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Modified exponential ratio estimator for ranked set sampling in the presence of tie information and application on COVID-19 real data

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Abstract

In this article, we develop the isotonic ratio mean estimator proposed by Kocyigit and Kadilar (2020) within the case of tie information in the Ranked Set Sampling (RSS). In addition, we take into consideration the population size being large and small. We offer a generalized exponential ratio estimator using the modified isotonic estimator for this situation. Simulation results show that the proposed estimator is more efficient than the estimators in the literature. In the real data study, the 34 regions of China since China is the country where the pandemic started are taken as a small population, and the data of 212 countries on a world scale is taken as a large population, then the estimators are calculated. In the real data study, the proposed estimator gives better results than the existing estimators thus the results in the simulation study are similar to the real data study.

Subject Classification: *62D05, 62F03.*

Keywords: *Ranked set sampling, Exponential ratio estimator, Tie, Isotonic estimator, COVID-19.*

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1. Introduction

The RSS methodology was first deliberated by McIntyre (1952) but his study didn't give any mathematical support. Takahasi and Wakimoto (1968) appointed the mathematical theory of RSS and mentioned the perfect ranking situation. This sampling technique has continued to be developed to this day and has been used in many fields by authors such as Dell and Clutter (1972), Muttlak and Mc Donald (1990), Samawi and Muttlak(1996), Yu and Lam(1998), Al-Saleh and Al-Kadiri (2000), Muttlak and Sabah (2003), Wang et al. (2004), Kadilar et al. (2009), Bouza (2013), Singh et al. (2014), Khan and Shabbir (2017), Khan et al. (2019), Rather and Kadilar (2021), and Bhushan and Kumar (2022).

In the RSS method, each set should be ranked from the smallest to largest by using an auxiliary variable. If there are equal observations (tie) throughout the ranking process, one of these tied observations ought to be chosen randomly. There are also cases involving the personal judgments of the researcher and visual sorting. This situations cause ranking errors and effect on the estimations. To cut back on this error, Ozturk (2011) developed the partially ranking method. Later, Frey (2012) developed another method, rather than partially ranking, by using a T matrix that covers the information of those values that are equal to each other.

In the RSS-t method, the ensuing ties are recorded in the T matrix with the assistance of the indicator variables. Then, the statistics are weighted with a result of the stratification obtained via this matrix and therefore the estimations are improved, thus the efficiency of the estimators increases. In studies using the T matrix, mean and proportion estimators have been obtained (Frey 2012; Zamanzade 2018; Kocyigit and Kadilar 2020).

In contrast to the finite and small population, the samples from an infinite population can be drawn with equal probability and independently. In Frey (2011), in small populations, every observation has an inclusion probability to enter the sample, and these probabilities are not equal. Thus, the sample is not drawn randomly. Therefore, the aim of this study is to perform a simulation and real data study on both large and small populations.

This study is organized as follows: Section 2 introduces the estimators in the literature. In Section 3 we suggest the modified exponential ratio estimator for the case of tie information in the ranked set sampling. Sections 4 and 5 calculate the relative efficiencies of the estimators and compare these estimators with each other in simulation and real data respectively. Section 6 concludes and offers for future study.

2. Estimators in Literature

In this section, simple mean estimators are given first for the Simple Random Sampling (SRS) and RSS, then ratio estimators, whether or not, they use the tie information or the isotonic parts.

The basic mean estimator in RSS,

$$\hat{\mu}_1 = \frac{1}{m}\sum_{i=1}^{m}\bar{Y}_{[i]}, \tag{1}$$

where $\bar{Y}_{[i]}$ shows the sample mean of the *i*-th ordered statistics.

The simplest estimator using the tie information with the *T* matrix is defined by MacEacern et al. (2004) as follows:

$$\hat{\mu}_2 = \frac{1}{m}\sum_{i=1}^{m}\bar{Y}'_{[i]} \tag{2}$$

where $\bar{Y}'_{[i]}$ is defined as the weighted sample mean for the *i*-th stratum. The logic of this estimator is based on splitting each tied unit among the strata in T matrix.

