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Reliability modelling of a parallel-cold standby system with proviso of repair priority

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Abstract

Here, reliability modelling of a parallel cold standby system of four units has been done with the provision of priority to repair. Initially, the system has two operative units in parallel mode (called phase-I units) and the other two units are taken as spare in the cold standby (called phase-II units). The phase-I units (main units) are assumed as more efficient than that of the phase-II units (duplicate units). The units in phase-I and phase-II are non-identical but they are identical within the phases. The units in phase-II work simultaneously at the failure of units of phase-I. The priority to repair the units of phase-II has been given over the repair of the units of the phase-I. However, no priority is given for operation of the units of both phases. The repair activities are handled by a single server who attends the system immediately when required. The failure rate of the units follows negative exponential distribution while the distribution for repair time of the phase-I and phase-II units is assumed as arbitrary. Some significant reliability characteristics such as mean sojourn times (MST), transition probabilities, mean time to system failures (MTSF), availability, expected number of repairs (for both phase-I and phase-II units), busy period of the server, expected number of visits of the server and finally the profit function are obtained in steady state by making use of well-known semi-Markov process (SMP) and regenerative point technique (RPT). The graphical representation of these reliability characteristics has been made for arbitrary values of the rates associated with failure and repair times. The application of the present study can be seen in the power distribution system.

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1. Introduction

The industrial growth plays a key role in economic development of any nation. With the advent of advanced technology, the functioning of systems has become easy and approachable to the users. As a result of this we are now in a position to enjoy quality of life in terms of faultless services of the equipments and suitable reliable products with optimum maintenance. On the other hand, a huge amount of cost is being spent by the manufacturers for developing highly reliable products in order to win the confidence of the customers. In such situations there is no option other than to use efficient performance improvement techniques suggested by the systems designers and reliability engineers. The researchers have suggested some reliability improvement techniques for the repairable systems including the provision of extra units, the units of high quality, and the appropriate configure aspect of the components and proper repair mechanism. Over the years several research papers have been written on stochastic modelling of redundant systems with different functioning modes of the units.

Goyal and Murari [1984] considered three types of repair facilities and compared the stochastic models developed for a cold standby system. Goel et al. [1984] discussed a two unit (multi-component) parallel system with standby unit. Rander et al. [1992] analyzed a two-unit cold standby system with imperfect and perfect repairman. Sherbeny [2010] studied a two non-identical unit parallel system with different types of failure and repairs. Bhardwaj et al. [2010] analyzed probabilistically a system of two non-identical parallel units with the concept of priority to repair. Leung et al. [2011] discussed two dissimilar component cold standby repairable systems. Chen et al. [2014] studied the reliability of cold standby system with imperfect repair and under Poisson shocks. Kakaar et al. [2015] analysed the identical parallel system. Upma and Malik [2016] considered the non-identical units while developing the system models. Kumar and Malik [2018] discussed a non-identical two-unit repairable system. Singh and Poonia [2019] find out probabilistic assessment of two-unit parallel system using RPT. Kadyan et al. [2020] discussed a non-identical repairable system of three units with the concept of simultaneous working of cold-standby units. Rathi et al. [2021] studied the reliability characteristics of parallel cold standby non-identical units with operational priority. The models discussed by different researchers focus either on the identical units in parallel or one non- identical unit in spare. But, there can be situations where one non-identical unit is not capable enough to work at place of failed unit and two or more units are required to work simultaneously in order to meet the system expectations. The studies referred above also indicate that not much work related to the reliability analysis of a system with simultaneous working of parallel and cold standby units in phases has been reported so far in the literature of reliability.

Here, reliability modelling of a parallel cold standby system of four units has been done with the provision of priority to repair. Initially, the system has two operative units in parallel mode (called phase-I units) and the other two units are taken as spare in the cold standby (called phase-II units). The phase-I units (main units) are assumed as more efficient than that of the phase-II units (duplicate units). The units in phases I and II are non-identical but they are identical within the phases. The units in phase-II work simultaneously at the failure of units of phase-I. The priority to repair the units of phase-II has been given over the repair of the units of the phase-I. However, no priority is given for operation of the units of both phases. The repair activities are handled by a single server who attends the system immediately when required. The failure rate of the units follows negative exponential distribution while the distribution for repair time of the phase I and phase-II units is assumed as arbitrary. Some significant reliability characteristics such as "mean sojourn times (MST), transition probabilities, mean time to system failure(MTSF), availability, expected number of repairs (for both phase-I and phase-II units), busy period of the server, expected number of visits of the server and finally the profit function are obtained in steady state by making use of well-known semi-Markov process (SMP) and regenerative point technique" (RPT). The graphical representation of these reliability characteristics has been made for arbitrary values of the rates associated with failure and repair times. The application of the present study can be seen in the power distribution system.

