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Bernoulli selection processes and power contractions in strategic operations

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Abstract

Bernoulli selection processes and power contractions constitute two classes of probabilistic concepts of theoretical and practical importance. The paper's theoretical contribution consists of the development of a characterization of power contractions through the use of Bernoulli selection processes. Moreover, the interpretations in risk management, stochastic discounting modelling, and investment evaluation of Bernoulli selection processes for power contractions constitute the practical contribution. In consequence, the presence of proactivity in these disciplines makes quite clear that the results of the paper facilitate the implementations of strategic thinking, decision making, and management operations.

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1. Introduction

Stochastic multiplicative models of two positive and independent random variables are particularly important in theory and practice [8, 12, 13]. If one of the positive random variables is defined on the standard unit interval then these models are called random contractions [6, 9, 10]. It is very well known that random contractions are strong tools for several scientific disciplines [5, 9, 10]. In addition, if the random variable, is defined on the standard unit interval, follows the power distribution then the random contractions are called power contractions [7]. The paper focuses on the introduction of the Bernoulli distribution for making selections from the class of power contractions. From the point of view of probability distribution theory such an introduction is equivalent to the investigation of the distribution function of a mixture of two power contractions. The implementation of this investigation is based on the corresponding characteristic function [1, 2].

In order to be more precise, the paper focuses on the realization of the following five purposes. The first purpose is the establishment of a characterization of a power contraction by incorporating a Bernoulli selection process. Such a characterization may facilitate research activities concentrating on the interconnections between Bernoulli selection processes and random contractions. The second purpose consists of application in risk management of the concept of Bernoulli selection process for random contractions. It is directly adopted that this concept can substantially facilitate the implementation of various risk management operations. Risk measurement, risk severity reduction, risk duration reduction, risk financing, competing risks treatment, extending catastrophic risk occurrence time, reactive treatment of ongoing risks are some risk management operations appropriate for development and investigation by making use of Bernoulli selection processes and power contractions. The interpretation in stochastic discounting modeling of Bernoulli selection processes for power contractions constitutes the third purpose. It is of some interest the representation of Bernoulli selection processes for present values of single random cash flows, under random time and constant force of interest, as Bernoulli selection processes for power contractions. The fourth purpose is the use of Bernoulli selection processes for power contractions in investment evaluation. More precisely, present value, cash flows, lifetime, payback period, total cost are the main stochastic factors required for making decisions concerning the acceptance or rejection of an investment. Equivalently, these stochastic factors are generally adopted as the structural elements of the discipline of investment evaluation. In consequence, the implementation of the fourth purpose of the paper is based on the structural elements of the discipline of investment evaluation. In particular, the payback period constitutes the most important stochastic factor in investment decision making since the payback period incorporates the most valuable information of an investment proposal. The implementation of the fifth purpose of the paper can be facilitated by the implementation of the second, third, and fourth purpose. The presence of proactivity in several strategic operations is suitable for the consideration of incorporating concepts of risk management, stochastic discounting modeling, and investment evaluation in strategic thinking, decision making and management. In consequence, Bernoulli selection processes for power contractions facilitate the introduction and application of various strategic operations. Research activities, concentrating on the formulation and interpretation in various disciplines of Bernoulli selection processes for various categories of stochastic models, are valuable [1, 3]. Such activities may be particularly useful in the development and implementation of efficient strategic operations [4].

2. Establishing a Characterization for Power Contractions

The second section focuses on the establishment of a characterization for power contractions by using two positive random variables, two random variables following the power distribution, and a discrete random variable following the Bernoulli distribution. Clearly, this section uses a Bernoulli selection process for the family of power contractions. Such a selection process strongly facilitates the investigation of the structural properties of a particular power contraction and the family of power contractions. Clearly, the theoretical result of this section is the introduction of a characterization incorporating a Bernoulli selection process on the family of power contractions. It is widely acknowledged that the structural elements supporting the theoretical contribution of this section also provide valuable information for the corresponding practical contribution.

Theorem: *We assume that X, L are positive random variables, S, V are random variables following the power distribution with parameter a and N is a random variable following the Bernoulli distribution with parameter p. We set*

$$Y = \begin{cases} XS, & N = 0 \\ LV, & N = 1 \end{cases} \tag{1}$$

and we assume that X, S, L, V, N are independent, $\varphi_X(u)$ *is the characteristic function of X, and* $\varphi_Y(u)$ *is the characteristic function of Y. If* $\varphi_Y(u)$ *is differentiable then*

$$\varphi_Y(u) = qa\int_0^1 \varphi_X(uw)w^{qa-1}dw$$

if, and only if

$$Y \overset{d}{=} L \tag{2}$$

where the symbol $\overset{d}{=}$ *is used to represent equality in distribution.*

Proof: We establish the sufficiency condition. Moreover, the reverse argument establishes the necessity condition.

