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Joint determination of quality investment and tolerance under the sampling inspection plan with quality protection

Chung-Ho Chen *
Tzu-Hsien Lee
Department of Industrial Management and Information
Southern Taiwan University of Science and Technology
Tainan
Taiwan (R.O.C.)

Abstract

In this study, we present the joint design for process mean, standard deviation, and tolerance. In the first phase, Taguchi's quality loss and process adjustment cost are used. In the second phase, the quality investment and sampling inspection are addressed with quality protection under the specified capability value and consumer's risk.

Subject Classification: *(2010) 60 Probability theory and stochastic processes.*

Keywords: *Process mean, Standard deviation, Tolerance, Quality investment, Process capability index, Taguchi's quality loss.*

1. Introduction

For the production process, we usually consider the process mean and standard deviation. They affect the bias and variability of product. One needs to control them in order to have the optimal output of product. One can adopt Taguchi's (1986) quality cost function for measuring the performance of quality. One of Huang's (2001) process adjustment cost considers that it is concerned with square of mean and reciprocal to the variability.

In 2010, Shin et al.'s process parameters setting model addresses Taguchi's quality and process costs. The tolerance design considers the trade-off problem for decreasing the probability of non-conforming product and increasing the manufacturing cost. In Shin et al.'s (2010)

**E-mail:* `chench@stust.edu.tw`

tolerance model, the product with 100% inspection considers quality loss, rejection, and tolerance costs. The tolerance cost considers for shortening the specification interval of product. From Case (1990) and Jeang (1994), the tolerance cost considers the exponential function of product tolerance.

Quality investment can be used for quality improvement. Tsou (2006), Chen and Tsou (2003), Ganeshan et al. (2001), and Hong et al. (1993) addressed it as the function of exponential reduction for mean and variability. The capability index C_{pmk} can be used for measuring the optimal value of product. Hence, the setting of specified C_{pmk} value can assure the customer's constrained loss.

Chen and Chou (2020) addressed the optimal process target setting problem by adopting rectifying sampling plan under the specified capability value. Their model wants to maximize the expected total profit that satisfies the specified consumer's risk and value of capability index. The product inspection under the specified tolerance can be used for assuring the shipment quality of product. In this work, we propose the modified Shin et al.'s (2010) model with quality investment, specified C_{pmk} value, and consumer's risk for product with quality protection. The optimal decision variables are to get the quality investment, tolerance, and parameters of sampling plan.

2. Modified Shin et al.'s Model

2.1 *Notations*

k	coefficient of quality loss
μ	unknown process mean
σ	unknown standard deviation
M	manufacturing target of process
C_1	coefficient of μ^2 in the process cost
C_2	coefficient of $1/\sigma^2$ in the process cost
$E[TC_1(\mu,\ \sigma)]$	expected total cost without process inspection
$E[TC_2(Inv,\delta,n,d_0)]$	expected total cost with sampling inspection
$\phi(\cdot)$	probability density function (p.d.f.) of standard normal distribution
$\Phi(\cdot)$	cumulative distribution function of standard normal distribution
C_R	rejection cost per unit

a_2 constant in the tolerance cost

a_3 coefficient of the tolerance range in the tolerance cost

LSL lower limit of product, $LSL = \mu_0 - \delta\sigma_0$

USL upper limit of product, $USL = \mu_0 + \delta\sigma_0$

μ_0 known value for process mean in the phase I

σ_0 known value for standard deviation in the phase I

Inv quality investment

μ_1 improved process mean through quality investment

σ_1 improved standard deviation through quality investment

β_1 exponential declined coefficient of process mean through quality investment

α_1 exponential declined coefficient of standard deviation through quality investment

Y quality characteristic of product

$f(y)$ p.d.f. of Y without quality investment,

$$f(y) = \frac{1}{2\sqrt{\pi}\sigma}\exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right), \quad -\infty < y < \infty$$

