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# Estimation of missing data in intuitionistic fuzzy soft matrix (IFSM) with application in MCDM

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**Abstract**

The theory of fuzzy soft as well as intuitionistic soft set generally depends on complete information. But there are situations in which the complete information is not available due to various different reasons like data erroneous, in sufficiency of data, lacking of evidence and illegibility of data. Missing data restrict fuzzy soft matrix and intuitionistic fuzzy soft matrix and that causes more ambiguous uncertainty in the process of decision making. Thus, estimation of incomplete information data plays an important role in multi criteria decision making process which has been developed recently.

Here, a novel method of missing data estimation in intuitionistic fuzzy soft matrix is described. Firstl, we estimate the missing values using the information given in data. After

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estimation the missing values in IFSM thus obtained is applied in a multi-criteria decision-making problem with illustration by an example.

***Subject Classification:*** *15B15 Fuzzy Matrices.*

***Keywords:*** *Fuzzy soft set, Incomplete fuzzy soft matrix, Intuitionistic fuzzy soft set, Incomplete fuzzy soft matrix and multi criteria decision making.*

## 1. Introduction

Zadeh [33] introduced fuzzy set theory and developed many several new concepts and methods to handle vagueness and imprecision in fuzzy environment. Thereafter, Molodtsov [22] defined soft set theory and that had proved an effective tool for solving realistic problems in economics, engineering, social science, medical science and in business management. Maji and Roy[20] also en et al. [5] presented the parameterization reduction in soft set and compared this definition to the associated concepts of attributes reduction in rough set theory. A new approach to fuzzy soft set on the basis of multi –criteria decision process was introduced and studied by Feng et al. [10] by using soft sets. Recently, Kirisci [17] and Phaengtan et al. [23] have studied fuzzy soft in medical and partial average of fuzzy soft set-in decision making respectively.

As we know that a coin has two sides one is head and another is tail. In the same way human intuition suggests that there is also a non-membership degree on an element of fuzzy set besides the membership degree. Thus, beside soft and fuzzy soft sets there is another concept introduced by Atanassov [2] popularly known as intuitionistic fuzzy set.That theory enriched the area of fuzzy theory and attracted many researchers to work in this field.

Hence, IFS could depict uncertainty in a better way than fuzzy set and that was why IFSs had invited more attention and had been applied to solve problems in many fields, for example, in uncertainty modelling by Song Y. et al. [27], Liu H. et.al. [18]. De et al. [7] presented an application of Intuitionistic fuzzy sets in medical diagnosis. Intuitionistic fuzzy set is the generalization of fuzzy set. Thus, it is capable of capturing the information that includes some degree of hesitation.

In 2001 Maji et al. [20] defined intuitionistic fuzzy soft set (IFSS) as the hybrid model of intuitionistic fuzzy set and soft set. IFSS deals with the data which are not crisp, precise and deterministic; rather those are vague and ambiguous in nature. An application of intuitionistic fuzzy soft set-in medical diagnosis was studied by Hooda et al. [14]. Xu et al. [30] gave the geometric aggregation operators based on IFSs. An application of similarity measures of Intuitionistic fuzzy set to a multi criteria decision making problem was given by Szmidt and Kacprzyk [28]. Burillo and Bustince [3] constructed the theorems for IFSs and Deli and Cagman [8] studied the intuitionistic fuzzy soft set theory and its application.

There are different ways of representing fuzzy set, soft set and IFSS, e.g., in the form of table, in matrix form, etc. However, the most common and useful among these is matrix form as that plays a very important role in arranging the data in economics, management, medical and engineering sciences. Since the classical matrices theory failed to resolve uncertain data, therefore, Kandasamy et al. [16] introduced the concept of fuzzy matrix and applied it to many social, medical and scientific problems in fuzzy environment. A technique was also applied by Meenakshi et al. [21] to study Sanchez approach for medical diagnosis of inter valued fuzzy matrices. Cagman and Enginogiti [4] defined fuzzy matrices and applied it in decision making problem. Later on, intuitionistic fuzzy soft matrix theory was developed and applied by Jalilul et al. [15].

It is easy to handle the data when complete information is available, but difficulty arises when there is loss of information due to missing values. This kind of loss of information occurs during process of collecting data, unavailability of information source, inaccurate measure of data, etc. Therefore, incomplete soft set and incomplete fuzzy soft sets are taken into account to deal with this kind of situation. Qin et al. [24] presented a way of data filling for incomplete soft set.A method for estimating the parameters of a Takagi-Sugeno fuzzy model in presence of incomplete data was given by Almeida et al. [1].

