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On the Poisson transmuted Ailamujia distribution with applications to dispersed and skewed count data

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Abstract

Assuming classical Poisson distribution for skewed and dispersed count observations may be misleading since it assumes equality for mean and variance. A new count distribution suitable for modelling dispersed and skewed count observations is proposed in this paper. This is achieved by assuming transmuted Ailamujia distribution for the parameter of the Poisson distribution and used mixed Poisson distribution process to obtain a new distribution. Mathematical properties of the new distribution are obtained and some measures of dispersion are assessed by assuming different parameter combinations for the distribution. Using the MLE for parameter estimation, the performance of the new

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proposition is assessed on four referred dispersed count data. Results obtained show that the new proposition provide better fit for all the datasets considered.

Subject Classification: *62G08, 62P20.*

Keywords: *Ailamujia distribution, Quadratic Transmutation, Mixed Poisson distribution, Maximum likelihood estimation.*

1. Introduction

Many real life phenomena are discrete in nature and the need to assume appropriate distribution in modelling them always arise. Among traditional count distributions is the Poisson distribution which assumes equality for mean and variance of observations. This assumption is rarely obtained in nature. In most times, count observations are dispersed (variance higher than mean)) and as such, assuming Poisson distribution may be misleading.

Unlike continuous distributions with positive supports, there are limits to the number of existing discrete distribution that can be assumed. This has given risen to different discretization procedures. Roy (2004) proposed a process using the survival function, $S(x)$, of the continuous distribution to obtain its discrete equivalent as:

$$P(X=x)=S(x)-S(x+1),\ x=0,1,2,\ldots$$

This technique has been utilized to develop many discrete distributions. See references in (Alghamdi et al., 2022) for details.

Another discretization method that has received patronage is the usage of the Poisson process popularized by (Willmot, 1986). This technique has received attentions especially with applications to discrete data with many zeros (Bhati et al., 2017; Iyer-Biswas & Jayaprakash, 2014; Simeunović et al., 2018).

Among univariate distributions, the Ailamujia distribution (Lv et al., 2002) is recently receiving attentions in lifetime data modelling (Aijaz et al., 2020; Ain et al., 2020; Jamal et al., 2021; Jayakumar & Elangovan, 2019; Rather et al., 2022). The mixed Poisson process has been used on the distribution to obtain a more flexible discrete distribution (Hassan et al., 2020).

In order to improve flexibility and applicability, this paper utilizes the quadratic transmutation map (Shaw & Buckley, 2007) on the Ailamujia distribution to obtain a new continuous distribution that is assumed as the mixing distribution for a new mixed Poisson distribution.

2. Methodology

If a random variable ***N*** assumes a Poisson distribution with parameter $\lambda > 0$, its PMF is given as:

$$P_n = \frac{e^{-\lambda}\lambda^n}{n!}, \; n = 0,1,2,\ldots \tag{1}$$

Also, if a continuous random variable ***X*** assumes Ailamujia distribution (Lv et al., 2002) with parameter $\theta > 0$, its distribution function is given as:

$$G(x) = 1-(1+\theta x)e^{-\theta x}, \; x \geq 0 \tag{2}$$

By assuming the quadratic transmutation map (Shaw & Buckley, 2007) with distribution function given as:

$$F(x) = (1+a)G(x) - a(G(x))^2; \qquad x \in \mathbb{R}, \; |a| \leq 1 \tag{3}$$

The Quadratic Transmutation Ailamujia Distribution (QTAD) with transmutation parameter $a| \leq 1$ is obtained with the distribution function given as:

$$F(x) = 1 - a(1+\theta x)^2 e^{-2\theta x} + (a-1)(1+\theta x)e^{-\theta x}; \qquad x \in \mathbb{R}, \; |a| \leq 1 \tag{4}$$

The corresponding probability distribution function of the QTAD is obtained as:

$$f(x) = \theta^2 x e^{-\theta x} - a\theta^2 x e^{-\theta x} + 2a\theta^2 x e^{-2\theta x} + 2a\theta^3 x^2 e^{-2\theta x} \tag{5}$$

Figure 1 shows that QTAD is unimodal and positively skewed.