The means of the ordered sampling statistic $\bar{Y}'_{[1]},\ldots,\bar{Y}'_{[m]}$ are expected to be in non-decreasing order. Nevertheless, this case may be broken because of RSS-associated reasons or in case of tie. Therefore, it can be made non-decreasing with the aid of using an isotonizing strategy. Wang et al (2008) suggested the isotonic mean estimator as

$$\hat{\mu}_3 = \frac{1}{m}\sum_{i=1}^{m}\bar{Y}'_{[i],iso} \tag{3}$$

where $\bar{Y}'_{[i],iso}$ is the isotonic version of the $\bar{Y}'_{[i]}$ with n'_i weighted sample sizes. Hereby, $\hat{\mu}_4$ is the isotonic versions of $\hat{\mu}_2$. These isotonic means in-stratum can be calculated easily by using the PAVA algorithm explained by Robertson et al. (1988).

The ratio estimator proposed by Prasad (1989) for SRS is defined by

$$\hat{\mu}_4 = \frac{\bar{y}}{\bar{x}}\bar{X}, \tag{4}$$

where $\bar{x}$ and $\bar{y}$ are the sample means of the auxiliary and study variables, respectively and $\bar{X}$ shows the population mean of the auxiliary variable. The estimator in (4) is adapted to the RSS by Samawi and Mutlak (1996) as follows:

$$\hat{\mu}_5 = \frac{\overline{y}_{[n]}}{\overline{x}_{[n]}} \overline{X}, \tag{5}$$

where $\overline{x}_{[n]}$ and $\overline{y}_{[n]}$ are the sample means of the auxiliary and study variables, respectively, obtained by the RSS.

The exponential ratio estimator was proposed by Bahl and Tuteja (1991) for the SRS and improved by Singh, Tailor, and Jatwa (2011) afterward Koçyiğit and Kadılar (2020) adapted that exponential ratio estimator for RSS in the presence of tie information using the isotonic estimator by Wang et al (2008) as

$$\hat{\mu}_6 = \hat{\mu}_3 \exp\left[\frac{\overline{X}_t - \hat{\mu}_{3[xt]}}{\overline{X}_t + \hat{\mu}_{3[xt]}}\right] \tag{6}$$

3. Proposed Estimators

We define the proposed ratio estimator for the case of tie information in the ranked set sampling motivated by Singh et al (2009) and Kocyigit and Kadilar (2020), as follows:

$$\hat{\mu}_7 = \hat{\mu}_3 \exp\left[\frac{(\alpha\overline{X}_t + \beta) - (\alpha\hat{\mu}_{3[xt]} + \beta)}{(\alpha\overline{X}_t + \beta) + (\alpha\hat{\mu}_{3[xt]} + \beta)}\right] \tag{7}$$

where $\hat{\mu}_3$ is the isotonic mean estimator proposed by Wang et al (2008), $\hat{\mu}_{3[xt]}$ is also the isotonic mean estimator of the auxiliary variable X_t and $\overline{X_t}$ is the population mean of X_t that includes tie information. α and β can be defined as 1, kurtosis of X_t, coefficient of variation or correlation coefficient. The proposed estimator in (7) is simply the generalization of $\hat{\mu}_6$ and combines the ratio estimator with the isotonic estimators. Also, if α=1 and β=0, the proposed estimator turns to the isotonic exponential ratio estimator $\hat{\mu}_6$ proposed by Koçyiğit and Kadılar (2020). Therefore, the generalized exponential ratio estimator gives at least as good results as it includes the exponential ratio estimator.

4. Simulation Study

There are two ways to simulate ties in data, which are TIC and DPS. DPS method was used in this study. The DPS method generally divides the values in the dataset by a specified value of e and rounds down the result. Although this method is easy to implement, it also comes with disadvantages. Frey (2012) examined these disadvantages, which may occur depending on the *e* and *m* values. Furthermore, when this method was used for simulating ties in RSS, Koçyiğit and Kadılar (2020) stated that as the value of *e* increases, the correlation between the auxiliary variable and the study variable decreases.