Si et al. [26] introduced a concept of estimating missing data using other incomplete fuzzy soft set which improved the performance of incomplete soft set. Another way to tackle the situation of incomplete information is to ignore the object with incomplete information, but it may be possible that the object we ignore contain important information. So, it’s better to estimate the data instead of ignoring it. Recently, Hooda and Barak [12] have described a technique to estimate missing values in fuzzy matrix and interval valued fuzzy matrix.

Thiesson [29] presented acceleration in quantification of Bayesian network with incomplete data and Quinlan [25] worked on unknown attributes values in induction. An object-parameter way for forecasting the unknown entries in fuzzy soft sets was presented by Deng and Wang [9]. Yan and Zhi [31] proposed data analysis approach of soft sets under incomplete information, which was based on weighted average of all possible choice values of objects. Estimation of missing data in design of experiment and contingency table was studied by Hooda and Barak [11].

A novel approach for estimating missing data in incomplete fuzzy soft and interval valued fuzzy soft sets was given by Das et al. [6].Extending such applications further, it would be interesting to consider Intuitionistic fuzzy soft matrix, because sometime it is possible that there are two different viewpoints on the same subject, one in favour and other not in favour. Thus, to handle such situation and to take proper decision on the basis of given information intuitionistic fuzzy soft matrix is chosen.

In this communication a new unknown data estimation method for intuitionistic fuzzy soft matrix using existing information is defined and explained explicitly. Firstly, we calculate mean values of each parameter and then average distances are estimated from the mean. The membership and non-membership values are also computed separately. The basic concepts required for the development of the paper are discussed in section 2. In section 3, a technique to estimate unknown data in intuitionistic fuzzy soft matrix is defined with example and an application of IFSM is studied in multi-criteria decision making with an illustration by a numerical example in section4. In section 5, discussion and conclusion are writen and the references are given in the end of the paper.

## 2. Preliminaries

Now, basic concepts relevant with this paper are given and explained by examples.

**Definition 2.1[14]:** Fuzzy Set

Let X is a non-empty universal set, then a function$\mu_A(x): X \to [0,1]$, defines fuzzy set on X as

$$A = \{(x_i, \mu_A(x_i)): \mu_A(x_i) \in [0,1]; \forall\, x_i \in X\}.$$

Where, $\mu_A(x_i)$ is called membership function and satisfies the following conditions:

$$\mu_A(x_i) = \begin{cases} 0, & \text{if } x_i \notin A \text{ and there is no ambiguity} \\ 1, & \text{if } x_i \in A \text{ and there is no ambiguity} \\ 0.5, & \text{there is max ambiguity whether } x_i \in A \text{ or } x_i \notin A \end{cases}$$

**Definition 2.2[19]:** Soft Set

Let's consider 'U' as the universal set and T be the parametric set, then the pair $(\pi, T)$ is defined as soft set over U iff $\pi$ is a function of T into power set of U i.e.

$$\pi : T \to P(U)$$

**Example2.1:** If $V = (f_1, f_2, f_3, f_4)$ is a set of four farm houses and $E = (e_1(green\ surrounding), e_2(cheap), e_3(wooden))$ is the set of parameters. Then, the soft sets $(F, E)$ over V is given by

$$(\mathrm{F}, \mathrm{E}) = \{\, F(e_1) = (f_1, f_3),\ F(e_2) = (f_2, f_4),\ F(e_3) = (f_1, f_2)\}$$

and is represented in Table 1 as given below:

**Table 1**
**Tabular Representation of Soft Set**

| $V/E$ | $e_1$ | $e_2$ | $e_3$ |
|---|---|---|---|
| $f_1$ | 1 | 0 | 1 |
| $f_2$ | 0 | 1 | 1 |
| $f_3$ | 1 | 0 | 0 |
| $f_4$ | 0 | 1 | 0 |

**Definition 2.3[4]:** Fuzzy Soft Matrix

Let X be a universal set and E be the set of parameters and $A \subset E$. If F(x) is the set of all fuzzy subsets of X and F is a mapping from A to fuzzy set F(x), then the pair (F, A) is known as fuzzy soft set of X.

