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Using kernel functions to estimate the probability density function for segmentation images

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Abstract

This paper used nonparametric kernel functions to segment images using the thresholding method. This was done by transforming the image to grey and then estimating the probability density function from the grey image data and taking the highest value of the core function vector as the threshold limit. The study also found that the bandwidth parameter affects the segmentation process because it affects the estimate of the probability density function. The bandwidth parameter smooths the curve and brings it closer to the true curve because of its significant effect on bias and variance. The beam width parameter is known to depend on the size of the sample. Thus, it was found that when the parameter is used to determine the image size into equal rows and columns, it gives good, satisfactory results for segmented images that contain the most important areas of interest with removing unhelpful or important areas. Finally, the uniform Gaussian, cosine, triangular, and logistic Silverman kernel functions proved their efficiency in extracting all image features.

Subject Classification: *05C42, 11R45.*

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1. Introduction

Statistics is a science with specialized scientific methods, advanced functions, and various laws and theories that form the basis for the emergence and development of other sciences. It has proved invaluable in many applications and fields as diverse as astronomy, biology, medicine, economics, and finance. A notable approach to image processing involves using statistical methods for tasks such as shredding, optimization, sorting, and various other digital image processing techniques. In the case of segmentation image [8], considered the most important form of image processing, most researchers use several parameter statistical scales, called parameter estimates [7], to obtain estimates of a population's parameters from the sample data taken from that population. For instance, the threshold technique that relies on image segmentation (pixel-based methods) to map each image to two or more classes by determining the threshold value can depend on the shape of the image histogram. One of those methods is the Otsu way, considered one of the universal threshold methods. The second method is Niblack's thresholding technique, one of the local threshold techniques that rely on parametric statistical abilities [5].

In this paper, nonparametric kernel functions were used for image fragmentation, and the most important functions were examined in the theoretical aspect. The practical aspect was the bandwidth parameter extracted by the plug-in method and observed to be affected by sample size. After estimating the bandwidth parameter and the probability density function, the highest values of the kernel vector were used, and the threshold for image segmentation was considered via the threshold method. All the functions yielded satisfactory results, and the average mean square error (AMSE) criterion was used to compare the results. In 2013, Suhre and Cetin [2] used a kernel density estimator to segment images using the image histogram threshold and a kernel density estimator to detect the edges of images.

The rest of this paper is organized as follows. Section 2 provides the segmentation image in image processing and the types of kernel functions. Section 3 applies the appropriate kernel function using the MATLAB platform and shows the algorithm implemented with the results listed. Section 4 sets out the conclusion.

2. Theoretical aspect

2.1 *Segmentation of image by a thresholding technique*

The most important step in image processing is segmentation, which divides images into homogeneous areas. The image segmentation process aims to transform the image into more prominent and important areas by extracting information with significant and useful characteristics from the original image. One of the most important segmentation techniques is threshold technology, which uses greyscale or colour images [6]. The threshold can be used to create binary images. Segmentation relies on pixel-based methods that assign each pixel in the image to two or more classes [1].

Furthermore, these methods designate areas with grey levels above the threshold for the objects or conversely. This paper relies on the threshold technique using a parameter statistic for segmentation images. This technique is a local threshold method [2] that uses the local threshold effectively when the gradient effect is small relative to the selected sub-image size. In the local threshold technique, the threshold value T depends on the grey levels of $f(x, y)$ and some local image properties of neighbouring image units, such as mean or variation ([3, 4]). The threshold value is chosen by relying on the k value. In this case, the k value is the bandwidth parameter h, which plays a key role in determining the kernel value [13]. In particular, a low value of h makes the kernel value large, and therefore the threshold value is small.

2.2 *Kernel functions*

Two types of kernel functions [4] can be used for null estimation: kernel optimal, which reduces the mean integrated squared errors (MISE) (i.e., MISE differentiation for the kernel function and the variance minimum kernel that reduces alignment variance), and kernel state, which is a finite continuous symmetric real function whose integrity is equal to 1. To find the kernel functions with the lowest variance and the optimal kernel functions, the kernel function with the lowest variance assumes that $\{x1, x2, \ldots Xn\}$ are independent and undistributed variables and that n is the sample observations taken from society X.