2. Notations

MST	Mean sojourn time
MTSF	Mean time to system failure
O	System is operative
Dc	Cold-standby unit
•	Regenerative point
M/D	Phase-I unit (main units)/ Phase-II unit (duplicate units)

Contd...

MFur/MFwr	Phase-I unit is failed and under repair/waiting for repair
MFUR/ MFWR	Phase-I unit is failed and under repair/waiting for repair continuously from the previous state
DFur/DFwr	Phase-II unit is failed and under repair//waiting for repair
DFUR/ DFWR	Phase-II unit is failed and under repair/waiting for repair continuously from the previous state
λ / λ_1	Failure rate of phase-I unit/ phase-II unit
$g(t)/G(t)$	pdf/cdf of the repair rate of the phase-I unit
$f(t)/F(t)$	pdf/cdf of the repair rate of the phase-II unit
$\overline{G(t)} / \overline{F(t)}$	cdf of repair rate of Phase I /phase II units that repair will not be completed in (0, t]
*/**	Symbol for Laplace transformation/Laplace-Stieltjes
$q_{ij}(t)/Q_{ij}(t)$	pdf/cdf of passage time from regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' without visiting any other regenerative state in (0, t]
$q_{ij.k,r}(t)$ / $Q_{ij.k,r}(t)$	pdf/cdf of direct transition time from regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' visiting state k, r once in (0, t]
$q_{ij.k(r,s)^n}$ / $Q_{ij.k(r,s)^n}$	pdf/cdf of direct transition time from regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' visiting state k once and n-times states r and s
s/ ©	Symbol for Stieltjes convolution / Laplace Convolution.
LIT/ LT/LST	Laplace Inverse Transform/ Laplace Transform/Laplace Stieltjes Transform
$A_i(t)$	Probability that the system is in up-state at instant time 't'
$V_i^S(t)$	Expected number of visits of server
$R_i^M(t)$ / $R_i^D(t)$	Expected number of repairs of phase-I /phase-II Units
$B_i^R(t)$	Busy period of server due to repair
W_i(t)	Probability that the server is busy in the state Si up to time 't' without making any transition to any other regenerative state or returning to the same state via one or more non-regenerative states

The possible state transition diagram for the model can be visualised as shown in Figure 1 below :

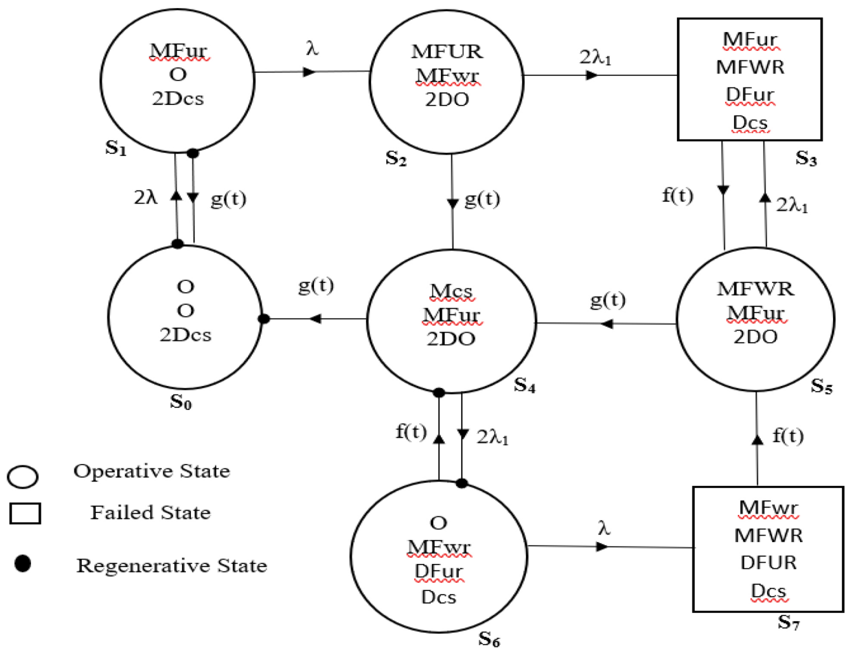

Figure 1

State transition diagram

3. System Descriptions

S_0	O O 2Dcs	The initial state in which the phase-I units are in operation and phase-II units are in spare
S_1	MFur O 2Dcs	Second state in which one of the phase-I units is in operation and other is failed under repair and phase-II units is in cold-standby
S_2	MFUR MFwr 2DO	The third state in which one of the phase-I units is continuously failed under repair and other is failed waiting for repair and the phase-II units are in operation mode
S_3	MFwr MFWR DFur Dcs	In fourth state system model is failed in which phase-I units is continuously failed waiting for repaired and other unit is waiting for repair and one of the phase-II units is failed under repair and other one is in cold-standby mode

Contd...

