Journal of Statistics & Management Systems
ISSN 0972-0510 (Print), ISSN 2169-0014 (Online)
Vol. 27 (2024), No. 3, pp. 673–680
DOI : 10.47974/JSMS-1219

Economic manufacturing quantity model with quality investment

Chung-Ho Chen *
Tzu-Hsien Lee
Department of Industrial Management and Information
Southern Taiwan University of Science and Technology
I Nan-Tai Street
Young-Kang Distri.
Taninan
Taiwan, R.O.C.

Abstract

In this article, we propose the modified economic production problem by considering acceptance inspection for obtaining production quantity, mean, and quality investment. One considers unknown mean and known variability under the normal quality. The quality investment function is the declining exponential function of variability. One adopts Taguchi's loss cost for evaluating quality.

Subject Classification: *(2010) 60 Probability theory & stochastic processes.*

Keywords: *Production problem, Mean, Standard deviation, Quality investment, Taguchi's loss function.*

1. Introduction

Traditional economic production problem assumes perfect quality. Some researchers, e.g., Porteus [1] and Rosenblatt & Lee [2, 3] have addressed the imperfect quality problem.

The modern production system addresses the control of product quality, production quantity, process promotion and improvement. For the inventory model, we need the optimization of expected total cost. For the machine equipment, one considers the trade-off problem of the reliability and maintenance, i.e., maximum availability.

* *E-mail:* **chench@stust.edu.tw**

Mean and variance affect the bias and variability. Taguchi [4] addressed the non-linear loss function for determining the quality.

Mean setting is an important topic for SPC. The known variance and unknown mean are the basic assumptions. Some researchers adopt Taguchi's for obtaining the optimal manufacturing target, e.g., Chan & Ibrahim [5], Chan et. al., [6, 7], Shin et. al., [8]., and Chen [9].

Quality investment is an available method for promoting quality. Hong et. al., [10], Ganeshan et. al., [11], Chen & Tsou [12], and Tsou [13] have proposed some works about this topic.

In this article, one addresses the modified problem with mean and investment cost. The solution is to get the quantity lot, quality investment, and mean based on sampling plan.

2. Modified EMQ Model

Consider the EMQ is the lot size of sampling plan. We adopt single attribute rectifying inspection for screening the product. All units are inspected when the lot is rejected. If the inspected unit is the defective, then it will be rejected and replaced. The conditional profit is

$$W_0 = \begin{cases} A_2 Q - nI_c - QC\mu_0 - npR_I - (Q-n)LF_0 - nLF_1, & \text{if } X \le d_0 \\ A_1 Q - R_I Qp - QI_c - QC\mu_0 - QLF_1, & \text{if } X > d_0 \end{cases} \tag{1}$$

where A_1 is sold price in the bad lot; A_2 is sold price in the good lot; Q is lot quantity; X is defective occurred in the inspection; n is inspection quantity; d_0 is largest number of defective unit; I_c is inspection cost; R_I is replacement cost; p is probability of defective unit; C is processing cost; μ_0 is mean; LF_0 is expected quality cost for uninspected product for good lot; LF_1 is expected quality cost for inspected product for bad lot.

Hence, the expected total profit is

$$\begin{aligned} W_1 = \{&(A_2 - A_1)Q + R_I \cdot Qp + (Q-n)I_c - npR_I + (Q-n)LF_1 - (Q-n)LF_0\} \\ &P(X \le d_0) + (A_1 Q - R_I \cdot Qp - QI_c - QC\mu_0 - QLF_1) \end{aligned} \tag{2}$$

where $P(X \le d_0)$ is accepted probability of inspected lot, $P(X \le d_0) = 1 - P(X > d_0)$.