Let $\{x1, x2, \ldots, xn\}$ be an independent and identically distributed (*iid*) sample of n observations taken from a population X with an unknown probability distribution function $f(x)$. Let $\hat{f}(x)$ be a kernel estimate of the original assigned to each $i-th$ sample data point x_i a function $K(x_i,h)$, called a kernel function in the following way:

$$\hat{f}_K(x)=\frac{1}{nh}\sum_{i=1}^{n}K\left(\frac{x-x_i}{h}\right) \tag{1}$$

K refers to the kernel function, which has the following properties:

$$\int K(u)du=1 \qquad \int uK(u)dx=0$$
$$\int u^2K(u)du=k_2\neq 0 \qquad \int K^2(u)du=I_2<0$$

$u=\frac{x-x_i}{h}$, h is the packet width parameter [12] and a function of the sample size. It greatly influences bias and contrast, where an increase in the preamble parameter increases the bias and decreases the variance and, conversely, affects the degree of the preamble of the estimation curve [9]. The preamble parameter [7], cited by Silverman, is elicited by using the integrated quadratic error average, which is the most widely used measure of accuracy $\hat{f}(x)$ and can be expressed as follows:

$$MISE(\hat{\hat{f}}(x))=\int E\{\hat{\hat{f}}(x)-f(x)\}2dx \tag{2}$$

The following formula obtains the bandwidth parameter [11]:

$$h=k_2^{2/5}\{\int k^2(u)du\}^{1/5}\{\int(f^2(x))^2dx\}^{-1/5}n^{-1/5} \tag{3}$$

2.2 *Kernel function choice*

Most scientific studies have shown that the choice of kernel functions is less important than the choice of bandwidth for density potentials because few kernel functions are used. The most common univariate non-normal kernel is a beta family, the best known of which is triweight and biweight, Epanechnikov and uniform. Let $k(x)$ be a function of the true value used to determine local weights of the linear weighter. It is a real value function used to determine true local weights for the condition $\int K(x)dx=1.$ Assuming h is an overall width parameter, the formula is

$$K(x,\alpha)=\frac{1}{B(0.5,\alpha+1)}(1-u^2)^{\alpha}\,I(|x|\leq 1),\ \alpha=0,1,2,3 \tag{4}$$

where $B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$, $\Gamma(a)=(a-1)!.$

Substituting α=0 gives us a uniform function, and substituting α=1,2,3 changes the beta function to Epanechnikov, biweight and triweight kernels. Moreover, when it becomes a large α, the beta function will be approximately close to the Gaussian kernel function.

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} \tag{5}$$

The most important kernel functions [5] used in the paper are listed below.

Table 1

The functions used in this paper.

Kernel Shape	K(u)	$\int u^2 K(u)du$	$\int k^2(u)du$
Uniform	$\frac{1}{2} I(\|u\| \le 1)$	$\frac{1}{3}$	$\frac{1}{2}$
Epanechnikov	$\frac{3}{4}(1-u^2) I(\|u\| \le 1)$	$\frac{1}{5}$	$\frac{3}{5}$
Biweight	$\frac{15}{16}(1-u^2)^2 I(\|u\| \le 1)$	$\frac{1}{7}$	$\frac{5}{7}$
Triweight	$\frac{35}{32}(1-u^2)^3 I(\|u\| \le 1)$	$\frac{1}{9}$	$\frac{350}{429}$
Triangular	$(1-\|u\|) I(\|u\| \le 1)$	$\frac{1}{6}$	$\frac{2}{3}$
Tricube	$\frac{70}{81}(1-\|u\|^3)^3 I(\|u\| \le 1)$	$\frac{35}{243}$	$\frac{175}{247}$
Cosine	$\frac{\pi}{4}\cos\left(\frac{\pi}{2}u\right) I(\|u\| \le 1)$	$1-\frac{8}{\pi^2}$	$\frac{\pi^2}{16}$
Gaussian	$\frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$	1	$\frac{1}{\sqrt{2\pi}}$
Logistic	$\frac{1}{e^u+e^{-u}+2}$	$\frac{\pi^2}{3}$	$\frac{1}{6}$
Sigmoid function	$\frac{2}{\pi(e^u+e^{-u})}$	$\frac{\pi^2}{4}$	$\frac{2}{\pi^2}$
Silverman kernel	$\frac{1}{2} e^{-\frac{\|u\|}{\sqrt{2}}} . \sin\left(\frac{\|u\|}{\sqrt{2}} + \frac{\pi}{4}\right)$	0	$\frac{3\sqrt{2}}{16}$