In Figure 1, it can be seen that the correlation decreases as "e" increases, as indicated in the literature. However, when examining a relatively small population, it has been observed that the correlation tends to decrease overall, but after a point, it does not follow a certain pattern as in the large population, as shown in Figure 2.

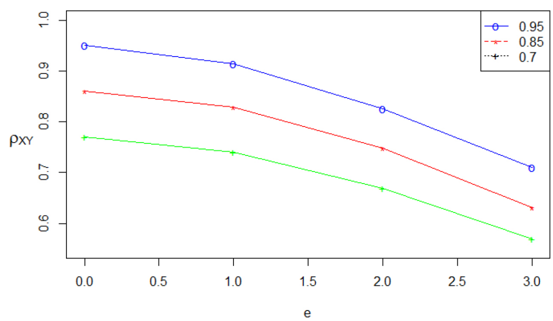

Figure 1

Correlation coefficient changes according to *e* when population size is large enough (N = 1000)

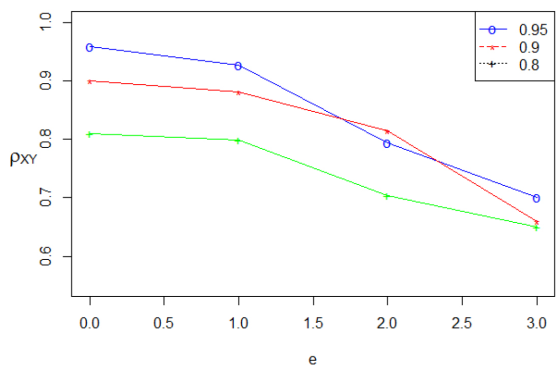

Figure 2

Correlation coefficient changes according to *e* when population size is small (N = 50)

From Frey's (2011) inclusion probability study, we know that RSS works differently in large and small populations. If there is a tie in the population, when we increase the number of ties by increasing the value of e, we see that the initial correlation coefficient of the population deteriorates more in a small population from Figures 1 and 2.

In the simulation coded by the R, the first population size of N=1000 and second population size of N=50 are derived from the bivariate normal distributions. Parameters are selected as ρ = 0.75, 0.85, and 0.95; m = 3, 4, and 5. The model was developed under the DPS method and the mean estimators were obtained for the number of trials of 100000. In order to see in which cases the proposed estimator $\hat{\mu}_7$ is better than $\hat{\mu}_6$, the Normal Distribution parameters were chosen as 5 and 1 in the same way as Koçyiğit and Kadılar (2020).

The relative efficiency values, where the reference estimator is $\hat{\mu}_1$, are calculated by proportioning the mean square errors as follows:

$$RE_i = \frac{MSE(\hat{\mu}_1)}{MSE(\hat{\mu}_i)}, \; i = 0,2,3,4,5,6,7$$

RE_0 shows efficiency of simple mean estimator for simple random sampling against $\hat{\mu}_1$.

The RE results for large population and small population are given in Tables 1 and 2, respectively.

Table 1

REs for N(5,1) and N=1000.

DPS	ρ_{XY}	**0.75**			**0.85**			**0.95**		
$X_t = \lfloor X / e \rfloor$	m	**3**	**4**	**5**	**3**	**4**	**5**	**3**	**4**	**5**
e=1	RE_0	0.7748	0.7094	0.6805	0.6998	0.6367	0.5921	0.6322	0.5571	0.4973
	RE_2	1.0236	1.0235	1.0291	1.0434	1.0517	1.065	1.0892	1.1201	1.1403
	RE_3	1.0673	1.0689	1.0748	1.0868	1.0989	1.1107	1.1301	1.1632	1.1842
	RE_4	1.125	1.0635	1.0331	1.5471	1.4597	1.3639	2.57	2.3116	2.0921
	RE_5	1.3412	1.2897	1.2882	1.116	1.7239	1.6376	**3.0204**	**2.7691**	**2.5933**
	RE_6	**1.5873**	**1.4779**	1.2437	1.9365	1.7964	1.6914	2.8058	2.5872	2.428
	RE_7	1.5842	1.4757	**1.4217**	**1.9632**	**1.8167**	**1.7069**	2.7975	2.5809	2.423

