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Bouc-Wen model with machine learning for SISO and MIMO nano-positioning system

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Abstract

Most of the application in present world have taken 'Nano' as a part of their near future implementation. In Nano-Positioning System (NPS) mechanical dynamics are handled at Nano scale. Designing a precision NPS is a greater challenge. Modified linearized Bouc-Wen(BW) model gives the better hysteresis, static and dynamic behavior. In this work SISO (Sıngle-input Sıngle-Output) and MIMO (Multiple-Input Multiple-Output) models of NPS are designed. Previously, system was basically designed to handle conventional macro level dynamics and the same has been extended to handle Nano scale in this work. Kalman Filtering is included to achieve very sustainable results in NPS. The NPS is designed in MATLAB/Simulink with 3 types of electrical signals. Each design is compared using various performance parameters. Machine learning process is included to suggest the appropriate input voltage level sui to obtain desired displacement.

Subject Classification: *00A71 General theory of mathematical modeling.*

Keywords: *Bouc-Wen, Kalman filtering, SISO, MIMO, NPS, Machine learning.*

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1. Introduction

Some of the application like semiconductor and ultra-precisioned machining industries reunites an high- speed high precision positioning systems like Scanning Tunnelling Microscope (STM) [1], Atomic Force Microscope (AFM) [2] etc. measures the mechanical, electrical and structural characteristics of the surface at atomic level. Suppose in an example with STM, the distance between metallic surface and the conductive tip of STM is at nano-scale (i.e. $<10^{-9}$m) and when it is electrically biased (i.e. Voltage is applied) then transfer of electrons may take place between tip and surface using piezoelectric actuators [3,4] for conductive surfaces and ATM [5] for non-conductive surfaces.

In this work entire NPS is modelled using MATLAB/Simulink. The linear systems are easy to design and handle but static. BW hysteresis model integrated to the NPS shows better results by linearising the non-linear model [9,10]. Also capacitive sensor used for detection in actuators [11] imposes nonlinearity and hence low SNR(Signal-to-Noise Ratio). Kalman Filter shows better precision by its self-adaptive nature to nullify the resonance noise caused by inherent frequency of mechanical stage [11].

In this paper, Section 1 introduces applicationa of NPS and the nedd of designing a highly preocisioned system. Section 2 focuses on the Literature review on similar work. Mathematical discussion about the BW model and Kalman filtering are done in Section 3 and 4. Section 5 contains Simulink implementation of the MIMO NPS. Section 6 has the summarised results compared over performance parameters.

2. Related Work

The piezo-electric actuators pose high-precision and solid- state fast actuation which are mainly used in the various applications like sono-chemistry application domain, vibration control domain, hydraulic flow control domain, energy harvesting, and so on. Usually the designed systems have nonlinearity hysteresis which degrades the controlling nature of the system [6].

Analyzing the system can be made by mathematical model of the hysteresis is to be modeled and hysteresis compensation model needs to be enforced using inverse modeling. BW model [11] is one such hysteresis model. Paper [7] deals with Single-Degree of Freedom systems and multi degree freedom. Paper [8] models hysteresis compensation using adaptive neural output feedback control.

Above mentioned papers predominantly designs the systems at macroscopic levels. The work [11-14] extends to design to nano-scale in which the positioning measures are < 1 nm. For the designed simulink model of SISO NPS is 3 input types- step, sine and pulse. To have an improved output, Kalman filtering is added. Further in this work MIMO modelling is done for NPS.

3. Hysteresis Modelling

BW model is a non-linear hysteretic model is used to capture analytically the various shapes of hysteretic cycle. The motion equation with one-degree freedom is,

$$f(t) = m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + F(t) \tag{1}$$

Here, f(t) and F(t) are Excitation and Restoring force, m is Mass[Kg] and x(t) is the Displacement[m].

The Restoring force is ,

$$F(t) = aK_i x(t) + (1-a)k_i z(t) \tag{2}$$

Here, $a = \frac{post-yield(k_f)}{pre-yield(k_i)}$, $z(t)$ is the hysteretic displacement

With zero initial condition $z(t)$ is represented as:

$$\frac{dz(t)}{dt} = \frac{dx(t)}{dt}\left\{\alpha - \left[\beta\, sign\left(z(t)\frac{dx(t)}{dt}\right) + \gamma\right]|z(t)|^n\right\} \tag{3}$$

Where, α, β, γ, n dimentionless quantities which decides the hysteresis loop shape, sign() signum function. Equation 5 is nonlinear differential equation.