**Example 2.2:** Let us consider X = $\{c_1, c_2, c_3\}$ be the set of three cars and E = $\{e_1 = costly, e_2 = getup, e_3 = colour\}$ be the set of parameters and let A =$\{e_1, e_2\} \subset E$, then

$$(F, A) = \begin{cases} F(e_1) = \{c_1/0.6,\ c_2/0.4,\ c_3/0.4 \\ F(e_2) = \{c_1/0.8,\ c_2/0.2,\ c_3/0.7 \end{cases}$$

(F, A) is a fuzzy soft set over X to give the "attractiveness of cars".
Rectangular form as given in Table 2 is called Fuzzy Sot Matrix

**Table 2**
**Matrix Form of Fuzzy Set**

| $X/E$ | $e_1$ | $e_2$ |
|---|---|---|
| $c_1$ | 0.6 | 0.8 |
| $c_2$ | 0.4 | 0.2 |
| $c_3$ | 0.4 | 0.7 |

**Definition 2.4:** Incomplete Fuzzy Soft Matrix

If the missing values in tabular form of fuzzy soft set are there, then it may be definedas incomplete fuzzy soft matrix and we mark unknown data by the sign" $*$ ".

**Example 2.3:** Let us consider $U = \{u_1, u_2, u_3, u_4, u_5, u_6\}$ is the universal set of six houses and the set of parameters is given by

$$E = \{e_1 = expensive, e_2 = beautiful, e_3 = wooden, e_4 = cheap, e_5 = in\ green\ surrounding, e_6 = modern, e_7 = in\ good\ repair, e_8 = in\ bad\ repair\}.$$

Mr X wants to purchase a house of his choice based on parameters given by $P = \{e_1, e_2, e_3, e_4, e_5\}$. Then, the incomplete fuzzy soft matrix (U, E) can be represented in tabular form as given below in Table 3:

**Table 3**
**Tabular Incomplete Fuzzy Soft Matrix**

| $U/E$ | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ |
|---|---|---|---|---|---|
| $u_1$ | 0.1 | 0.5 | 0.3 | 0.4 | 0.3 |
| $u_2$ | $*$ | 0.5 | 0.2 | 0.3 | 0.6 |
| $u_3$ | 0.1 | $*$ | 0.4 | 0.5 | 0.1 |
| $u_4$ | 0.7 | 0.2 | 0.2 | 0.2 | 0.3 |
| $u_5$ | 0.2 | 0.6 | 0.3 | $*$ | 0.3 |
| $u_6$ | 0.9 | 0.2 | 0.1 | 0.1 | 0.8 |

**Definition 2.5 [14]:** Intuitionistic Fuzzy Set

Let X be a universal set, then intuitionistic fuzzy set A in X is given by

$$A = \{(x, \mu_A(x), \nu_A(x)) : x \epsilon X\},$$

where $\mu_A : X \to [0, 1]$ $and$ $\nu_A : X \to [0, 1]$ such that $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X. \mu_A(x)$ $and$ $\nu_A(x)$ denotes the membership and non-membership degree of the element$x \in X$ respectively. There is the intuitionistic fuzzy index denoted by$x \in X$ in A and the degree of hesitancy is given by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$.

**Definition 2.6 [15]:** Intuitionistic Fuzzy Soft Matrix

Let X be a universal set, E be the set of parameters and $A \subseteq E$. Let $I(x)$ denotes the power set containing intuitionistic fuzzy subset of X, and F is a mapping from A to $I(x)$, then the pair $(F, A)$ is called an intuitionistic fuzzy soft matrix over X,

**Example 2.4:** Let X = $\{c_1, c_2, c_3\}$ be the set of three cars, E = $\{e_1 = costly, e_2 = getup, e_3 = colour\}$ be the parametric set and A =$\{e_1, e_2\} \subset E$, then the intuitionistic fuzzy soft set (F, A) is given by

$$(F, A) = \begin{cases} F(e_1) = \{c_1/(0.6, 0.3), c_2/(0.4, 0.5), c_3/(0.3, 0.6)\\ F(e_2) = \{c_1/(0.6, 0.2), c_2/(0.7, 0.4), c_3/(0.5, 0.4)\end{cases}.$$

Rectangular form as given in Table 4 represents Intuitionistic Fuzzy Soft Matrix.

**Table 4**
**Intuitionistic Fuzzy Soft Matrix**

| $X/E$ | $e_1$ | $e_2$ |
|---|---|---|
| $c_1$ | (0.6, 0.3) | (0.6,0.2) |
| $c_2$ | (0.4,0.5) | (0.7,0.4) |
| $c_3$ | (0.3,0.6) | (0.5,0.4) |

**Definition 2.7:** In case some values are missing in intuitionistic fuzzy soft matrix, then it is known as incomplete IFSM with unknown values marked by the sign " $*$ ".