S_4	Mcs MFur 2DO	The fifth state in which one of the phase-I unit is in cold-standby mode and other phase-I unit is failed under repair and phase-II units are in operation
S_5	MFWR MFur 2DO	The sixth state in which phase-I units continuously failed waiting for repair and other phase-I unit is failed under repair and phase-II units are in operation
S_6	O MFwr DFur Dcs	The seventh state in which phase-I units is continuously failed waiting for repaired and other unit is waiting for repair and one of the phase-II units is failed under repair and other one is in cold-standby mode
S_7	MFwr MFWR DFUR Dcs	The eighth state in which phase-I units is failed waiting for repaired and other phase-I unit is failed continuously waiting for repair and one of the phase-II units is failed continuously under repair and other one is in cold-standby mode

4. Transition Probabilities and Mean Sojourn Times

The expressions for transition probabilities from state i to j as follow:

$$p_{ij} = Q_{ij}(\infty) = \int_0^\infty q_{ij}(t)\,dt \qquad p_{ij} = \lim_{n\to\infty} Q_{ij}(t)dt$$

$$dQ_{01}(t) = 2\lambda e^{-2\lambda t}\mathrm{dt} \qquad dQ_{10}(t) = e^{-\lambda t} g(t)\mathrm{dt} \qquad dQ_{12}(t) = \lambda e^{-\lambda t}\overline{G(t)}\mathrm{dt}$$

$$dQ_{23}(t) = 2\lambda_1 e^{-2\lambda_1 t}\overline{G(t)}\mathrm{dt} \qquad dQ_{24}(t) = e^{-2\lambda_1 t} g(t)\mathrm{dt} \qquad dQ_{35}(t) = f(t)\mathrm{dt}$$

$$dQ_{40}(t) = e^{-2\lambda_1 t} g(t)\mathrm{dt} \qquad dQ_{46}(t) = 2\lambda_1 e^{-2\lambda_1 t}\overline{G(t)}\mathrm{dt}$$

$$dQ_{53}(t) = 2\lambda_1 e^{-2\lambda_1 t}\overline{G(t)}\mathrm{dt} \qquad dQ_{54}(t) = e^{-2\lambda_1 t} g(t)\mathrm{dt}$$

$$dQ_{64}(t) = e^{-\ddot{e}\mathrm{t}} f(t)\mathrm{dt} dQ_{14.2}(t) = dQ_{12}(t)\mathbf{s}dQ_{24}(t)$$

$$dQ_{14.2(3,5)^n}(t) = \frac{dQ_{12}(t)\mathbf{s}dQ_{23}(t)\mathbf{s}dQ_{35}(t)\mathbf{s}dQ_{54}(t)(t)}{1-dQ_{35}(t)dQ_{53}(t)}$$

$$dQ_{64.75}(t) = dQ_{67}(t)\mathbf{s}dQ_{75}(t)\mathbf{s}dQ_{54}(t)$$

$$dQ_{64.75(3,5)^n}(t) = \frac{dQ_{67}(t)\mathbf{s}dQ_{75}(t)\mathbf{s}dQ_{53}(t)\mathbf{s}dQ_{35}\mathbf{s}dQ_{54}(t)}{1-dQ_{35}(t)dQ_{53}(t)}$$

$$p_{ij} = \lim_{t\to\infty} Q_{ij}(t)\mathrm{dt} = \lim_{s\to 0} Q_{ij}^{**}(s)$$

$p_{01} = p_{35} = p_{75} = f^*(0) = 1$

$p_{10} = g^*(\lambda)$ $\quad p_{12} = 1 - g^*(\lambda)$ $\quad p_{23} = p_{46} = p_{53} = 1 - g^*(2\lambda_1)$

$p_{24} = p_{40} = p_{54} = g^*(2\lambda_1)$ $\quad p_{67} = 1 - f^*(\lambda)$

$p_{14.2} = p_{12}p_{24}$ $\quad p_{14.2(3,5)^n} = \frac{p_{12}p_{23}p_{35}p_{54}}{1-p_{35}p_{53}} = p_{12}p_{23}$

$p_{64.75(3,5)^n} = \frac{p_{67}p_{75}p_{53}p_{35}p_{54}}{1-p_{35}p_{53}} = p_{67}p_{75}p_{53}$ $\quad p_{64.75} = p_{67}p_{75}p_{54}$