$f(y, Inv)$ p.d.f. of Y with quality investment,

$$f(y, Inv) = \frac{1}{2\sqrt{\pi}\sigma_1}\exp\left(-\frac{1}{2}\left(\frac{y-\mu_1}{\sigma_1}\right)^2\right), \quad -\infty < y < \infty$$

σ_T target value of standard deviation

δ coefficient of tolerance

I_c inspection cost per unit

C_{pmk} process capability index

K_m specified C_{pmk} value

β customer's risk

n sample size

X number of non-conformance

d_0 clearance number, the lot is accepted when $X \le d_0$

N lot size

$P(X \le d_0)$ accepted probability of inspected lot, $P(X \le d_0) = 1 - P(X > d_0)$

$LTPD$ lot tolerance percent defective

$P_a(p = LTPD)$ the accepted probability for lot with non-conforming probability $p = LTPD$.

2.2 *Assumptions*

1. The characteristic of quality is normal distribution.
2. The quality measurement adopts quadratic quality loss function.
3. The declining exponential reduction for mean and variability is assumed for the quality investment function.
4. The symmetric specification limits are considered.
5. Consider C_{pmk} value as the quality protection when one adopts the sampling inspection in the quality investment and specification limits settings model.

2.3 *Phase I--- Process Parameters Setting*

According to Shin et al. (2010) and Huang (2001), the modified process parameters setting model is as follows:

$$\text{Minimize} \quad E[TC_1(\mu,\ \sigma)] = \int_{-\infty}^{\infty} k(y-M)^2 f(y)dy + C_1\mu^2 + C_2\frac{1}{\sigma^2}$$

$$= k[(\mu - M)^2 + \sigma^2] + C_1\mu^2 + C_2\frac{1}{\sigma^2} \tag{1}$$

Set the first derivative for process mean and standard deviation as zero. We obtain the optimal $\mu_0 = \frac{kM}{k+C_1}$ and $\sigma_0 = \sqrt[4]{\frac{C_2}{k}}$. The specified values of μ_0 and σ_0 are the input values of phase II.

2.4 *Phase II--- Quality Investment and Specification Limits Settings Model*

All units are inspected when the lot is rejected. If the inspected unit is the defective, then it will be rejected and replaced by conforming one. The expected total cost of phase II considers the quality loss of the accepted lot, quality loss of the rejected lot, rejection cost, manufacturing cost, inspection cost, and quality investment. The specified K_m and β are applied for assuring the shipment quality of product. From Shin et al. (2010), Chase (1990), and Jeang (1994), the modified tolerance setting model is as follows:

Minimize

$$
\begin{aligned}
&E[TC_2(Inv,\ \delta, n, d_0)] \\
&= \Big\{[nI_c + npC_R + (N-n)LF_0 + nLF_1]P(X \le d_0) + [NI_c + NpC_R \\
&\quad + NLF_1]P(X > d_0) + \left[a_2 \exp\left(-a_3 \frac{USL-LSL}{2}\right)\right]N + Inv\Big\} / N \\
&= \Big\{[(N-n)LF_0 + nLF_1]P(X \le d_0) + NLF_1 P(X > d_0) \\
&\quad + NC_R\left[1 - \Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) + \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right] \cdot [N - (N-n)P(X \le d_0)] \\
&\quad + a_2 \exp(-a_3 \delta\sigma_0)N + Inv + [NI_c - I_c(N-n)P(X \le d_0)]\Big\} / N \\
&= \left[\left(1-\frac{n}{N}\right)LF_0 + \frac{n}{N}LF_1\right]P(X \le d_0) + LF_1 \cdot P(X > d_0) \\
&\quad + C_R\left[1 - \Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) + \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\left[1 - \left(1-\frac{n}{N}\right)P(X \le d_0)\right. \\
&\quad + a_2 \exp(-a_3 \delta\sigma_0) + \frac{Inv}{N} + \left[I_c - I_c\left(1-\frac{n}{N}\right)P(X \le d_0)\right] \qquad (2)
\end{aligned}
$$