Contd...

e=2	RE_0	0.855	0.8263	0.7866	0.814	0.7759	0.7538	0.7747	0.7197	0.6835
	RE_2	1.0794	1.0819	1.0843	1.1009	1.1088	1.1132	1.1682	1.1822	1.1829
	RE_3	1.1197	1.1218	1.1224	1.1374	1.147	1.1504	1.1997	1.2173	1.2206
	RE_4	0.6276	0.6396	0.6317	0.7661	0.7708	0.7649	0.9299	0.9125	0.9014
	RE_5	0.8278	0.8682	0.8775	1.007	1.0274	1.0468	1.2404	1.2694	1.2776
	RE_6	1.4635	1.4187	1.3837	**1.6886**	1.6228	1.5753	**2.1764**	**2.0579**	**1.9727**
	RE_7	**1.4744**	**1.4264**	**1.3913**	**1.6886**	**1.6229**	**1.5754**	2.1736	2.0555	**1.9706**
e=3	RE_0	0.9265	0.8987	0.8901	0.8942	0.8769	0.8619	0.8798	0.8517	0.8288
	RE_2	1.0664	1.0669	1.0718	1.0911	1.0949	1.0994	1.1316	1.1376	1.139
	RE_3	1.0985	1.0971	1.099	1.1216	1.1234	1.1266	1.1636	1.1698	1.1698
	RE_4	0.3979	0.4107	0.4118	0.0001	0.4029	0.4058	0.0001	0.4573	0.4537
	RE_5	0.471	0.5041	0.5293	0.4633	0.5019	0.5321	0.5386	0.5695	0.602
	RE_6	1.1717	1.1609	1.1585	1.2547	1.2368	1.2381	1.5019	1.4625	1.4488
	RE_7	**1.2980**	1.2751	1.2635	1.3831	1.3551	1.3422	1.5971	1.5527	1.527

Table 2

REs for N(5,1) and N=50.

DPS $X_t = \lfloor X/e \rfloor$	ρ_{XY}	**0.75**			**0.85**			**0.95**		
	m	**3**	**4**	**5**	**3**	**4**	**5**	**3**	**4**	**5**
e=1	RE_0	0.7819	0.7391	0.6972	0.6378	0.5602	0.4899	0.6078	0.5153	0.4534
	RE_2	0.9892	0.9881	0.9877	1.0792	1.1084	1.1248	1.1393	1.2066	1.2769
	RE_3	1.0434	1.0513	1.0563	1.1229	1.1573	1.1782	1.1821	1.2562	1.329
	RE_4	0.8348	0.795	0.7596	1.9541	1.7414	1.5412	3.71	3.2115	2.8281
	RE_5	1.0588	1.0838	1.0819	2.3091	2.1111	1.9187	**3.9463**	**3.4682**	**3.0244**
	RE_6	1.3918	**1.3143**	**1.2563**	2.32	2.0841	1.8771	2.7571	2.5709	2.4038
	RE_7	**1.3921**	1.3142	1.2556	**2.4614**	**2.1959**	**1.9526**	2.7411	2.5591	2.3955
e=2	RE_0	0.8191	0.7831	0.7396	0.7741	0.7251	0.6756	0.8014	0.7474	0.7069
	RE_2	1.0748	1.0838	1.0889	1.1631	1.1735	1.1857	1.1921	1.2053	1.2072
	RE_3	1.1226	1.1358	1.1397	1.1971	1.2172	1.2348	1.227	1.2465	1.2543
	RE_4	0.4643	0.4693	0.4624	0.8607	0.8444	0.8086	1.1244	1.107	1.0765
	RE_5	0.6768	0.7294	0.7555	1.1357	1.1769	1.1597	1.3851	1.3994	1.3702
	RE_6	1.4659	1.4155	1.3465	2	1.8804	1.7806	1.9262	1.8338	1.7364
	RE_7	**1.5072**	**1.4453**	**1.3704**	**2.0026**	**1.8824**	**1.7814**	**1.9462**	**1.8499**	**1.7475**
e=3	RE_0	0.9224	0.8816	0.8897	0.9197	0.8834	0.8613	0.9096	0.8918	0.8698
	RE_2	0.9961	1.0002	1.0104	1.1153	1.1179	1.1204	1.2076	1.2143	1.2216
	RE_3	1.0449	1.0455	1.0503	1.1459	1.1501	1.1534	1.2312	1.2395	1.2478
	RE_4	0.3746	0.3554	0.3463	0.4392	0.4436	0.4352	0.7228	0.7091	0.6945
	RE_5	0.4344	0.4491	0.4846	0.4987	0.5228	0.5471	0.7802	0.8051	0.8149
	RE_6	1.005	0.9918	1.0038	1.3216	1.2948	1.2653	1.567	1.5428	1.5132
	RE_7	**1.1988**	**1.171**	**1.1532**	**1.4296**	**1.3981**	**1.3641**	**1.5986**	**1.5736**	**1.5466**