In Piezo-electric actuator, f(t) is an input voltage, hysteretic state variable z(t) is length, and x(t) is an output positioning displacement. The equation (1) and z(t) is rewritten as,

$$m\frac{d^2x(t)}{dt^2} + c\frac{dx(t)}{dt} + kx(t) = k(d\,f(t) - z(t)) \tag{4}$$

$$\frac{dz(t)}{dt} = \alpha d\frac{dx(t)}{dt} - \beta\left|\frac{df(t)}{dt}\right| - \gamma f(t)\left|\frac{df(t)}{dt}\right| \tag{5}$$

Here, effective piezoelectric coefficient (m/V) is denoted by d. Figure 1 shows the simulink representaion of BW.

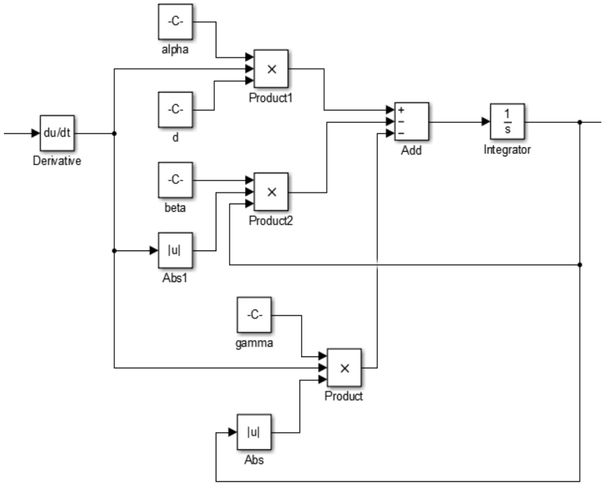

Figure 1

Simulink representation of Bouc –Wen

Inverse model nullifies the hysteresis imposed in the previous stage. The inverse model outputs a displacement:

$$f(t)=\left[\frac{m}{kd}\right]\frac{d^2x(t)}{dt^2}+\left[\frac{b}{kd}\right]\frac{dx(t)}{dt}+\left[\frac{1}{d}+\frac{k}{kd}\right]\frac{dx(t)}{dt}+\left[\frac{z(t)}{d}\right] \tag{6}$$

The designed system poses non-linear hysteresis but as linear designs are easy, the designed system is linearized and made static by reducing the difference between estimated linear and non-linear values

$$\tilde{e}=\alpha d\frac{dx(t)}{dt}-\beta\left|\frac{dx(t)}{dt}\right|z(t)-\left(k_1\frac{dx(t)}{dt}+k_1 z(t)\right) \tag{6}$$

The Figure 2 depicts SISO model of NPS using Simulink model using various parameters. The following design specifications are used to model NPS [11], A, B and C are [1 *T* 0 1], [0 1], and [1 0] respectively, T is 100nS, Q and R are 0.01 and $4x10^{-9}$ respectively.

The overall transfer function G(s) of the Piezo-Electric Actuator) is,

$$G(s)=\frac{7.2x10^{13}s^2+2.3x10^{16}s+3.2x10^{31}}{s^6+1.1x10^4s^5+9.5x10^7s^4+7x10^{13}s^3+2x10^{15}s^2+5.6x10^{18}s+10^{22}} \tag{7}$$

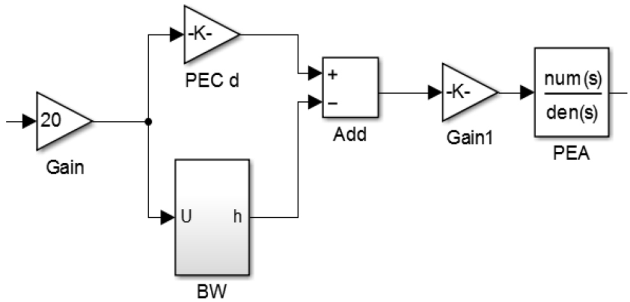

Figure 2

Simulink model of SISO NPS

4. Design of Kalman Filter

Kalman filtering or Linear Quadratic Estimation is a recursive filter which is both linear and optimal. During the filter operation, an estimate is about probable outputs is done. Measure values are compared recursively with estimate. If required the self-adaptive corrections are made for future estimations. Kalman filter is modelled using state equation model which has two equations,

1. State Equation: $X(k) = AX(k\ \text{-}1) + BW(k\text{-}1)$
2. Measurement equation: $Y(k)=CX(k)+V(k)$

Here, X(k) and $X(k\ \text{-}1)$ indicates Present and Previous State values respectively, and Y(k) is Present observed Output, Noise during the Process is W(k-1) , Matrices related to State Transformation are A for previous state, B for Noise part, C for amount of input and V(k) ☉ Observation noise. Kalman Filter design has following 2 stages:

- Time update stage:
  - o Propagate State:

$$x^*_{k+1} = f(\widehat{x_k}, u_k, 0) = A\hat{x}^-_{k-1} + Bu_k$$

  - o Propagate Error Covariances:

$$P^*_{k+1} = A\hat{P}^-_{k-1} A^T + Q = A_{k+1} P_k A^T_{k+1} + W_{k+1} Q_k W^T_{k+1}$$

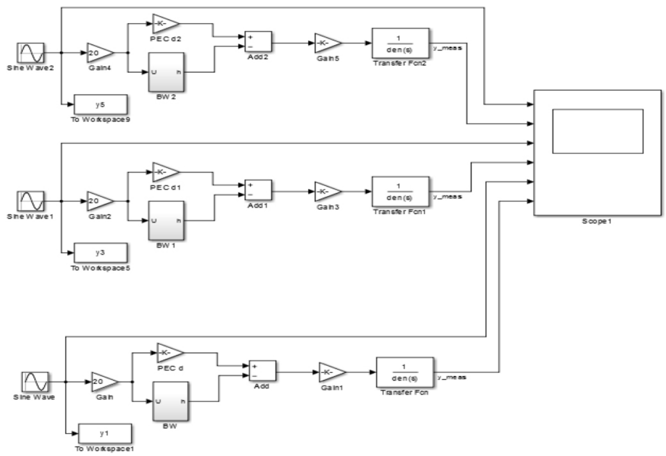

Figure 3

Simulink model of MIMO model

- Measurement stage:
  - o Kalman Gain : $K_{k+1} = \hat{P}_k^{-} H^T (H\hat{P}_k^{-} H^T + R)^{-1}$
    $$= P_{k+1}^{*} C_{k+1}^{T} (C_{k+1} P_{k+1}^{*} C_{k+1}^{T} + V_{k+1} R_{k+1} V_{k+1}^{T})^{-1}$$
  - o Estimate State: $\widehat{x_{k+1}} = \hat{x}_k^{-} + K_k (z_k - H\hat{x}_k^{-}) = x_{k+1}^{*} + K_{k+1}$
  - o Estimate Error Covariances: $P_{k+1} = (I - K_k H)\ P_k^{-} = (I - K_{k+1} C_{k+1})$

5. MIMO Modeling

The NPS model designed in the previous section is an example of SISO. This SISO is used as subsystem in MIMO NPS. In SISO the movement is along one axis. MIMO system can control the movements along three axes. The MIMO is designed by decomposing the MIMO system into several SISO-like subsystems. Therefore here 3 voltage levels acts as multiple inputs and 3-axial displacements forms multiple output. Along with this if any noise is found in output, it is rectified using Kalman filter. Figure 3 is Simulink model of SISO like model of MIMO.

6. Machine learning:

Machine learning process is included to suggest the appropriate voltage levels among various voltage levels. Machine learning deals with the automation of programming where the computer program modifies itself based on the new dataset.

These can used to suggest appropriate voltage while considering all the performance parameters as the features for the train and test dataset.

7. Results and discussion:

NPS gives displacement as output as a response to electrical signal. The designed SISO model as in Figure 2 is checked with step, sine and pulse inputs. The scope output of the Figure 2 is shown in Figure 4. The displacement output is taken along single axis in SISO and along three axes in MIMO NPS whose scope result for sine input. Some samples of the output are taken in Table 1. One can observe the stability in the output with the linearization.

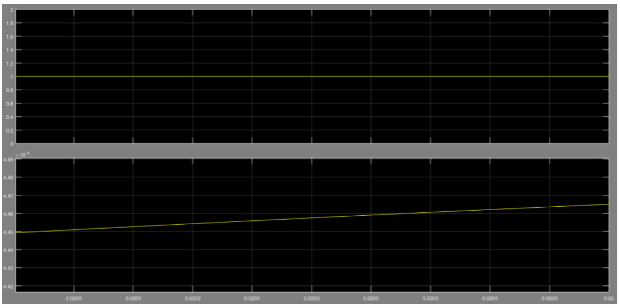

Figure 4

Simulink Scope output for Sine

Table 1

Displacement for different models of SISO NPS

Overall SISO module with	Displacement (nm)
Bouc-Wen	42 to 54
Gain considerations of Bouc-Wen	45 to 52
Linearised BW	12 to 20

To compare the results with various amplitude levels, the frequency is fixed to 5 rad/S for sine input. 2 show the comparison of the possible displacement with and without Kalman filtering. As a notice one can observe better results with the Kalman filter.