**Example 2.5:** Let U be the set of medicine denoted as $U = \{m_1,\ m_2,\ m_3,\ m_4\}$ and $A = \{fever(f), chest\ pain(p), cough(c)\}$ be the set of disease. For each disease, given medicines are prescribed with some degree of membership except for the few for which degree of membership are unknown. This data is presented in the form of a matrix with unknown entries represented by sign "* '" and is called as incomplete intuitionistic fuzzy soft matrix as shown in Table 5.

**Table 5**
**Incomplete Intuitionistic Fuzzy Soft Matrix**

| $U/A$ | $f$ | $p$ | $c$ |
|---|---|---|---|
| $m_1$ | (0.9, 0.05) | (0.3, 0.6) | $*$ |
| $m_2$ | $*$ | (0.25, 0.6) | (0.3, 0.6) |
| $m_3$ | (0.65, 0.2) | $*$ | (0.9, 0.4) |
| $m_4$ | (0.8, 0.1) | (0.3, 0.65) | (0.85, 0.3) |

## 3. Estimation of Missing Data in IFSM

In this section we estimate the missing values in intuitionistic fuzzy soft matrix and for that an algorithm is proposed for estimating the unknown data in IFSM by estimating membership and non-membership values separately. Also, we illustrate its application by considering a numerical problem.

3.1 *Steps to estimate unknown data in IFSM*

Let $H = (U, E) = (\mu_{ij}, \nu_{ij})_{m \times n}, i = 1\ to\ m, j = 1\ to\ n$ be an incomplete intuitionistic fuzzy soft set, where $U = \{u_1, u_2, u_3, \dots \dots .., u_m\}$be the set of objects and $E = \{e_1, e_2, e_3, \dots \dots .., e_n\}$be the set of parameters. Each unknown value in $(\mu_{ij}, \nu_{ij})$ is denoted by "*".

**Step 1:** Calculate mean value for both membership and non-membership function respectively $X_j\ and\ X_j{}'$for each parameter $e_j$ , where,$j = 1\ to\ n$ as

$X_j = \sum_{i=1}^{m} \mu_{ij}/|U_i|$ and $X_j' = \sum_{i=1}^{m} \nu_{ij}/|U_i|$, where $U_i = \{i/\mu_{ij} \neq *, \nu_{ij} \neq *, 1 \leq i \leq m\}$.

**Step 2:** Calculate average distance of $\mu_{ij}$ $and$ $\nu_{ij}$ from respective mean $X_j$ and $X_j'$, denoted by $D_{avg}(X_j)$ and $D_{avg}(X_j')$, where $D_{avg}(X_j) = \sum_{i=1}^{m} |X_j - \mu_{ij}|/|U_i|$ and $D_{avg}(X_j') = \sum_{i=1}^{m} |X_j' - \nu_{ij}|/|U_i|$.

**Step 3:** Calculate distance information for membership and non-membership function respectively as $\{d_m Y_j^+, d_m Y_j^-\}$ and $\{d_n Y_j^+, d_n Y_j^-\}$ from mean $X_j$ and $X_j'$as $d_m Y_j^+ = X_j + D_{avg}(X_j), d_m Y_j^- = X_j - D_{avg}(X_j)$, and $d_n Y_j^+ = X_j' + D_{avg}(X_j'), d_n Y_j^- = X_j' - D_{avg}(X_j')$.

**Step 4:** For each object $U_i(i = 1\ to\ m), \mu_{ij}\ and\ \nu_{ij}$ $(j = 1\ to\ n)$ are compared with mean $X_j$ and $X_j'$respectively ($\mu_{ij} \neq *, \nu_{ij} \neq *$). Let $n_L$ and $n_L'$be the number of count for which $\mu_{ij} < X_j$ and $\nu_{ij} < X_j'$and $n_G$ and $n_G'$be the number of count for which $\mu_{ij} \geq X_j$ and $\nu_{ij} \geq X_j'$for corresponding parameter $e_j(j = 1\ to\ n)$.

**Step 5:** Using $(n_L, n_G)$ and $(n_L', n_G')$ the probabilistic weight $(w_{L_i}, w_{G_i})$ and $(w_{L_i}', w_{G_i}')$ of each object $U_i(i = 1\ to\ m)$are respectively given as $w_{L_i} = n_L/(n_L + n_G), w_{G_i} = n_G/(n_L + n_G)$, and $w_{L_i}' = n_L'/(n_L' + n_G'), w_{G_i}' = n_G'/(n_L' + n_G')$.