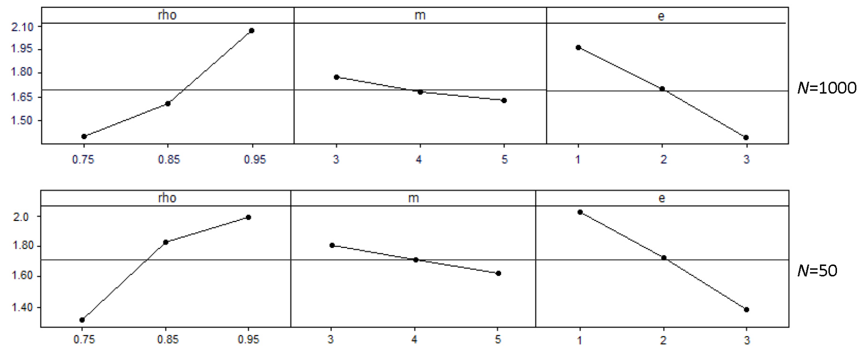

Figure 3

Graphs of REs according to the correlation coefficients, set sizes and *e* levels for large and small population

According to the simulation results, given in Figure 3, which are MINITAB main effects plots, while the correlation coefficient increases and while the set sizes of m and e values decrease, the relative efficiency values increase as the similar to the literature.

5. Real COVID-19 Data Results

The coronavirus (COVID 19), which started in China in late 2019 and hit the world in no time, is the fastest and most difficult respiratory virus to control. Since the disease became a global crisis, it began to be used and examined frequently in current statistical studies, such as Sing et al. (2020), Bhatnagar et al. (2021). In this section, we consider a small real data population to calculate the MSE values of the estimators by confirmed and suspected patients of COVID-19 acute respiratory disease reported by regions in China, harmoniously Koçyiğit and Kadılar (2020). In the simulation coded by the R and set size selected as $m = 3$, 4, and 5. The mean estimators were obtained for the number of trials of 100000. The relative efficiency values are calculated, as given in Section 4. For the proposed estimator $\hat{\mu}_7$, α and β are defined as kurtosis of the auxiliary variable and 1, respectively. The RE results for the small population data are given in Table 3.