subject to

$$LSL = \mu_0 - \delta\sigma_0 \qquad (3)$$

$$USL = \mu_0 + \delta\sigma_0 \qquad (4)$$

$$f(y, Inv) = \frac{1}{\sqrt{2\pi}\sigma_I} \exp\left(-\frac{1}{2}\left(\frac{y-\mu_1}{\sigma_1}\right)^2\right), \quad LSL < y < USL \qquad (5)$$

$$\mu_1^2 = M^2 + (\mu_0^2 - M^2)\exp(-\beta_1 Inv) \qquad (6)$$

$$\sigma_1^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha_1 Inv) \qquad (7)$$

$$LF_0 = \int_{-\infty}^{\infty} k(y-M)^2 f(y, Inv)dy = k[(\mu_1 - M)^2 + \sigma_1^2] \qquad (8)$$

$$P_a(p = LTPD) = \beta \qquad (9)$$

$$C_{pmk} = \left(1 - \frac{|\mu_1 - M|}{\frac{USL-LSL}{2}}\right)\frac{USL-LSL}{6\sqrt{(\mu_1-M)^2 + \sigma_1^2}} = \frac{(\delta\sigma_0 - |\mu_1 - M|)}{3\sqrt{(\mu_1-M)^2 + \sigma_1^2}} = K_m \qquad (10)$$

$$LF_1 = \frac{\int_{LSL}^{USL} k(y-M)^2 f(y,\ Inv)dy}{\int_{LSL}^{USL} f(y,\ Inv)dy}$$

$$= \{k[(\mu_1 - M)^2]\left[\Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) - \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]$$

$$-2k\sigma_1(\mu_1 - M)\left[\phi\left(\frac{USL-\mu_1}{\sigma_1}\right) - \phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]$$

$$-k\sigma_1^2\left[\left(\frac{USL-\mu_1}{\sigma_1}\right)\phi\left(\frac{USL-\mu_1}{\sigma_1}\right) - \left(\frac{LSL-\mu_1}{\sigma_1}\right)\phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right.$$

$$\left.\left. -\Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) + \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right]\right\} \Big/ \left[\Phi\left(\frac{USL-\mu_1}{\sigma_1}\right) - \Phi\left(\frac{LSL-\mu_1}{\sigma_1}\right)\right] \quad (11)$$

One needs to obtain the combination $(Inv,\ \delta,\ n,\ d_0)$ with minimum value of $E[TC_2(Inv,\delta,\ n,\ d_0)]$ under the K_m and β. There are five steps for solution:

Step 1. Set the upper value d_0 as $d_{\max}$ and upper value *Inv* as $Inv_{\max}$. Set the initial $Inv = 0$ and $d_0 = 0$.

Step 2. For the given *LTPD*, β, and d_0, we can obtain the sample size satisfying Eq. (9).

Step 3. One can compute the corresponding δ satisfying Eq. (10) and $E[TC_2(Inv,\delta,\ n,\ d_0)]$ from Eq. (2).

Step 4. Let $Inv = Inv + 1$, repeat Step 3 until $Inv = Inv_{\max}$.

Step 5. Let $d_0 = d_0 + 1$, repeat Step 2 until $d_0 = d_{\max}$. The optimal solution is the combination $(Inv,\ \delta,\ n,\ d_0)$ with minimum value of Eq. (2).

3. Numerical Example

Assume some parameters with $k = 200$, $M = 50$, $C_1 = 2$, $C_2 = 6$, $a_2 = 2$, $a_3 = 30$, $C_R = 50$, $\alpha = 0.5$, $\beta = 0.1$, $\sigma_T = 0.25$, $K_m = 1$, and $I_c = 1$.

The optimal solution of Eq. (1) is $\mu_0 = 49.505$ and $\sigma_0 = 0.416$. By solving Eqs. (2)-(11), the optimal solution for sampling inspection is $\mu_1 = 49.818$, $\sigma_1 = 0.252$, $Inv = 9.97$, $\delta = 2.68$, $d_0 = 0$, $n = 46$, and $E[TC_2(Inv,\delta,d_0,n) = 27.520$.