**Step 6:** The unknown data for parameter $e_j$ is computed as $\mu_{ij} = (d_m Y_j^+ \times w_{G_i}) + (d_m Y_j^- \times w_{L_i})$ and$\nu_{ij} = (d_n Y_j^+ \times w_{G_i}') + (d_n Y_j^- \times w_{L_i}^-)$, where $(\mu_{ij}\ and\ \nu_{ij})$are unknown entries in IFSS (U, E).

3.2 *Example for illustration:*

Next, let us consider an incomplete intuitionistic fuzzy soft matrix as given in Table no 6:

**Table 6**
**Incomplete Intuitionistic Fuzzy Soft Matrix**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ |
|---|---|---|---|---|---|---|
| $u_1$ | (0.6, 0.3) | (0.7, 0.3) | * | (0.3, 0.7) | (0.8, 0.2) | (0.4, 0.6) |
| $u_2$ | (0.3, 0.5) | (0.4, 0.5) | (0.6, 0.3) | (0.5, 0.4) | (0.4, 0.4) | (0.6, 0.4) |
| $u_3$ | (0.5, 0.4) | (0.8, 0.2) | (0.5, 0.4) | (0.8, 0.1) | (0.7, 0.3) | (0.7, 0.3) |
| $u_4$ | (0.7, 0.2) | * | * | (0.9, 0.1) | (0.3, 0.5) | (0.3, 0.5) |
| $u_5$ | (0.8, 0.2) | (0.6, 0.3) | (0.8, 0.2) | * | (0.5, 0.3) | (0.5, 0.4) |
| $u_6$ | (0.4, 0.5) | (0.7, 0.2) | (0.7, 0.2) | (0.4, 0.4) | (0.4, 0.5) | (0.4, 0.3) |
| $u_7$ | (0.5, 0.5) | (0.3, 0.6) | (0.3, 0.6) | (0.4, 0.5) | * | (0.6, 0.3) |

Here the missing value are $(\mu_{13}, \nu_{13})$, $(\mu_{42}, \nu_{42})$, $(\mu_{43}, \nu_{43})$, $(\mu_{54}, \nu_{54})$, $(\mu_{75}, \nu_{75})$ which are denoted by "*" and are estimated by the algorithm 3.1.

**Step 1:** Mean of membership and non-membership values for parameter $e_1$ is estimated and denoted by $\mathrm{X1} = (0.6 + 0.3 + 0.5 + 0.7 + 0.8 + 0.4 + 0.5)/7 = 0.543, X_1{}' = (0.3 + 0.5 + 0.4 + 0.2 + 0.2 + 0.5 + 0.5)/7 = 0.371$. Similarly, $X_2 = 0.583, X_2{}' = 0.35, X_3 = 0.58, X_3{}' = 0.34, X_4 = 0.55, X_4{}' = 0.367, X_5 = 0.517, X_5{}' = 0.367, X_6 = 0.5, X_6{}' = 0.4$.

**Step 2:** Average distances of membership and non-membership function $D_{avg}(X_j)$ $and$ $D_{avg}(X_j{}')$ respectively are given as

$$D_{avg}(X_1) = (|0.6 - 0.543| + |0.3 - 0.543| + |0.5 - 0.543| + |0.7 - 0.543| + |0.8 - 0.543| + |0.4 - 0.543| + |0.5 - 0.543|)/7 = 0.135$$

$$D_{avg}(X_1{}') = (|0.3 - 0.371| + |0.5 - 0.371| + |0.4 - 0.371| + |0.2 - 0.371| + |0.2 - 0.371| + |0.5 - 0.371| + |0.5 - 0.371|)/7 = 0.118$$

Similarly, remaining average distance are

$$D_{avg}(X_2) = 0.156, D_{avg}(X_2{}') = 0.133, D_{avg}(X_3) = 0.144, D_{avg}(X_3{}') = 0.128, D_{avg}(X_4) = 0.2, D_{avg}(X_4{}') = 0.178, D_{avg}(X_5) = 0.156, D_{avg}(X_5{}') = 0.1, D_{avg}(X_6) = 0.114, D_{avg}(X_6{}') = 0.086.$$