From Eq. (3), the expected total profit per item is

$$\begin{aligned} W_2 = &\left\{(A_2 - A_1) + R_I \cdot p + \left(1 - \frac{n}{Q}\right) I_c + \left(1 - \frac{n}{Q}\right)(LF_1 - LF_0) - \frac{npR_I}{Q}\right\} P(X \le d_0) \\ &+ (A_1 - R_I \cdot p - I_c - C\mu_0 - LF_1) \end{aligned} \tag{3}$$

The expected total profit is
Maximize

$$ETP(Q,\ \mu_0,\ Inv) = M_1 \cdot \left(W_2 - \frac{Inv}{Q}\right) - S_t \cdot \frac{M_1}{Q} - 0.5 \cdot Q \cdot \left(1 - \frac{O_1}{I_1}\right) h \tag{4}$$

subject to

$$\begin{aligned} W_2 = & \left\{(A_2 - A_1) + R_I \cdot p + \left(1 - \frac{n}{Q}\right) I_c + \left(1 - \frac{n}{Q}\right)(LF_1 - LF_0) - \frac{npR_I}{Q}\right\} P(X \le d_0) \\ & + (A_1 - R_I \cdot p - I_c - C\mu_I - LF_1) \end{aligned} \tag{5}$$

$$\begin{aligned} LF_0 &= \int_{-\infty}^{T} k_1 (y - T_1)^2 f(y, Inv) dy + \int_{T}^{\infty} k_2 (y - T_1)^2 f(y, Inv) dy \\ &= k_1 \left\{ [(\mu_0 - T_1)^2 + \sigma_I^2] \Phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right) - \sigma_I (\mu_0 - T_1) \phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right) \right\} + \\ & \quad k_2 \left\{ [(\mu_0 - T_1)^2 + \sigma_I^2] \left[1 - \Phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right)\right] + \sigma_I (\mu_0 - T_1) \phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right) \right\} \end{aligned} \tag{6}$$

$$\begin{aligned} LF_1 &= \frac{\int_{LSL}^{T} k_1 (y - T_1)^2 f(y,\ Inv) dy + \int_{T}^{USL} k_2 (y - T_1)^2 f(y,\ Inv) dy}{\int_{LSL}^{USL} f(y,\ Inv) dy} \\ &= \Bigg\{ \Bigg\{ k_1 [(\mu_0 - T_1)^2 + \sigma_I^2] \left[\Phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right) - \Phi\left(\frac{LSL - \mu_0}{\sigma_I}\right) \right] - \\ & \quad \sigma_I \left[(\mu_0 - T_1) \phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right) - (\mu_0 - 2T_1 + LSL) \phi\left(\frac{LSL - \mu_0}{\sigma_I}\right) \right] \Bigg\} + \\ & \quad \Bigg\{ k_2 [(\mu_0 - T_1)^2 + \sigma_I^2] \left[\Phi\left(\frac{USL - \mu_0}{\sigma_I}\right) - \Phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right) \right] - \\ & \quad \sigma_I \left[-(\mu_0 - T_1) \phi\left(\frac{T_1 - \mu_0}{\sigma_I}\right) + (\mu_0 - 2T_1 + USL) \phi\left(\frac{USL - \mu_0}{\sigma_I}\right) \right] \Bigg\} \Bigg\} \\ & \quad / \left[\Phi\left(\frac{USL - \mu_0}{\sigma_I}\right) - \Phi\left(\frac{LSL - \mu_0}{\sigma_I}\right) \right] \end{aligned} \tag{7}$$

$$f(y, Inv) = \frac{1}{\sqrt{2\pi}\sigma_I} \exp\left(-\frac{1}{2} \left(\frac{y - \mu_I}{\sigma_I} \right)^2 \right),\ \ LSL < y < USL \tag{8}$$

$$\sigma_I^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2) \exp(-\alpha_1 Inv) \tag{9}$$

$$p = 1 - \Phi\left(\frac{USL - \mu_0}{\sigma_I}\right) + \Phi\left(\frac{LSL - \mu_0}{\sigma_I}\right) \tag{10}$$

$ETP(Q,\ \mu_0, Inv)]$ is expected total profit; $\phi(\cdot)$ is p.d.f. of standard n.r.v.; $\Phi(\cdot)$ is c.d.f. of standard n.r.v.; LSL is lowest value of conformance;