If $\varphi_L(u)$ is the characteristic function of L then

$$\varphi_Y(u) = pa\int_0^1 \varphi_L(uw)w^{a-1}dw + qa\int_0^1 \varphi_X(uw)w^{a-1}dw$$

is the characteristic function of Y. It easily follows that the above characteristic function has the representation

$$\varphi_Y(u) = p\frac{a}{u^a}\int_0^u \varphi_L(w)w^{a-1}dw + q\frac{a}{u^a}\int_0^u \varphi_X(w)w^{a-1}dw. \tag{3}$$

Through the use of (2) in (3) we get the integral equation

$$\varphi_Y(u) = p\frac{a}{u^a}\int_0^u \varphi_Y(w)w^{a-1}dw + q\frac{a}{u^a}\int_0^u \varphi_X(w)w^{a-1}dw$$

or equivalently the integral equation

$$u^a\varphi_Y(u) = pa\int_0^u \varphi_Y(w)w^{a-1}dw + qa\int_0^u \varphi_X(w)w^{a-1}dw. \tag{4}$$

Differentiation of (4) implies that

$$au^{a-1}\varphi_Y(u) + u^a\frac{d\varphi_Y(u)}{du} = pa\varphi_Y(u)u^{a-1} + qa\varphi_X(u)u^{a-1}$$

The above equation has the following representation

$$\varphi_Y(u) + \frac{u}{a}\frac{d\varphi_Y(u)}{du} = p\varphi_Y(u) + q\varphi_X(u) \tag{5}$$

for $u \neq 0$. It is quite obvious that (5) has the equivalent representation

$$\varphi_Y(u) + \frac{u}{aq}\frac{d\varphi_Y(u)}{du} = \varphi_X(u). \tag{6}$$

From (6) we get the integral equation

$$\frac{qa}{u^{qa}}\int_0^u \varphi_Y(w)w^{qa-1}dw + \frac{1}{qa}\frac{qa}{u^{qa}}\int_0^u w^{qa}d\varphi_Y(w) = \frac{qa}{u^{qa}}\int_0^u \varphi_X(w)w^{qa-1}dw. \tag{7}$$

Integrating by parts the second term in the left side of (7) we get that

$$\frac{qa}{u^{qa}}\int_0^u \varphi_Y(w)w^{qa-1}dw + \varphi_Y(w) - \frac{qa}{u^{qa}}\int_0^u \varphi_Y(w)w^{qa-1}dw = \frac{qa}{u^{qa}}\int_0^u \varphi_X(w)w^{qa-1}dw. \tag{8}$$

From (8) we get the characteristic function

$$\varphi_Y(u) = \frac{qa}{u^{qa}}\int_0^u \varphi_X(w)w^{qa-1}dw \tag{9}$$

From (9) we get that

$$\varphi_Y(u) = qa\int_0^1 \varphi_X(uw)w^{qa-1}dw. \tag{10}$$

Moreover (10) implies that Y is a power contraction of X.

The following sections of the paper concentrate on the application in practice of the theoretical contribution of this section in several scientific disciplines. More clearly, the following sections provide practical interpretation in proactive risk management decisions, stochastic discounting activities, and strategic operations.

3. Severity and Duration Reduction of Risk

Risk severity reduction is widely recognized as the most significant proactive treatment operations of risk. The third section provides an interpretation of the theoretical results of the previous section in proactive risk severity reduction operations [4].

We assume that X represents the severity of a risk with probability q and L represents the severity of the same risk with probability p. If we use a risk severity reduction operation then the power contraction

$$XS$$

can be considered as the severity of the risk with probability q after the application of the risk severity reduction operation and the power contraction

$$LV$$

can be considered as the severity of the risk with probability p after the application of the severity reduction operation to the risk. In consequence, the random variable Y in (1) represents the severity of the risk after applying the risk severity reduction operation. It is obvious that Y is a Bernoulli selection process for power contraction. Moreover, the previous section establishes that the independence of X, S, L, V and the equality in distribution of Y and L imply that Y is also a power contraction if L, V follow the power distribution with parameter a.

It is also generally adopted that risk duration reduction constitutes a significant proactive treatment operation of risk. It is also of some particular theoretical and practical importance to mention that the above interpretation of the results of the previous section in proactive risk severity reduction operations is also valid for proactive risk duration reduction operations [4].

The facilitation of risk management objectives is significantly enhanced by the proactive approach in risk treatment operations. More clearly, it is a valuable result the introduction of proactivity in risk severity reduction and risk duration reduction operations by making use of Bernoulli selection process for power contractions. In consequence, it seems to be of some practical importance the consideration of Bernoulli selection processes for power contraction of functions of positive random variables. In particular, the incorporation of a random sum of positive random variables in a Bernoulli selection process for power contractions strongly supports the practical applicability of such a process [11].

Various techniques and methodologies used in the field of risk management also serve as structural elements within the field of global risk governance. In this case, it is quite useful to take into consideration the possibility of using stochastic models of risk management in global risk governance. It is readily comprehensible that the stochastic models for risk identification, risk measurement, and risk treatment can be used in global risk identification, global risk measurement, and global risk treatment. In addition, it is quite obvious that Bernoulli selection processes for power contractions can be useful in global risk severity and global risk treatment operations. It seems to be very useful the introduction of proactivity in global risk severity reduction and global risk duration reduction operations by making use of Bernoulli selection processes for power contractions incorporating random sums, random maxima, or random minima of positive random variables. Such selection processes are particularly valuable for the consideration of proactivity as a powerful concept for global risk governance decision making [11].