Also, it is verified that

$$p_{01} = p_{10} + p_{12} = p_{23} + p_{24} = p_{35} = p_{40} + p_{46} = p_{53} + p_{54} = p_{64} + p_{67}$$
$$= p_{75} = p_{10} + p_{14.2} + p_{14.2(3,5)^n} = p_{64} + p_{64.75} + p_{64.75(3,5)^n} = 1$$

Mean Sojourn Times

$\mu_0 = m_{01} = \frac{1}{2\lambda}$ $\quad \mu_1 = m_{10} + m_{12} = \frac{1}{\lambda}[1 - g^*(\lambda)]$

$\mu_2 = m_{23} + m_{24} = \frac{1}{2\lambda_1}[1 - g^*(2\lambda_1)]$ $\quad \mu_3 = m_{35} = -f^{*\prime}(0)$

$\mu_4 = m_{40} + m_{46} = \frac{1}{2\lambda_1}[1 - g^*(2\lambda_1)]$ $\quad \mu_5 = m_{53} + m_{54} = \frac{1}{2\lambda_1}[1 - g^*(2\lambda_1)]$

$\mu_6 = m_{64} + m_{67} = \frac{1}{\lambda}[1 - f^*(\lambda)]$ $\quad \mu_7 = m_{35} = -f^{*\prime}(0)$

$\mu_1' = \mu_1 + \mu_2 p_{12} + \mu_3 p_{12}p_{23} + \frac{p_{12}p_{23}(\mu_3+\mu_5)}{p_{46}}$

$\mu_6' = \mu_6 + \mu_2 p_{67}(\mu_5 + \mu_7) + \frac{p_{67}p_{53}(\mu_3+\mu_5)}{p_{54}}$

5. Reliability and MTSF

Let $\phi_i(t)$ be the cdf of the first passage time from regenerative state i to a failed state. Regarding failed state as absorbing state, we have

$\phi_0(t) = Q_{01}(t)\mathbf{s}\phi_1(t)$

$\phi_1(t) = Q_{10}(t)\mathbf{s}\phi_0(t) + Q_{14.2}(t)\mathbf{s}\phi_4(t) + Q_{13.2}(t)$

$\phi_4(t) = Q_{40}(t) \mathbf{s} \phi_0(t) + Q_{46}(t) \mathbf{s} \phi_6(t)$

$\phi_6(t) = Q_{64}(t) \mathbf{s} \phi_4(t) + Q_{67}(t)$

By taking LST of the above expressions, the reliability of the system model can be obtained as:

$$\phi_0^{**}(s) = \frac{\Delta_1}{\Delta} = \frac{Q_{01}^{**}(s)\ [Q_{13.2}^{**}(s)(1-Q_{46}^{**}(s)Q_{64}^{**}(s))+Q_{14.2}^{**}(s)(Q_{46}^{**}(s)Q_{67}^{**}(s))]}{(1-Q_{46}^{**}(s)Q_{64}^{**}(s))(1-Q_{01}^{**}(s)Q_{10}^{**}(s))+Q_{14.2}^{**}(s)Q_{40}^{**}(s)}$$

$$MTSF = \lim_{s\to 0} \frac{1-\phi_0^{**}(s)}{s} = \frac{N_1}{D_1}$$

$$\begin{aligned} N_1 = {} & p_{40}m_{46} + p_{46}m_{46} - \mu_0 p_{10}p_{46}p_{64} - m_{10}p_{46}p_{64} - p_{10}m_{46}p_{64} - p_{10}p_{46}m_{64} \\ & + \mu_0 p_{10} + m_{10} + \mu_0 p_{10}p_{24}p_{40} + m_{12}p_{24}p_{40} + m_{24}p_{12}p_{40} + p_{12}p_{24}m_{40} \\ & + \mu_0 p_{12}p_{23} + m_{12}p_{23} + p_{12}m_{23} - \mu_0 p_{12}p_{23}p_{46}p_{64} - m_{12}p_{23}p_{46}p_{64} \\ & - p_{12}m_{23}p_{46}p_{64} - p_{12}p_{23}m_{46}p_{64} - p_{12}p_{23}p_{46}m_{64} + \mu_0 p_{12}p_{24}p_{46}p_{67} \\ & + m_{12}p_{24}p_{46}p_{67} + p_{12}m_{24}p_{46}p_{67} + p_{12}p_{24}m_{46}p_{67} + p_{12}p_{24}p_{46}m_{67} \end{aligned}$$

$D_1 = (1 - p_{46}p_{64})(1 - p_{10} - p_{12}p_{24}p_{40})$

The reliability of the system is determined as:

$$\boldsymbol{R}^*(\boldsymbol{s}) = \frac{\mathbf{1} - \boldsymbol{\phi}_0^{**}(s)}{\boldsymbol{s}} \text{ and } \boldsymbol{R}(\boldsymbol{t}) = \boldsymbol{L}^{-1}(\boldsymbol{R}^*(\boldsymbol{s}))$$