Table 3

REs for China COVID-19 data set (N = 34, ρ = 0.84)

	m=5	m=4	m=3
RE_0	1.0644	1.1437	1.2053
RE_2	1.4639	1.5829	1.6697
RE_3	1.4641	1.5829	1.6696
RE_4	0	0	0
RE_5	0	0	0
RE_6	2.53	2.9016	3.6516
RE_7	**2.54**	**2.93**	**3.67**

As the second real population, the number of confirmed and suspected patients revealed by the EU Open Data Portal (EU Open data Portal) for 14 Dec 2020 from all countries within the world was used (Data Europa, 2020) which is given in Appendix 1. Suppose we tend to are attempting to estimate the number of patients who die from the disease in the given population. it has been observed that there's a positive high correlation between the number of cases caught and the number of patients who died. Therefore, the number of confirmed cases in the last twenty-four hours was used as an auxiliary variable. It will be seen within the bar graph in Figure 4 that there are ties on the auxiliary variable in this population too. For example, according to the chart provided, from the frequency value of the no patients column, there are 60 countries in the world with no confirmed cases in the last twenty-four hours. The parameters of the population are given in Table 4.

Table 4

Global COVID-19 data set parameters

N=212	β_{XT} = 166.6721	$\bar{X}_T$ = 2550.278
ρ_{XTY} = 0.923 (p-value<0.05)		$\bar{Y}$ = 33.4198

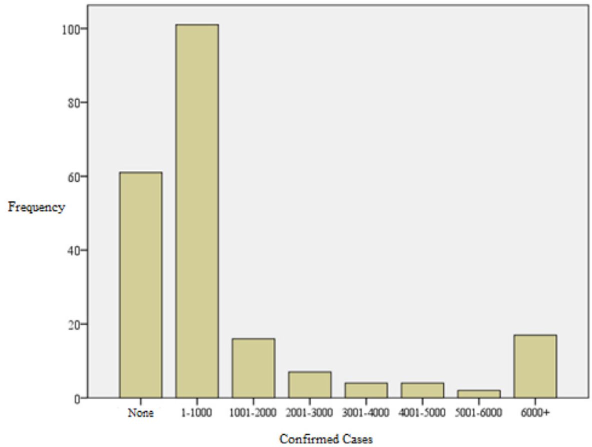

Figure 4

Histograms of auxiliary variable number of confirmed patients which has tie in global COVID-19 data set

All calculations were made in the same way as for China COVID-19 data, and RE value results are given in Table 5.

Table 5

Relative efficiencies for Global COVID-19 data set (N = 212, ρ = 0.92)

	$m = 5$	$m = 4$	$m = 3$
RE_0	0.8202	0.8516	0.8896
RE_2	1.0055	1.0103	1.0265
RE_3	1.0052	1.01	1.0262
RE_4	-	-	-
RE_5	1.9067	-	-
RE_6	**3.6518**	**3.678**	**3.8617**
RE_7	**3.6518**	**3.678**	**3.8617**

6. Conclusion

The proposed estimator has been seen as the most effective estimator altogether in simulation and real data studies, the details of that are given within the previous sections. Though the proposed estimator may be a ratio estimator, it additionally offers effective results in low correlations. Whereas the efficiencies of other estimators were inconsistent with respect to correlation, set size, or the number of ties, no spikes or decreases were observed in the effectiveness of the proposed estimator. It has been calculated as the most effective estimator in real data studies. This is a result of the number of ties in the actual data is very high.

It is observed that the proposed estimator is more efficient than the isotonic exponential ratio estimator by Koçyiğit and Kadılar (2020). As in the literature, ratio estimators perform poorly when correlation decreases and the number of links increases. However, the proposed estimator, despite being a ratio estimator, gives better results at low correlations. In cases where ratio estimators cannot be calculated, such as when working with zero-average variables or in cases where the auxiliary variable can be zero as real data sets, the proposed estimator gives realistic results.

Small population simulation results were similar with COVID-19 data. Proposed estimators are not affected by population size. As the e value changes in the small population, the correlation coefficient changes more than that in the large population. However, despite this inconsistency in the correlation coefficient, it did not affect the relative efficiency of the proposed estimator. It is observed that other ratio estimators deteriorate as the correlation decreased in the small population.