**Step 3:** Distance value for membership function $\{d_m Y_j{}^+, d_m Y_j{}^-\}$ and non-membership function $\{d_n Y_j{}^+, d_n Y_j{}^-\}$ on either side of mean $X_j$ $and$ $X_j{}'$ are given below:

$$d_m Y_1{}^+ = 0.543 + 0.135 = 0.678, d_m Y_1{}^- = 0.543 - 0.135 = 0.408$$

$$d_n Y_1{}^+ = 0.371 + 0.118 = 0.489, d_n Y_1{}^- = 0.253$$

Similarly, $d_m Y_2{}^+ = 0.739, d_m Y_2{}^- = 0.427$, $d_n Y_2{}^+ = 0.483, d_n Y_2{}^- = 0.217$, $d_m Y_3{}^+ = 0.724$, $d_m Y_3{}^- = 0.436$, $d_n Y_3{}^+ = 0.468, d_n Y_3{}^- = 0.212$, $d_m Y_4{}^+ = 0.75, d_m Y_4{}^- = 0.35, d_n Y_4{}^+ = 0.545, d_n Y_4{}^- = 0.18, d_m Y_5{}^+ = 0.673, d_m Y_5{}^- = 0.361$, $d_n Y_5{}^+ = 0.467, d_n Y_5{}^- = 0.267, d_m Y_6{}^+ = 0.614, d_m Y_6{}^- = 0.386$, $d_n Y_6{}^+ = 0.486, d_n Y_6{}^- = 0.314$.

**Step 4:** Now compare each $\mu_{ij}$ $and \nu_{ij}$ with mean $X_j$ $and$ $X_j{}'$ respectively to compute $(n_L, n_G)$ and $(n_L{}', n_G{}')$, the calculation for $u_1$ is given below:

For membership functions; $X_1 < \mu_{11}(0.543 < 0.6), X_2 < \mu_{12}(0.583 < 0.7), X_4 > \mu_{14}(0.55 > 0.3), X_5 < \mu_{15}(0.517 < 0.8), X_6 > \mu_{16}(0.5 > 0.4), (n_L = 2, n_G = 3)$.

For non-membership function; $X_1{}' > \nu_{11}(0.371 > 0.3)$, $X_2{}' > \nu_{12}(0.35 > 0.3), X_4{}' < \nu_{14}(0.367 < 0.7), X_5{}' > \nu_{15}(0.367 > 0.2), X_6{}' < \nu_{16}(0.4 < 0.6), (n_L{}' = 3, n_G{}' = 2)$.

Similarly, for$u_4$; $(n_L = 2, n_G = 2), (n_L{}' = 2, n_G{}' = 2)$, for $u_5$; $(n_L = 1, n_G = 4), (n_L{}' = 4, n_G{}' = 1)$, for $u_7$; $(n_L = 4, n_G = 1), (n_L{}' = 1, n_G{}' = 4)$.

**Step 5:** Probability weights$(w_{L_i}, w_{G_i})$and $(w_{L_i}{}', w_{G_i}{}')$for membership and non-membership function respectively are computed as

For $u_1$,$w_{L_1} = 2/5,\ w_{G_1} = 3/5, w_{L_1}{}' = 3/5,\ w_{G_1}{}' = 2/5$
For $u_4$,$w_{L_4} = 2/4,\ w_{G_1} = 2/4, w_{L_4}{}' = 2/4,\ w_{G_4}{}' = 2/4$
For $u_5$,$w_{L_5} = 1/5,\ w_{G_1} = 4/5, w_{L_5}{}' = 4/5,\ w_{G_5}{}' = 1/5$
For $u_7$,$w_{L_7} = 4/5,\ w_{G_7} = 1/5, w_{L_7}{}' = 1/5,\ w_{G_7}{}' = 4/5.$