6. Steady State Availability

Let $A_i(t)$ be the probability that the system is in up-state at instant 't' given that the system entered regenerative state 'i' at time instant t=0. We have

$A_0(t) = M_0(t) + q_{01}(t) © A_1(t)$

$A_1(t) = M_1(t) + q_{10}(t) © A_0(t) + [q_{14.2}(t) + q_{14.2(3,5)^n}(t)] © A_4(t)$

$A_4(t) = M_4(t) + q_{40}(t) © A_0(t) + q_{46}(t) © A_6(t)$

$A_6(t) = M_6(t) + [q_{64}(t) + q_{64.75}(t) + q_{64.7,5(3,5)^n}(t)] © A_4(t)$

Where $M_i(t)$ is the probability that the system is up initially in state i is up at time t without visiting to any other regenerative state.

$M_0(t) = e^{-2\lambda t}\mathrm{dt},\ M_1(t) = e^{-\lambda t}\overline{G(t)}\mathrm{dt}\ \ M_4(t) = e^{-2\lambda_1 t}\overline{G(t)}\mathrm{dt}\ \ M_6(t) = e^{-\text{ë}t}\overline{F(t)}\mathrm{dt}$

Taking LT of above equations and solving for $A_0^*(s)$, the steady state availability is given by:

$$A_0(\infty) = \lim_{s\to 0} sA_0^*(s) = \frac{N_2}{D_2}$$

where,

$N_2 = [\mu_0 + \mu_1]p_{40} + p_{12}[\mu_4 + \mu_6 p_{46}]$ and

$D_2 = [\mu_0 + \mu_1']p_{40} + p_{12}[\mu_4 + \mu_6' p_{46}]$

7. Expected Number of Repairs of Phase-I Units

Let $R_i^M(t)$ be the expected number of repairs of phase-I units given to the server in (0, t] such that the system entered regenerative state 'i' at t=0. We have

$$R_0^M(t) = Q_{01}(t)\mathbf{s}R_1^M(t)$$

$$R_1^M(t) = Q_{10}(t)\mathbf{s}[1 + R_0^M(t)] + [Q_{14.2}(t) + Q_{14.2(3,5)^n}(t)]\mathbf{s}[1 + R_4^M(t)]$$

$$R_4^M(t) = Q_{40}(t)\mathbf{s}[1 + R_0^M(t)] + Q_{46}(t)\mathbf{s}R_6^M(t)$$

$$R_6^M(t) = Q_{64}(t)\mathbf{s}R_4^M(t) + [Q_{64.75}(t) + Q_{64.7,5(3,5)^n}(t)]\mathbf{s}[1 + R_4^M(t)]$$

Taking LST of above equations and solving for $R_0^{M**}(s)$, by using Cramer's Rule we get the expected number of repairs of phase-I unit as:

$$R_0^M(\infty) = \lim_{s\to 0} sR_0^{M**}(t) = \frac{\Delta_1}{\Delta_1'} = \frac{N_3}{D_2}$$

where,

$N_3 = p_{40} + p_{12}\ [p_{40} + p_{46}p_{67}]$ and

$D_2 = [\mu_0 + \mu_1']p_{40} + p_{12}[\mu_4 + \mu_6' p_{46}]$

8. Expected Number of Repairs of Phase-II Units

Let $R_i^D(t)$ be the expected number of repairs of phase-II unit given to the server in (0, t] such that the system entered regenerative state 'i' at t=0. We have

$$R_0^D(t) = Q_{01}(t)\text{s}R_1^D(t)$$

$$R_1^D(t) = Q_{10}(t)\text{s}R_0^D(t) + Q_{14.2}(t)\text{s}R_4^D(t) + [Q_{14.2(3,5)^n}(t)]\text{s}[1 + R_4^D(t)]$$

$$R_4^D(t) = Q_{40}(t)\text{s}R_0^D(t) + Q_{46}(t)\text{s}R_6^D(t)$$

$$R_6^D(t) = Q_{64}(t)\text{s}R_4^D(t) + [Q_{64.75}(t) + Q_{64.7,5(3,5)^n}(t)]\text{s}[1 + R_4^D(t)]$$