The numerical result of sensitivity analysis is listed. According to Table 1, one has some results as follows:

1. The parameters, β_1 and K_m, have a significant effect for quality investment.
2. The parameters, k, σ_T, and K_m, have a significant effect for coefficient of tolerance.
3. The parameters, k, C_R, σ_T, K_m, $LTPD$, β, have a significant effect for expected total cost.
4. The parameters, K_m, $LTPD$, and β, have a significant effect for parameters of sampling plan.

4. Conclusions

This paper have proposed a modified Shin et al.'s (2010) work considering quality investment, specified C_{pmk} value, and consumer's risk for quality protection with sampling inspection. Numerical results show that the quality loss coefficient, the exponential declined coefficient for the process mean through quality investment, and the target value of standard deviation have a significant effect for optimal solution. Further study should consider economic production quantity for formulating the integrated inventory model.

Table 1

Sensitivity analysis of parameters for numerical example

C_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
2.4	49.407	0.416	49.794	0.251	10.53	2.84	29.466	(0, 46)
1.6	49.603	0.416	49.843	0.252	9.28	2.52	25.771	(0, 46)
C_2	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
7.2	49.505	0.436	49.818	0.252	9.98	2.56	27.546	(0, 46)
4.8	49.505	0.394	49.818	0.251	9.96	2.83	27.491	(0, 46)
k	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
240	49.587	0.398	49.843	0.252	9.63	2.63	29.561	(0, 46)
160	49.383	0.440	49.784	0.251	10.45	2.75	25.646	(0, 46)
T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
51	50.495	0.416	50.816	0.251	10.04	2.69	27.708	(0, 46)

Contd...

49	48.515	0.416	48.820	0.252	9.91	2.66	27.334	(0, 46)
C_R	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
60	49.505	0.416	49.814	0.252	9.78	2.70	28.715	(0, 46)
40	49.505	0.416	49.822	0.251	10.22	2.65	26.264	(0, 46)
α_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
0.6	49.505	0.416	49.817	0.251	9.94	2.67	27.409	(0, 46)
0.4	49.505	0.416	49.819	0.254	10.04	2.68	27.815	(0, 46)
β_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
0.12	49.505	0.416	49.819	0.254	8.36	2.68	27.740	(0, 46)
0.08	49.505	0.416	49.817	0.250	12.42	2.67	27.399	(0, 46)
σ_T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
0.30	49.505	0.416	49.815	0.301	9.82	2.99	33.651	(0, 46)
0.20	49.505	0.416	49.824	0.202	10.29	2.36	22.191	(0, 46)
a_2	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
2.4	49.505	0.416	49.818	0.252	9.97	2.68	27.920	(0, 46)
1.6	49.505	0.416	49.818	0.252	9.97	2.68	27.120	(0, 46)
a_3	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
36	49.505	0.416	49.818	0.252	9.97	2.68	27.520	(0, 46)
24	49.505	0.416	49.818	0.252	9.97	2.68	27.520	(0, 46)
K_m	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
1.2	49.505	0.416	49.859	0.250	12.54	2.82	24.325	(0, 46)
0.8	49.505	0.416	49.797	0.253	8.90	2.35	32.094	(1, 78)
LTPD	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
0.06	49.505	0.416	49.821	0.251	10.15	2.65	26.444	(0, 38)
0.04	49.505	0.416	49.814	0.252	9.77	2.70	29.052	(0, 58)
β	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
0.05	49.505	0.416	49.813	0.252	9.73	2.71	29.300	(0, 60)
0.01	49.505	0.416	49.807	0.252	9.39	2.75	33.097	(0, 92)
N	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
600	49.505	0.416	49.818	0.252	9.96	2.68	26.761	(0, 46)
400	49.505	0.416	49.818	0.251	10.00	2.67	28.657	(0, 46)
I_c	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, d, d_0, n)]$	(d_0, n)
1.2	49.505	0.416	49.818	0.252	9.96	2.68	27.544	(0, 46)
0.8	49.5053	0.416	49.818	0.252	9.98	2.67	27.495	(0, 46)

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