine the ratios of veritable median life α to stipulated median life α_0 for the acceptance of the lot with producer's risk ∇ = 0.05.

Case study

Suppose an experimenter receives large lots of bakeries fresh and decides to establish a group sampling plan (GASP) to know to ensure that the veritable median life is at least α_0= 1000 hours based on the time of experiment at z_0 = 750h. The GASP is studied at parameters $\mathfrak{Z}^* = 0.99$, c = 2, and Λ = 0.75 Then, for the group size 4 items, we have

i. The EIR distribution: the number of group x_{EIR} = 40.

ii. The WGIW distribution: the number of group x_{WGIW} = 57.

iii. The GIW distribution: the number of groupx_{GIW} = 6.

Consequently, the samples sizes are drawn are 24, 228 and 160 items and assign 4 items to 6, 57 and 40 groups for EIR, WGIW, GIW distributions respectively have to be put on test at 750 hours. Accept the lot, if there are two or less failures items occurred during 750 hours, the lot is accepted and the experimenter can ensure with confidence level 0.99 that the median life is at least 1000 hours. Otherwise, stop the experiment and reject the lot. On the other hand, the minimum number of group increases quite rapidly as the shape parameter increase when the termination time ratio is short for three distributions.

The OC values with group size a = 4 for GASP $\left(x, c, \frac{z}{\alpha_0}\right)$ = (40, 2, 0.75), (57, 2, 0.75) and (6, 2, 0.75) for EIR, WGIW and GIW respectively can be provided in Tables 4, 5 and 6. This means, if $\frac{\alpha}{\alpha_0}$ = 2, the producer's risk for EIR and GIW distributions will be 0.039 and 0.094 whereas the producer's risk for WGIW distribution is almost zero. When $\frac{\alpha}{\alpha_0}$ = 4 the producer's risk for three distributions will be zero, then the producer's risk decreases when the median ratio increases. Tables 7, 8 and 9 present the values of ratio $\frac{\alpha}{\alpha_0}$ for different values of $\frac{z}{\alpha_0}$ and c in order to keep the producer's risk at 5%. For example, when Λ = 0.75, c = 2, a = 4 and x_{EIR} = 40, x_{WGIW} = 57 and x_{WGIW} = 6 the required ratio for specified distributions are 1.944, 1.503 and 2.124 respectively. Namely, the experimenter of the product should have a median life of more than or equal to 1994, 1503 and 2124 hours respectively in order that the products to be accepted with a probability of more than or equal to 95%.

7. Comparisons of Plans

To determine the group sampling plan efficiency, we compare the group sampling plan with the ordinary acceptance sampling plan proposed by Mahdy et al. (2018) and consider Λ = 0.75, c = 2, $\mathfrak{Z}^*$ = 0.99, and quality level $\frac{\alpha}{\alpha_0}$ = 2. Table 10 illustrates this comparison as follows

Table 10

Comparison between single sampling and group sampling plan

Distributions	**single sampling plan**				**Group sampling plan**			
	n	**OC(Π)**	**PR(Π)**	α/α_0	n	**OC(Π)**	**PR(Π)**	α/α_0
EIR	22	0.837	0.163	2.361	160	0.961	0.039	1.944
WGIW	25	0.999	0.001	1.664	228	1	0	1.503
GIW	9	0.801	0.199	2.306	24	0.906	0.094	2.124

The previous Table shows that the group sampling plan gives better fit than the ordinary sampling plan although the sample size in group sampling plan more than sample size at ordinary sampling plan. Also, the SGIW distribution is considered the best of the three distributions as it yields less producer's risk and less minimum ratio.

8. Concluding Remarks

This work proposes a group sampling plan based on a truncated life test when the product's lifetime follows the EIR, WGIW, or GIW distributions. The minimal number of groups for the WGIW distribution plan is slightly higher than for the GIW distribution despite the fact that the WGIW distribution is a strong competitor to the GIW distribution. Furthermore, when compared to the GIW distribution, the WGIW distribution fits this design better.

Declarations

Data Availability: The data is given in the paper.
Conflict of Interest: none
Authors' contribution: M.M, B.A and M.A wrote the paper.

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