It has been seen that the proposed generalized estimator is better than all the estimators in the literature if the population is small in the real data study. However, when we consider the large population, the proposed estimator gives the same result as the Kocyigit and Kadilar (2020) estimator. This is already expected since the proposed estimator is a generalized version of the other estimator in the literature. It is noteworthy, however, that these estimators are equal when studied on large populations. In addition, Frey's (2012) information, which RSS samples drawn from small and large populations give different results, has been observed in real data studies.

In order for the DPS method to be applied, it is sufficient to select the number of "e" determined other than zero. However, there is no other option to determine "e" value, and this value that causes undesirable behavior should be chosen carefully. A method can be developed to facilitate the selection of "e" in future studies. In addition, it is remarkable that there is a correlation between the variables even in cases where the population is taken as small or large in the real data related to Covid-19.

Appendix 1

Global COVID-19 Data Set

No	State/ Country	Confirmed Cases	Total Death
1	Afghanistan	746	6
2	Albania	788	14
3	Algeria	464	12
4	Andorra	50	1
5	Angola	27	5
6	Anguilla	0	0
7	Antigua_and_Barbuda	0	0
8	Argentina	3558	98
9	Armenia	357	17
10	Aruba	5	0
11	Australia	6	0
12	Austria	3005	45
13	Azerbaijan	4451	39
14	Bahamas	15	0
15	Bahrain	178	0
16	Bangladesh	1355	32
17	Barbados	0	0
18	Belarus	1961	9
19	Belgium	2	6
20	Belize	122	3
21	Benin	0	0
22	Bermuda	40	0
23	Bhutan	1	0
24	Bolivia	120	4
25	Bonaire, Saint Eustatius and Saba	0	0
26	Bosnia_and_Herzegovina	811	38
27	Botswana	0	0
28	Brazil	21825	279
29	British_Virgin_Islands	0	0
30	Brunei_Darussalam	0	0

Contd...

31	Bulgaria	497	62
32	Burkina_Faso	136	0
33	Burundi	1	0
34	Cambodia	0	0
35	Cameroon	0	0
36	Canada	5891	81
37	Cape_Verde	55	0
38	Cayman_Islands	0	0
39	Central_African_Republic	0	0
40	Chad	19	0
41	Chile	2138	40
42	China	12	0
43	Colombia	8702	187
44	Comoros	0	0
45	Congo	0	0
46	Costa_Rica	0	0
47	Cote_dIvoire	41	0
48	Croatia	3363	78
49	Cuba	69	0
50	CuraÃ§ao	48	1
51	Cyprus	301	1
52	Czechia	2000	74
53	Democratic_Republic_of_the_ Congo	52	4
54	Denmark	2642	6
55	Djibouti	3	0
56	Dominica	0	0
57	Dominican_Republic	1107	1
58	Ecuador	586	1
59	Egypt	486	22
60	El_Salvador	486	5
61	Equatorial_Guinea	0	0
62	Eritrea	0	0
63	Estonia	342	1
64	Eswatini	54	0

Contd...

65	Ethiopia	472	3
66	Falkland_Islands_(Malvinas)	0	0
67	Faroe_Islands	3	0
68	Fiji	2	0
69	Finland	360	0
70	France	11533	150
71	French_Polynesia	0	0
72	Gabon	0	0
73	Gambia	2	0
74	Georgia	1337	49
75	Germany	16362	188
76	Ghana	81	0
77	Gibraltar	3	0
78	Greece	692	85
79	Greenland	0	0
80	Grenada	26	0
81	Guam	16	1
82	Guatemala	183	18
83	Guernsey	0	0
84	Guinea	0	0
85	Guinea_Bissau	0	0
86	Guyana	41	1
87	Haiti	74	1
88	Holy_See	0	0
89	Honduras	316	1
90	Hungary	3470	165
91	Iceland	5	0
92	India	27071	336
93	Indonesia	6189	166
94	Iran	7451	247
95	Iraq	1012	14
96	Ireland	429	1
97	Isle_of_Man	0	0
98	Israel	1844	16
99	Italy	17937	484

Contd...