Taking LST of above equations and solving for $R_0^{D**}(s)$, by using this we get expected number of repairs of phase-II units as:

$$R_0^D(\infty) = \lim_{s \to 0} sR_0^{D**}(s) = \frac{N_4}{D_2}$$

Where $N_4 = p_{46}p_{14.2(3,5)^n}$ and

$$D_2 = [\mu_0 + \mu_1']p_{40} + p_{12}[\mu_4 + \mu_6' p_{46}]$$

9. Expected Number of Visits by the Serve

Let V_0^S be the expected number of repairs of duplicate unit by the repairman in (0, t] such that the system entered regenerative state 'i' at t=0. We have

$$V_0^S(t) = Q_{01}(t)\text{s}[1 + V_1^S(t)]$$

$$V_1^S(t) = Q_{10}(t)\text{s}V_0^S(t) + [Q_{14.2}(t)\text{s} + Q_{14.2(3,5)^n}(t)\text{s}]V_4^S(t)$$

$$V_4^S(t) = Q_{40}(t)\text{s}V_0^S(t) + Q_{46}(t)\text{s}V_6^S(t)$$

$$V_6^S(t) = [Q_{64}(t)\text{s}V_4^S(t) + [Q_{64.75}(t) + Q_{64.7,5(3,5)^n}(t)\text{s}V_4^S(t)]$$

Taking LST of above equations and solving for $V_0^{s**}(s)$, by using this we get expected number of visits of the server:

$$V_0^s(\infty) = \lim_{s \to 0} sV_0^{s**}(s) = \frac{N_5}{D_2}$$

Where $N_5 = p_{60}[1 - p_{12}p_{21}]$ and

$$D_2 = [\mu_0 + \mu_1']p_{40} + p_{12}[\mu_4 + \mu_6' p_{46}]$$

10. Busy Period Analysis for the Server due to Repair

Let $B_i^R(t)$ be the probability that a server is busy at the time point given that the system entered in the regenerative state 'i' at t=0. We have

$$B_0^R(t) = q_{01}(t)©B_1^R(t)$$

$$B_1^R(t) = W_1^R(t) + q_{10}(t)©B_0^R(t) + q_{11.2}(t)©B_1^R(t) + [q_{16.2,3,4}(t) + q_{16.2,3(4,5)^n}(t)]©B_6^R(t)$$

$$B_6^R(t) = W_6^R(t) + q_{60}(t)©B_0^R(t) + q_{67}(t)©B_7^R(t)$$

$$B_7^R(t) = W_7^R(t) + [q_{76}(t) + q_{76.(5,4)^n}(t)]©B_6^R(t)$$

Where,

$W_1^R(t) = e^{-\lambda t}\overline{G(t)},\ \ W_6^R(t) = e^{-\lambda t}\overline{F(t)},$ and $W_7^R(t) = e^{-\lambda t}\overline{G(t)}$

Taking LT of the above expressions and solving for $B_0^{R^*}(s),$ we have

$$B_0^R(\infty) = \lim_{s\to 0} sB_0^{R^*}(s) = \frac{N_6}{D_2}$$

where,

$$N_6 = W_0^{R^*}(0)p_{60} + p_{12}p_{23}[W_6^{R^*}(0) + W_6^{R^*}(0)P_{67}]$$

$$D_2 = [\mu_0 + \mu_1']p_{40} + p_{12}[\mu_4 + \mu_6' p_{46}]$$

11. Profit Analysis

The profit (P) incurred to the system model in steady state can be obtained as:

$$P = K_0A_0 - K_1B_0^R - K_2R_0^D - K_3R_0^M - K_4V_0^S$$

Where,

K_0 = Revenue per unit up time of the system

K_1 = Cost per unit time for which server is busy to repair

K_3 = Cost per unit time repairs of phase-I unit

K_2 = Cost per unit time repairs of phase-II unit

K_4 = Cost per unit time visit of the server

12. Numerical and Graphical Representations of Reliability Measures

The reliability characteristics of the system model have been obtained by assuming arbitrary distributions for repairs rates of the units. The results for these measures have also been obtained for the particular situations where the repair rates of the units follow negative exponential distribution. The behaviour of MTSF, Availability and Profit function have been shown numerically and graphically respectively in the Tables 1, 2, 3 and in the Figures 2, 3 and 4.

Here, we take the repair time distribution as negative exponential: $g(t) = \xi e^{-\xi t}$ and $f(t) = \Psi e^{-\Psi t}$

Table 1

MTSF vs Failure rate of phase-I unit

λ	$\lambda 1 = 0.001$, $\Psi = 2,\ \xi = 1.5$	$\lambda 1 = 0.002$, $\Psi = 2.0,\ \xi = 1.5$	$\lambda 1 = 0.001$, $\Psi = 2,\ \xi = 2.5$	$\lambda 1 = 0.001$, $\Psi = 2.5,\ \xi = 1.5$
0.1	401188.22	106900.20	698961.61	401201.98
0.11	396450.08	104520.96	685642.54	396462.97
0.12	392856.68	102716.54	675529.64	392868.87
0.13	390067.87	101316.14	667672.38	390079.50
0.14	387860.85	100207.87	661447.73	387872.02
0.15	386084.76	99316.00	656433.53	386095.56
0.16	384634.54	98587.76	652335.60	384645.05
0.17	383435.25	97985.52	648943.84	383445.50
0.18	382432.21	97481.83	646105.01	382442.26
0.19	381584.86	97056.33	643705.22	381594.75