4. Stochastic Models in Continuous Discounting

The section concentrates on interpreting the theoretical results of the second section in stochastic discounting modelling.

We assume that

$$T, H$$

follow the exponential distribution with parameter λ and r is a positive constant. We set $a = \lambda/r$. Hence S has the representation

$$S = e^{-rT}$$

and V has the representation

$$V = e^{-rH}.$$

We consider the random variable

$$Y = \begin{cases} Xe^{-rT}, & N = 0 \\ Le^{-rH}, & N = 1 \end{cases}.$$

It is obvious that Y is a Bernoulli selection process for power contraction being stochastic discounting models. The above process is incorporated in stochastic discounting modelling in the following way.

We assume that X represents a cash flow arising at the random time T with probability q. If the constant r represents the force of interest in continuous compounding, then

$$Xe^{-rT}$$

represents the present value of X as viewed from the time point 0.

We also assume that L represents a cash flow arising at the random time point H with probability p. Hence the random variable

$$Le^{-rH}$$

represents the present value of L as viewed from the time point 0. It is quite obvious that Y in (1) is a Bernoulli selection process for power contractions being present values of cash flows. Moreover, the independence of X, T, L, H and the equality in distribution of Y and L imply that Y is also a power contraction. It is of particular significance to consider extensions of the Bernoulli selection process Y in (1). Below we consider three such extensions based on replacements of X by functions of positive random variables.

The first extension of Y in (1) is based on the replacement of X by a random sum of positive random variables. The practical applications of random sums of positive random variables in several scientific areas is a very good reason for considering and investigating the first extension of the Bernoulli selection process Y in (1). It is readily understood that the first extension of Y is suitable for Bernoulli selection processes incorporating a choice consisting of a sequence of positive random variables and a discrete random variable.

The second extension of Y in (1) is based on the replacement of X by a maximum of a random number of positive random variables. Moreover, the second extension of Y in (1) is suitable for Bernoulli selection processes incorporating a worst scenario.

The third extension of Y in (1) is based on the replacement of X by a minimum of a random number of positive random variables. Moreover, the third extension of Y in (1) is suitable for Bernoulli selection processes incorporating a best scenario.

5. Stochastic Concepts in Investment Evaluation

It is a valuable recognition that investment evaluation is a very important operation of any organization. Moreover, it is also recognized the presence of stochastic factors in evaluating investments. The section concentrates on the introduction of stochastic concepts in the operation of investment evaluation. More precisely, this section establishes an interpretation in investment evaluation of the Bernoulli selection process Y in (1).

We consider an investment at the time point 0. We assume that the positive random variable X denotes the residual value of the investment with choice probability q. We also suppose that L denotes the residual value of the investment with choice probability p. It is clear that the power contraction XS represents the remaining value of the investment with choice probability q and the power contraction LV represents the remaining value of the investment with choice probability p. The effects of suitable operations on the remaining value of the investment are described by the random variables

$$S, V$$

following the power distribution with parameter a. In consequence, the independence of X, S, L, V and the equality in distribution of Y and L imply that the Bernoulli selection process Y in (1) is also a power contraction.

In some cases it is highly beneficial to modify the above interpretation of the Bernoulli selection process for random contractions Y in (1) by making the following assumption. More clearly, we consider two investments at the time point 0 instead of one investment. It is easily seen that such a modification strongly supports the applicability of the interpretation in investment evaluation of the Bernoulli selection process of power contractions Y in (1). It seems of some interest the implementation of research work concentrating on the consideration of Bernoulli selection processes for stochastic discounting models, incorporating the payback period T of the investment having probability of choice q and the payback period H of the investment having probability of choice p. More precisely, the consideration of Bernoulli selection processes of the form

$$Y = \begin{cases} Xe^{-rT}, & N = 0 \\ Le^{-rH}, & N = 1 \end{cases}. \tag{11}$$

where T represents the payback period of the investment with probability of choice q, Xdenotes the residual value of the investment at the random time T, H is the payback period of the investment with probability of choice p, L represents the residual value of the investment at the time L, and the constant r represents the force of interest in continuous compounding. From a practical point of view the interpretations of the residual value X and the residual value Lin various situations related to the time value of money are readily recognized as activities strongly supporting the applicability of the Bernoulli selection process Y in (11).

6. Conclusions

Bernoulli selection processes and power contractions are generally considered as probabilistic concepts with useful applications in theory and practice. The paper establishes an interconnection between these two probabilistic concepts. More precisely, the paper introduces a characterization of a class of power contractions by incorporating the concept of Bernoulli selection processes for power contraction. From a practical point of view it is shown that the new concept is useful in risk management, stochastic discounting modeling, and investment evaluation. Since proactivity is a structural concept of these disciplines it easily follows that the concept of Bernoulli selection process for power contractions can facilitate strategic thinking, decision making, and management.

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