To compare between functions, the AMSE criterion [10] is used as follows:

$$AMSE = \left(\frac{\sum (y(i,j) - \hat{y}(i,j))^2}{mn} \right) \quad (6)$$

Figure 1 shows the algorithm for segmentation images using kernel functions.

1. Initially, insert the images and extract the arithmetic mean m and standard deviation s from each image.
2. Calculate $X_L = m - 4s$ and $X_U = m + 4s$ and then calculate $X_L = m - 4s$ and $= \{X_L : \frac{X_U - X_L}{n} : X_U\}$.
3. Extract the beam width value for each image.
4. Calculate $u = \frac{y - x_i}{h}$ and then estimate the probability density function for each image data using nonparametric core functions.
5. Use the pulp estimator in cutting the image K(u); if $x(i,j) >$ K(u), then y(i, j) = 1, but if $x(i,j) <$ k(u), then y(i, j) = 0.
6. Extract the cut image.
7. Extract AMSE for all functions for each image and choose the best function with the least AMSE.

The following is the research outline:

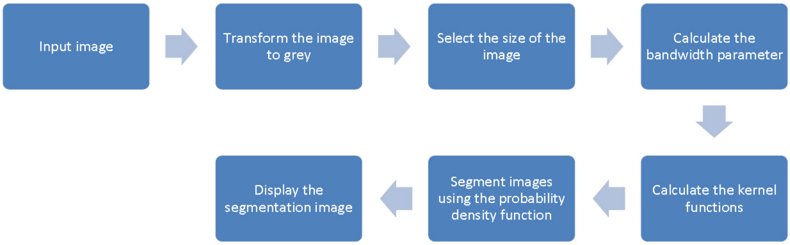

Figure 1

Flow chart of an algorithm for segmentation images using kernel functions

3. Practical aspect

Kernel functions were applied using MATLAB packages, showing the importance of using the kernel function for segmentation images. The method is applied via 11 kernel functions by taking the kernel vector's