USL is largest value of conformance; μ_0 is mean; σ_0^2 is known variance; Inv is quality investment; σ_I is improved standard deviation; α_1 is exponential declined coefficient for the improved standard deviation; Y is normal quality characteristic; $f(y, Inv)$ is p.d.f. of Y,

$$f(y, Inv) = \frac{1}{2\sqrt{\pi}\sigma_I}\exp\left(-\frac{1}{2}\left(\frac{y-\mu_0}{\sigma_I}\right)^2\right), \ -\infty < y < \infty;$$

σ_T is nominal value of variability; M_1 is demand rate in the period; O_1 is demand rate per day; I_1 is production quantity per day; S_t is setup cost; h is handing cost; k_1 and k_2 are loss coefficients; C is material cost; T_1 is target value of mean.

The decision variables $(Q,\ \mu_0, Inv)$ need to be obtained by maximizing $ETP(Q,\ \mu_0, Inv)$]. The procedure of solution is to do:

Step 1: Let maximum Q be $Q_{\max}$ and maximum Inv be $Inv_{\max}$. Set initial $Inv = 0$ and $Q = n$.

Step 2: Search interval $LSL < \mu_0 < USL$, one can obtain the corresponding $ETP(Q,\ \mu_0, Inv)$ from Eq. (4).

Step 3: For $Inv = Inv + 1$, continue Step 2 until $Inv = Inv_{\max}$.

Step 4: For $Q = Q + 1$, continue the above steps until $Q = Q_{\max}$. The parameters (Q, μ_0, Inv) with maximum $ETP(Q, \mu_0, Inv)$ is the solution.

3. Numerical Example

Consider some parameters: $k_1 = 200$, $k_2 = 300$, $T_1 = 50$, $\alpha_1 = 0.5$, $\sigma_T = 0.3$, $A_1 = 67.5$, $A_2 = 80$, $R_I = 30.5$, $M = 800$, $O_1 = 80$, $I_1 = 100$, $h = 10$, $c = 1$, $S_t = 20$, $LSL = 48$, $USL = 52$, $\sigma_0 = 0.5$, $n = 17$, $d_0 = 0$, and $I_c = 0.5$. By solving Eqs. (4)-(10), the optimal solution is $Q = 189$, $\mu_0 = 49.95$, $Inv = 16$, and $ETP(Q,\ \mu_0, Inv)$ = 6114.165.

Table 1 shows the sensitivity analysis. According to Table 1, one has some results:

1. The k_2, T_1, and σ_T have an important effect on the mean.
2. The σ_0 and α_1 have an important effect on the quality investment.
3. The M_1, O_1, I_1, S_t, h, and α_1 have an important effect on the EMQ.
4. The C, A_2, M_1, O_1, k_1, k_2, T_1, and σ_T have an important contribution on the expected profit.

4. Conclusions

The authors have proposed the extended production problem with quality investment and mean. Further study should consider the problem with reliability parameter.

Table 1

Sensitivity analysis of parameters

c	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
0.8	49.95	16	188	14106.140
1.2	49.95	16	189	-1877.811
A_2	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
64	49.95	16	189	-6685.835
96	49.95	16	189	18914.170
A_1	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
54	49.95	16	189	6114.165
81	49.95	16	189	6114.165
M	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
640	49.95	16	169	4855.691
960	49.95	17	209	7376.774
O	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
64	49.95	16	141	5985.243
96	49.95	18	431	6325.623
I	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
90	49.95	17	256	6211.231
120	49.95	16	146	6004.356
S_t	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
16	49.95	16	180	6131.525
24	49.95	16	199	6097.646
h	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
8	49.95	17	213	6154.349
12	49.95	16	172	6078.149
R_I	μ_0	Inv	Q	$ETP\ (Q, \mu_0, Inv)$
24.4	49.95	16	189	6114.165
36.6	49.95	16	189	6114.165

Contd...