2

Amplitude V/S Displacement for SISO (Sine input: Frequency= 5 rad/S)

Amplitude (V)	Possible displacement range without Kalman filter (in nm)	Possible displacement range with Kalman filter (in nm)
0.25	0.004-62	0.014-110
0.5	0.013-110	0.029-214
0.75	0.016-172	0.032-358
1	0.036-219	0.061-435

All the systems are compared using performance parameters like accuracy, allowable Input Range, bandwidth, resolution, response Time and stability. 5 summarises the comparison of SISO model with or without Kalman filtering for different performance parameters. One can observe the following inferences based on the 3.

- Allowable range of the voltage is extended with use Kalman filter
- Accuracy of the system is more in system with Kalman filter.
- Same amount of the displacement can be achieved even with lower frequency signal with Kalman filtering
- Resolution of the system is more in system designed with Kalman.

3

Summary of Results for different performance parameters for sine input

	Without Kalman filter	With Kalman filter
Allowable input voltage	Upto 2.06V (f=5rad/S)	Upto 2.15V (f=5rad/S)
Accuracy (With 3V)	88 to 106 nm	90-100 nm
Allowable Frequency in rad/S (with 0.7V)	0-30	0-20
Resolution	339nm	553nm

4 summarizes the output of MIMO with sine input given to all 3 input with variety voltage range

4

MIMO output with Filtering (Sine Input)

Axis	Input(f=5 rad/S)	Output (in nm)
X	0.25V	5
Y	1V	200
Z	1.5V	290

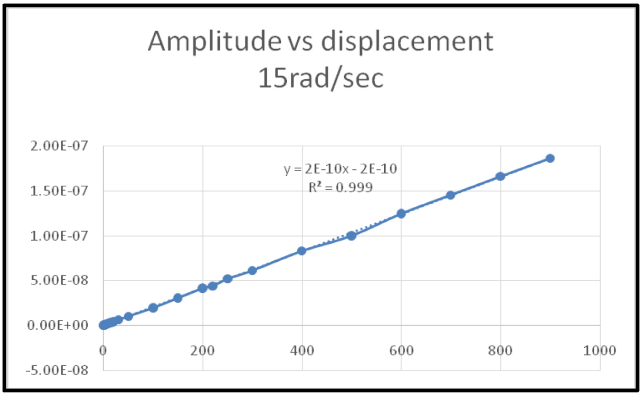

Figure 5

Amplitude versus displacement with fixed frequency of 15 rad/S

- To understand the interrelationship between the variables the graph was drawn as shown in the figure for the angular frequency of 15 rad/sec.
- It was found that there was excellent correlation between two variables, Amplitude vs. Displacement with the correlation coefficient of 0.999.
- Also from the graph it is found that the amplitude, from 100v to 150 v the displacement vary from 1.97E-07 to 3.07 E-07 with a maximum significant displacement effect. It is clear that as amplitude increases there is a significant increase in displacement. It is found that Amplitude and Displacement are excellently correlated. All these have been summarized in Figure 5.

  Example of the output is as follows:

```
Enter the Required Distance(in nm):23
For 23 nm , adjust to 1.17 V (Frequency 5 rad/S)
```

In this machine learning is used to estimate:

1. Displacements as per given electrical signal
2. Electrical signal for a given displacement.

There are two regression models

- Linear
- Quadratic

Estimated displacements equations are summarised in 5 for fixed frequency of 5 rad/S. In this one can observe estimation done by quadratic model is more approximate with the displacements given the NPS model for the same electrical signal. Similarly the experiment was extended to estimate displacement

- For any frequency with fixed amplitude.
- For any amplitude and frequency

In similar fashion, amplitude or frequency or both is/are estimated for any displacement with other electrical element to be fixed. For all these a set of 667 samples of displacements for various electrical signals are taken.

5

Output of 2 ML models

	Linear	**Quadratic**	**Model**
In V	D=5.05E-10+6.71E-11*A	D=-4.9E-11+6.98E-11*A-1.2E-15*A^2	
0.25	5.22E-10	3.11E-11	1.74E-11
0.5	5.38E-10	1.36E-11	3.75E-11
0.75	5.55E-10	3.83E-12	5.22E-11
1	5.72E-10	2.13E-11	6.82E-11

Conclusion

- Summarization of the work indicates that BW model is more efficient among many non-linear models especially for positioning system.
- By inhibiting the error occurred between expected linear result and non-linear obtained result, even the non-linear system can be linearized
- More stabilized displacement movement can be observed with linearized BW model

- The system has been extended to MIMO to handle 3- axial displacement and also offer low noise using filtering.
- Appropriate Input voltage level to maintain is suggested by the system for required displacement.
- Two Machine learning model (linear and quadratic regression models are applied to estimate displacement for a given electrical signal and vice versa
- In both condition Quadratic estimated more approximately to the results taken in dataset.

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