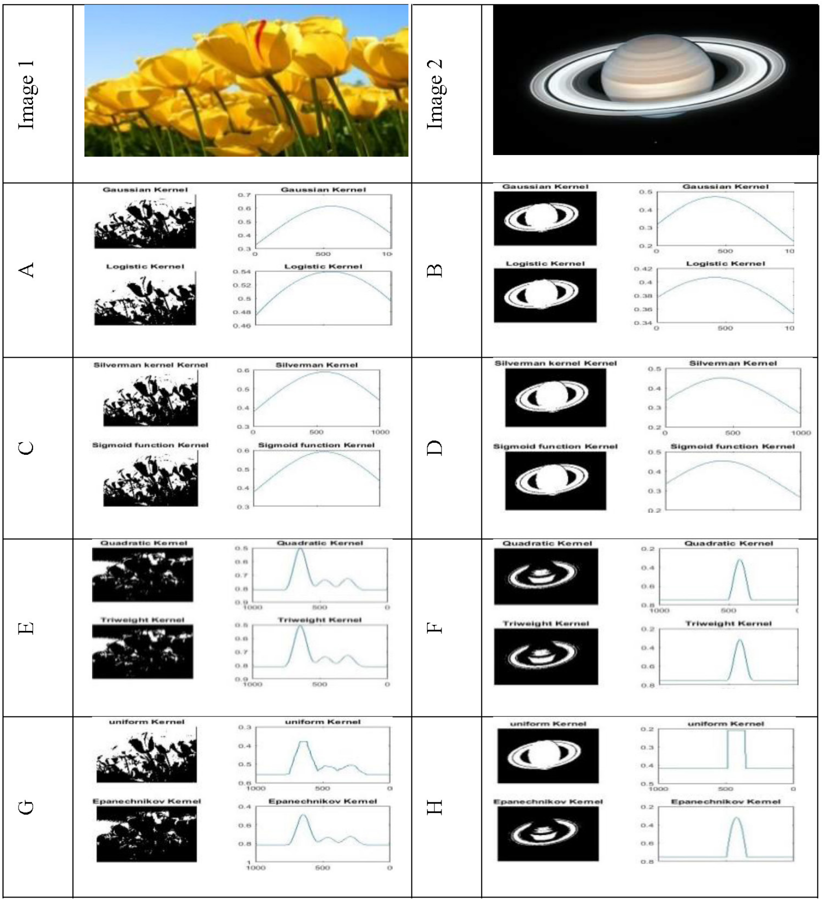

Figure 2

Image1 and Image 2 are original images, and the rest of the images are segmented using nonparametric kernel functions

largest value and considering it the threshold limit. The package parameter was extracted, as it is important in determining the direction of the function. Thus, the function depends on the sample size after the image is converted to a matrix, and then the size of the image equalized through (imsize) and taking the row as a sample size. The algorithm was implemented in two forms, and the results are illustrated below in Figure 2.

In Figure 2, the original two images (Image 1, Image 2) were converted to grey and then cut down with the kernel algorithm. We note that when a set of images are inserted, these are not shown in practice. Some functions give clear and good features in the segmentation process, and others do not. Each function tends to be closer to normal when the image has simple features, and when the image is complex, the curvature oscillates because it influences the size of the stored image. In sum, all countries showed satisfactory results in image processing but dissected images with uniform Gaussian, cosine, triangular and logistic functions were the best. These functions provided dissected images that contain important milestones while removing unimportant ones. In the results shown in Table (2), these functions are the best.

Table 2

The highest (Max) value for kernel function products and bandwidth parameter and average mean square error (AMSE) for image 1 and image 2

Functions	**Image 1**			**Image 2**		
Kernel functions	max value	bandwidth value	AMSE	max value	bandwidth value	AMAE
Uniform	0.4185	0.1658	1.0e+04 *2.5676	0.5562	0.1248	1.0e+05*1.0558
Epanechnikov	0.7533	0.1882	1.0e+04 *6.6962	0.8171	0.1416	1.0e+05*2.7717
Biweight	0.7509	0.2229	1.0e+04 *6.5491	0.8138	0.1677	1.0e+05*2.7177
Triweight	0.7497	0.2531	1.0e+04 *6.5491	0.8123	0.1904	1.0e+05*2.7177
Triangular	0.4303	0.2318	1.0e+04 *6.8488	0.5720	0.1744	1.0e+05*2.8675
Tricube	0.7604	0.2487	1.0e+04 *2.5809	0.8265	0.1871	1.0e+05*1.0393
Cosine	0.4386	0.2168	1.0e+04 *2.5205	0.5831	0.1631	1.0e+05*1.0971
Gaussian	0.4706	1.2008	1.0e+04 *2.6030	0.6139	0.9032	1.0e+05*1.0176
Logistic	0.4067	1.9157	1.0e+04 *2.6260	0.5394	1.4410	1.0e+05*0.9960
Sigmoid function	0.4520	0.0941	1.0e+04 *2.5446	0.5894	0.0708	1.0e+05*1.0614
Silverman kernel	0.4527	1.3060	1.0e+04 *2.5446	0.5910	0.9824	1.0e+05*1.0670

4. Conclusions

We used kernel to estimate the probability density function based on what was stated in the practical side and when using the kernel functions on the images used. It was noted that the functions uniform, Gaussian, cosine, triangular, logistic and Silverman kernel have given cut images containing the most important features with the removal of unimportant features. Those functions proved their efficiency in extracting all the features of images.

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