k_1	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
160	49.95	16	189	8109.704
240	49.95	17	191	4346.257
k_2	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
240	49.98	16	189	7872.403
360	49.93	17	191	4574.220
T	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
49	49.00	17	187	6342.760
51	50.95	16	186	5278.604
α	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
0.4	49.95	20	197	6097.571
0.6	49.95	14	184	6126.190
σ_T	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
0.24	49.96	17	190	12418.50
0.36	49.94	16	189	-1592.003
LSL	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
47	49.95	16	189	6114.165
49	49.95	16	183	5991.052
USL	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
51	49.95	16	186	6078.511
53	49.95	16	189	6114.165
σ_0	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
0.4	49.95	15	186	6121.346
0.6	49.95	18	193	6109.749
I_c	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
0.4	49.95	16	185	6121.443
0.6	49.95	16	192	6107.027
(d_0,n)	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
(0, 10)	49.95	17	183	6124.676
(0, 17)	49.95	16	189	6114.165
(0, 25)	49.95	18	201	6093.501
(d_0, n)	μ_0	*Inv*	*Q*	*ETP* (Q, μ_0, Inv)
(1, 10)	49.95	17	183	6124.676
(1, 17)	49.95	18	193	6109.749
(1, 25)	49.95	18	201	6093.501

References

[1] Chan, W.M. and Ibrahim, R.N., Evaluating the quality level of a product with multiple quality characteristics, *International Journal of Advanced Manufacturing Technology*, 24, 738-742 (2004).

[2] Chan, W.M., Ibrahim, R.N., and Lochert, P.B., Quality evaluation model using loss function for multiple S-type quality characteristics, *International Journal of Advanced Manufacturing Technology*, 26, 98-101 (2005a).

[3] Chan, W. M., Ibrahim, R.N., and Lochert, P.B., Evaluating the product quality level under multiple L-type quality characteristics, *International Journal of Advanced Manufacturing Technology*, 27, 90-95 (2005b).

[4] Chase, K.M., Least cost tolerance allocation for mechanical assemblies with automated process selection, *Manufacturing Review*, 3, 49-59 (1990).

[5] Chen, C.H., Process mean and production quantity settings by considering modified economic manufacturing quantity model, *Journal of Information and Optimization Sciences*, 39, 1187-1198 (2018).

[6] Chen, J.M. and Tsou, J.C., An optimal design for process quality improvement: modeling and application, *Production Planning & Control*, 14, 603-612 (2003).

[7] Ganeshan, R., Kulkarni, S., and Boone, T., Production economics and process quality: a Taguchi perspective, *International Journal of Production Economics*, 71, 343-350 (2001).

[8] Hong, J.D., Xu, S.H., and Hayya, J.C., Process quality improvement and setup reduction in dynamic lot-sizing, *International Journal of Production Research*, 31, 2693-2708 (1993).

[9] Jeang, A., tolerance design: choosing optimal tolerance specification in the design machined parts, *Quality and Reliability Engineering International*, 10, 27-35 (1994).

[10] Porteus, E.L., Optimal lot sizing, process quality improvement and set-up cost reduction, *Operations Research*, 34, 137-144 (1986).

[11] Rosenblatt, M.J. and Lee, H.L., Economic production cycles with imperfect production processes, *IIE Transactions*, 17, 48-54 (1986a).

[12] Rosenblatt, M.J. and Lee, H.L., A comparative study of continuous and periodic inspection policies in deteriorating production systems, *IIE Transactions*, 18, 2-9 (1986b).

[13] Shin, S., Kongsuwon, P., and Cho, B.R., Development of the parametric tolerance modeling and optimization schemes and cost-effective solutions, *European Journal of Operational Research*, 207, 1728-1741 (2010).

[14] Taguchi, G., Introduction to Quality Engineering, Asian Productivity Organization, Tokyo (1986).

[15] Tsou, J.C., Decision-making on quality investment in a dynamic lot sizing production system, *International Journal of Systems Science*, 37, 463-475 (2006).

[16] *Industrial Engineering*, 72, 239-246, 2014.

Received April, 2023