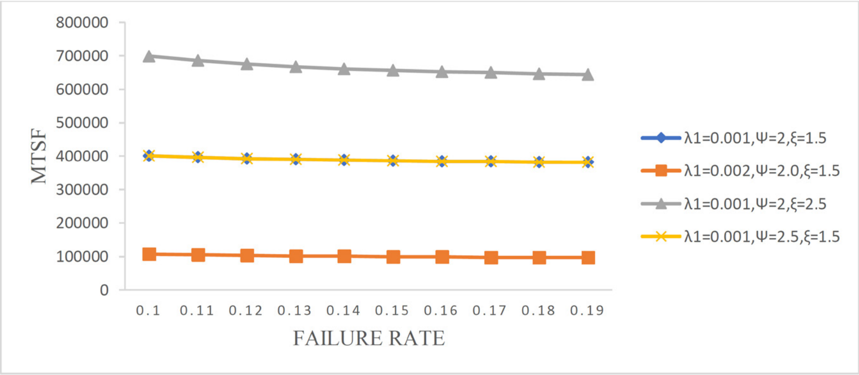

Figure 2

MTSF Vs Failure rate of phase-I unit

Table 2

Availability Vs Failure rate of phase-I unit

λ	$\lambda 1 = 0.001$, $\Psi = 2$, $\xi = 1.5$	$\lambda 1 = 0.002$, $\Psi = 2.0$, $\xi = 1.5$	$\lambda 1 = 0.001$, $\Psi = 2.0$, $\xi = 2.5$	$\lambda 1 = 0.001$, $\Psi = 2.5$, $\xi = 1.5$
0.1	0.9926855	0.9926703	0.9971532	0.9926886
0.11	0.9913186	0.9913005	0.9965957	0.9913223
0.12	0.989865	0.9898438	0.9959959	0.9898694
0.13	0.9883308	0.9883063	0.9953555	0.9883358
0.14	0.9867217	0.9866938	0.994676	0.9867274
0.15	0.9850431	0.9850116	0.9939592	0.9850496
0.16	0.9833003	0.983265	0.9932064	0.9833075
0.17	0.981498	0.9814589	0.9924191	0.9815061
0.18	0.9796409	0.9795978	0.9915988	0.9796498
0.19	0.9777333	0.9776861	0.9907469	0.9777431

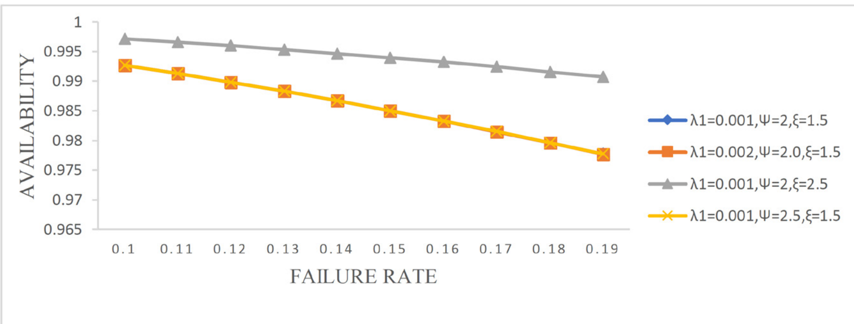

Figure 3

Availability Vs Failure rate of phase-I unit

Table 3

Profit Vs Failure rate of phase-I unit

λ	$\lambda 1 = 0.001$, $\Psi = 2.0$, $\xi = 1.5$	$\lambda 1 = 0.002$, $\Psi = 2.0$, $\xi = 1.5$	$\lambda 1 = 0.001$, $\Psi = 2.0$, $\xi = 2.5$	$\lambda 1 = 0.001$, $\Psi = 2.5$, $\xi = 1.5$
0.1	14049.39	14049.16	14212.97	14049.44
0.11	13956.81	13956.53	14136.06	13956.87

Contd...