**Step 6:** In the last step calculate the unknown values using distance values and probabilistic weight as follows:

$$\mu_{13} = \left(d_m Y_3{}^+ \times w_{G_1}\right) + \left(d_m Y_3{}^- \times w_{L_1}\right)$$
$$= (0.724 \times 3/5) + (0.436 \times 2/5) = 0.434 + 0.174$$
$$= 0.608$$
$$\nu_{13} = \left(d_n Y_3{}^+ \times w_{G_1}{}'\right) + \left(d_n Y_3{}^- \times w_{L_1}{}'\right)$$
$$= (0.468 \times 2/5) + (0.212 \times 3/5) = 0.187 + 0.127$$
$$= 0.314$$
$$\mu_{42} = \left(d_m Y_2{}^+ \times w_{G_4}\right) + \left(d_m Y_2{}^- \times w_{L_4}\right)$$
$$= (0.739 \times 2/4) + (0.427 \times 2/4) = 0.370 + 0.214$$
$$= 0.584$$
$$\nu_{42} = \left(d_n Y_2{}^+ \times w_{G_4}{}'\right) + \left(d_n Y_2{}^- \times w_{L_4}{}'\right)$$
$$= (0.483 \times 2/4) + (0.217 \times 2/4) = 0.242 + 0.109$$
$$= 0.351$$
$$\mu_{43} = \left(d_m Y_3{}^+ \times w_{G_4}\right) + \left(d_m Y_3{}^- \times w_{L_4}\right)$$
$$= (0.724 \times 2/4) + (0.436 \times 2/4) = 0.362 + 0.218$$
$$= 0.58$$
$$\nu_{43} = \left(d_n Y_3{}^+ \times w_{G_4}{}'\right) + \left(d_n Y_3{}^- \times w_{L_4}{}'\right)$$
$$= (0.468 \times 2/4) + (0.212 \times 2/4) = 0.234 + 0.106$$
$$= 0.34$$
$$\mu_{54} = \left(d_m Y_4{}^+ \times w_{G_5}\right) + \left(d_m Y_4{}^- \times w_{L_5}\right) = (0.75 \times 4/5) + (0.35 \times 1/5)$$
$$= 0.6 + 0.07 = 0.67$$
$$\nu_{54} = \left(d_n Y_4{}^+ \times w_{G_5}{}'\right) + \left(d_n Y_4{}^- \times w_{L_5}{}'\right)$$
$$= (0.545 \times 1/5) + (0.189 \times 4/5) = 0.109 + 0.151$$
$$= 0.26$$
$$\mu_{75} = \left(d_m Y_5{}^+ \times w_{G_7}\right) + \left(d_m Y_5{}^- \times w_{L_7}\right)$$
$$= (0.673 \times 1/5) + (0.361 \times 4/5) = 0.135 + 0.289$$
$$= 0.424$$
$$\nu_{75} = \left(d_n Y_5{}^+ \times w_{G_7}{}'\right) + \left(d_n Y_5{}^- \times w_{L_7}{}'\right)$$
$$= (0.467 \times 4/5) + (0.267 \times 1/5) = 0.374 + 0.053$$
$$= 0.427$$

Thus, the missing entries $(\mu_{13}, \nu_{13}), (\mu_{42}, \nu_{42}), (\mu_{43}, \nu_{43}), (\mu_{54}, \nu_{54}), (\mu_{75}, \nu_{75})$ are $((0.608, 0.314), (0.584, 0.351), (0.58, 0.34), (0.67, 0.26), (0.424, 0.427))$ respectively.

Hence, on filling the missing values, we get Intuitionistic Fuzzy Soft matrix as given below in Table 7.The estimated values entered in Table 7 are approximated up to one decimal point for convenience.

**Table 7**
**Intuitionistic Fuzzy Soft Matrix**

| | $e_1$ | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ |
|---|---|---|---|---|---|---|
| $u_1$ | (0.6, 0.3) | (0.7, 0.3) | **(0.6, 0.3)** | (0.3, 0.7) | (0.8, 0.2) | (0.4, 0.6) |
| $u_2$ | (0.3, 0.5) | (0.4, 0.5) | (0.6, 0.3) | (0.5, 0.4) | (0.4, 0.4) | (0.6, 0.4) |
| $u_3$ | (0.5, 0.4) | (0.8, 0.2) | (0.5, 0.4) | (0.8, 0.1) | (0.7, 0.3) | (0.7, 0.3) |
| $u_4$ | (0.7, 0.2) | **(0.6, 0.4)** | **(0.6, 0.3)** | (0.9, 0.1) | (0.3, 0.5) | (0.3, 0.5) |
| $u_5$ | (0.8, 0.2) | (0.6, 0.3) | (0.8, 0.2) | **(0.7, 0.3)** | (0.5, 0.3) | (0.5, 0.4) |
| $u_6$ | (0.4, 0.5) | (0.7, 0.2) | (0.7, 0.2) | (0.4, 0.4) | (0.4, 0.5) | (0.4, 0.3) |
| $u_7$ | (0.5, 0.5) | (0.3, 0.6) | (0.3, 0.6) | (0.4, 0.5) | **(0.4, 0.4)** | (0.6, 0.3) |

## 4. Application of IFSM in MCDM

In this section we describe an algorithm for application of Intuitionistic Fuzzy Soft matrix in decision making problem. Firstly, an algorithm is proposed based on membership and non-membership value of intuitionistic fuzzy soft matrix followed by an example for illustration. The method proposed here for using IFSM is much simpler than other method for MCDM problems.