100	Jamaica	102	2
101	Japan	2366	23
102	Jersey	91	0
103	Jordan	2339	30
104	Kazakhstan	764	0
105	Kenya	366	1
106	Kosovo	462	12
107	Kuwait	174	0
108	Kyrgyzstan	318	5
109	Laos	0	0
110	Latvia	629	25
111	Lebanon	1275	10
112	Lesotho	0	0
113	Liberia	0	0
114	Libya	899	21
115	Liechtenstein	12	0
116	Lithuania	1920	10
117	Luxembourg	517	4
118	Madagascar	0	0
119	Malawi	3	1
120	Malaysia	1229	4
121	Maldives	20	0
122	Mali	93	5
123	Malta	0	0
124	Marshall_Islands	0	0
125	Mauritania	0	0
126	Mauritius	0	0
127	Mexico	8608	249
128	Moldova	795	25
129	Monaco	0	0
130	Mongolia	5	0
131	Montenegro	478	4
132	Montserrat	0	0
133	Morocco	2012	35
134	Mozambique	142	2

Contd...

135	Myanmar	1127	23
136	Namibia	267	0
137	Nepal	830	9
138	Netherlands	9868	29
139	New_Caledonia	0	0
140	New_Zealand	0	0
141	Nicaragua	0	0
142	Niger	59	0
143	Nigeria	418	3
144	North_Macedonia	613	25
145	Northern_Mariana_Islands	0	0
146	Norway	0	0
147	Oman	571	8
148	Pakistan	2362	36
149	Palestine	1639	17
150	Panama	2422	25
151	Papua_New_Guinea	5	0
152	Paraguay	669	16
153	Peru	1928	68
154	Philippines	1069	3
155	Poland	8976	188
156	Portugal	4044	98
157	Puerto_Rico	1104	6
158	Qatar	0	0
159	Romania	4435	121
160	Russia	28080	488
161	Rwanda	131	0
162	Saint_Kitts_and_Nevis	0	0
163	Saint_Lucia	1	0
164	Saint_Vincent_and_the_ Grenadines	4	0
165	San_Marino	15	0
166	Sao_Tome_and_Principe	0	0
167	Saudi_Arabia	139	12
168	Senegal	78	0

Contd...

169	Serbia	4995	56
170	Seychelles	0	0
171	Sierra_Leone	0	0
172	Singapore	7	0
173	Sint_Maarten	0	0
174	Slovakia	5897	53
175	Slovenia	833	11
176	Solomon_Islands	0	0
177	Somalia	0	0
178	South_Africa	7999	170
179	South_Korea	718	7
180	South_Sudan	0	0
181	Sri_Lanka	655	3
182	Sudan	239	3
183	Suriname	4	0
184	Sweden	0	0
185	Switzerland	0	0
186	Syria	125	12
187	Taiwan	4	0
188	Tajikistan	38	0
189	Thailand	28	0
190	Timor_Leste	0	0
191	Togo	30	0
192	Trinidad_and_Tobago	15	0
193	Tunisia	968	58
194	Turkey	29136	222
195	Turks_and_Caicos_islands	0	0
196	Uganda	461	1
197	Ukraine	15627	249
198	United_Arab_Emirates	1194	8
199	United_Kingdom	18447	144
200	United_Republic_of_Tanzania	0	0
201	United_States_of_America	189723	1340
202	United_States_Virgin_Islands	0	0

Contd...

203	Uruguay	528	1
204	Uzbekistan	138	0
205	Vanuatu	0	0
206	Venezuela	462	5
207	Vietnam	2	0
208	Wallis_and_Futuna	0	0
209	Western_Sahara	0	0
210	Yemen	0	0
211	Zambia	57	1
212	Zimbabwe	27	0

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