0.12	13864.90	13864.58	14059.53	13864.97
0.13	13773.70	13773.33	13983.39	13773.77
0.14	13683.24	13682.82	13907.66	13683.32
0.15	13593.55	13593.08	13832.35	13593.65
0.16	13504.66	13504.13	13757.47	13504.77
0.17	13416.60	13416.02	13683.03	13416.72
0.18	13329.38	13328.74	13609.02	13329.51
0.19	13243.03	13242.33	13535.47	13243.17

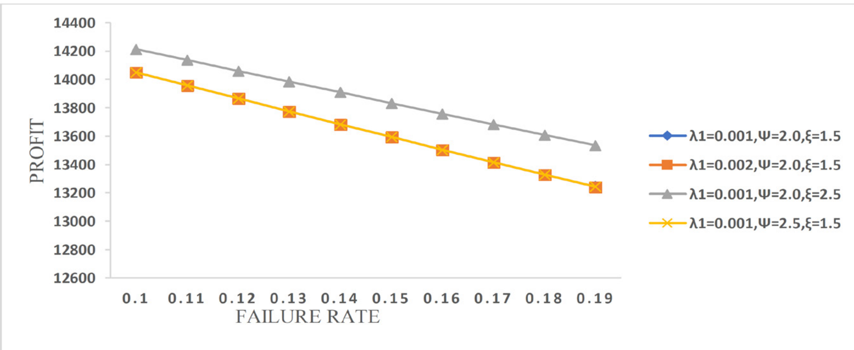

Figure 4

Profit Vs Failure rate of phase-I unit

13. Application

The present study has the application in the drainage system where flooded water is required to be drained out from low lying fields to the adjoining drain. During rainy season water accumulates in the fields which ultimately damage the field crops. To drain out the same, electric pumping sets are to be installed at site. In case of electric/mechanical pumps (Phase-I units) failure additional arrangement of two diesel pumping set (Phase-II units) are required to be taken in the set up. Sometimes it becomes necessary to provide to priority in repairs of the units in order to reduce down time and also to minimize the maintenance cost. Thus, in this case priority to repair the diesel pumping units (phase-II units) is given over the electric pumps (phase-I units). The system can be shown in the following figure 5:

14. Conclusion

A Parallel-Cold Standby system of four units has been analysed stochastically with the idea of priority for repair to the phase-II units.

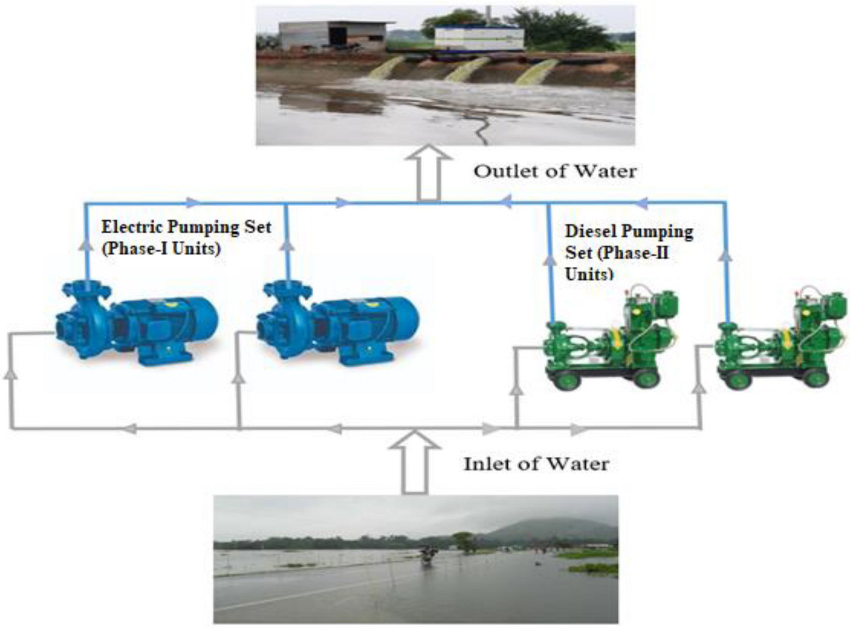

Figure 5

Drainage system

There are four units in system comprising two units as phase-I units which work initially in parallel mode and the other two units (called phase-II units) which remain as spare in cold standby mode. The phase-II units can be installed to work simultaneously at the failure of the phase-I units. The important reliability characteristics have been obtained and analysed for arbitrary values of the parameters. Graphical and tabulated presentations have been studied by taking exponential distributions for the repair time i.e. $g(t) = \xi e^{-\xi t}$ and $f(t) = \Psi e^{-\Psi t}$. The results are shown graphically in the Figures 2, 3, and 4 respectively. This study depicts that MTSF, availability and profit of a system have downward trend with the increase of failure rate of the main unit and duplicate unit while they increase with the increase in repair rate of main unit. There is little change (almost negligible) in the values of availability with the increase of repair rate of Phase-II units. We conclude that a repairable parallel-cold standby system of four units can be made to use in a better way in terms of reliability by increasing repair rate of phase-I units. Finally, the present study suggests that spending money on increasing repair rate of phase-II units will not be fruitful and thus, one should avoid the use of low-quality units in standby.

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