4.1 *Algorithm*

Here we describe the steps involved in algorithm for decision making.

**Step 1:** Firstly, we compute the intuitionistic fuzzy soft matrix

$$K_{if} = \left\{\left(u, \mu_{K_{if}}(u), \nu_{K_{if}}(u)\right) : u \in U\right\}$$

where $\mu_{K_{if}}(u) = \frac{1}{|U|}\sum_{x\in E, u\in U} \mu_K(x)\chi_{f_K(x)}(u)$ and

$$\nu_{K_{if}}(u) = \frac{1}{|U|}\sum_{x\in E, u\in U} \nu_K(x)\chi_{f_K(x)}(u), \text{ where } \chi_{f_K(x)}(u) = \begin{cases} 1, & u \in f_K(x) \\ 0, & u \notin f_K(x) \end{cases}$$

where $\mu_K(x)$& $\nu_K(x)$ respectively are membership and non-membership values of IFSM.

**Step 2:** Next, find fuzzy matrix$K_f = \left\{\frac{\gamma_{K_f}(u)}{u} : u \in U\right\}$, where

$$\gamma_{K_f}(u) = \mu_{K_{if}}(u)(1 - \nu_{K_{if}}(u)).$$

**Step 3:** Lastly**, c**hoose first three maximum membership degrees and that is the required decision.

**Example 4.2:** There are three scholarships in statistics department of a university X for a doctoral degree. For a scholarship many students apply, but on the basis of initial conditions on cumulative grade point average (CGPA) only seven students are short listed for further evaluations. Let $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$ represents the seven students and

$$E = \{e_1 = CGPA, e_2 = \text{number of research papers}, e_3 = \text{research quality}, e_4 = \text{research proposal}, e_5 = \text{personal statement}, e_6 = interview\}, \text{ be the set of attributes.}$$

For selection of students, VC of a university set up an expert committee for evaluation and the committee was to evaluate students on the basis of attributes. But when data was arranged in matrix form and scrutinised as given below in table no 6. Then some values were found missing, so it became imperative to estimate these missing values before taking any decision about award of scholarships. So, the algorithm 3.1 was applied for estimating the missing data in IFSM as given in table 7.

**Solution:** By using above algorithm, the calculation for decision making is given below:

$$K_{if} = \{(u_1, (0.49, 0.34)), (u_2, (0.4,0.36)), (u_3, (0.57,0.24)), (u_4, (0.49,0.29)), (u_5, (0.56,0.24)), (u_6, (0.43,0.3)), (u_7, (0.36\ ,0.41))\}.$$

Now, fuzzy set $K_f = \{0.2112/u_1, 0.256/u_2, \mathbf{0.4332/u_3}, \mathbf{0.3036/u_4}, \mathbf{0.343/u_5}, 0.301/u_6, 0.2301/u_7\}$.

Three students with maximum membership degree from the fuzzy set $K_f$ are chosen and that is the final decision about the students for scholarship. Thus students $u_3, u_4, u_5$ are finally chosen.

## 5. Discussion and Conclusion

Sometime entries in data arranged in matrix forms are found missing or illegible due to natural calamity or some other uncontrollable reason. In case it is entirely impossible to repeat or predict the missing values, then the techniques need to be developed to estimate these values. Thus, in the fields the estimation of missing data is very important area like agriculture, medical diagnosis, economics, management, scientific observatory laboratories as experiment can't be repeated in the same climatic and physical conditions. Various scientists and technocrats have suggested different methods to estimate missing values in data.

First of all, Yates [32] suggested a method which was also described and improved by Hooda [13]. Further, the estimation of missing data in fuzzy contingency table was studied by Hooda and Barak [11]. They have also described a method of estimation of missing values in fuzzy and interval valued fuzzy matrices [12]. Das et al. [6] have defined an algorithm for estimating missing values in incomplete fuzzy soft and interval valued soft matrices.

Thus, it is interesting and valuable to study the estimation of missing data in intuitionistic fuzzy soft matrix which was defined by Jalilul et al. [15]. In the present study a new approach to estimate unknown data in intuitionistic fuzzy soft matrix is described. An algorithm for application of intuitionistic fuzzy soft matrix in multi criteria decision making problem is described with illustration by an example.

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