Assuming the parameter of the Poisson distribution λ in (1) follows the QTAD in (5), then the density function of λ is obtained as:

$$\pi(\lambda) = \theta^2 \lambda e^{-\theta\lambda} - a\theta^2 \lambda e^{-\theta\lambda} + 2a\theta^2 \lambda e^{-2\theta\lambda} + 2a\theta^3 \lambda^2 e^{-2\theta\lambda} \tag{6}$$

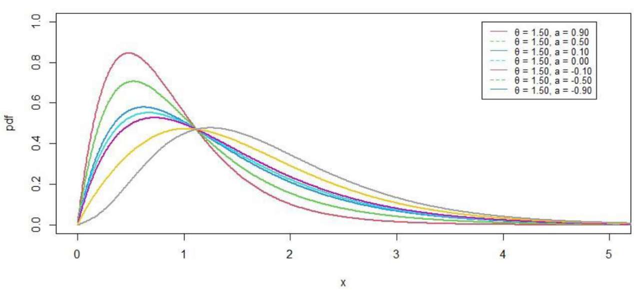

Figure 1

Shapes of the PDF of QTAD for different values of *a* when θ = 1.50

Hence, the marginal of $n\,|\,\lambda$ is obtained by solving

$$P_n = \int_0^\infty \frac{\lambda^n e^{-\lambda}}{n!}\pi(\lambda)d\lambda$$

The resulting P_n when it exists is the mixed Poisson distribution with mixing distribution $\pi(\lambda)$.

3. New Proposition

3.1 *Poisson QTAD*

The Poisson Quadratic Transmuted Ailamujia Distribution (QTAD) involves using the quadratic transmutation of map (Shaw & Buckley, 2007) to compound the Ailamujia Distribution (AD).

3.2 *The PMF of the Poisson QTAD*

Proposition 1: If a random variable $N \sim Poisson(\lambda)$ and $\lambda \sim QTAD(a,\theta)$, then PMF of a discrete random variable ***N*** has a Poisson-QTAD if its PMF is defined as:

$$P_n = \frac{\theta^2(n+1)}{(1+\theta)^{n+2}} - \frac{a\theta^2(n+1)}{(1+\theta)^{n+2}} + \frac{2a\theta^2(n+1)}{(1+2\theta)^{n+2}} + \frac{2a\theta^3(n+1)(n+2)}{(1+2\theta)^{n+3}};x=0,1,2,\ldots,\theta>0,-1\le a\le 1 \tag{7}$$

Proof: Let $N \sim Poisson(\lambda)$, its PMF is given as:

$$f(n\,|\,\lambda) = \frac{\lambda^n e^{-\lambda}}{n!},\; E(N\,|\,\lambda) = Var(N\,|\,\lambda) = \lambda$$

Therefore,

$$\begin{aligned}
P_n &= \int_0^\infty f(n|\,\lambda)\pi(\lambda)d\lambda \\
&= \int_0^\infty \frac{\lambda^n e^{-\lambda}}{n!}.(\theta^2\lambda e^{-\theta\lambda} - a\theta^2\lambda e^{-\theta\lambda} + 2a\theta^2\lambda e^{-2\theta\lambda} + 2a\theta^3\lambda^2 e^{-2\theta\lambda})d\lambda \\
&= \frac{\theta^2}{n!}\int_0^\infty \lambda^{n+1}e^{-(1+\theta)\lambda}d\lambda - \frac{a\theta^2}{n!}\int_0^\infty \lambda^{n+1}e^{-(1+\theta)\lambda}d\lambda + \frac{2a\theta^2}{n!}\int_0^\infty \lambda^{n+1}e^{-(1+2\theta)\lambda}d\lambda \\
&\quad + \frac{2a\theta^3}{n!}\int_0^\infty \lambda^{n+2}e^{-(1+2\theta)\lambda}d\lambda
\end{aligned}$$

Assuming

$$u=(1+\theta)\lambda \rightarrow \lambda=\frac{u}{1+\theta} \rightarrow d\lambda=\frac{du}{1+\theta}$$
$$v=(1+2\theta)\lambda \rightarrow \lambda=\frac{v}{1+2\theta} \rightarrow d\lambda=\frac{dv}{1+2\theta}$$

Then,

$$P_n=\frac{\theta^2}{n!(1+\theta)^{n+2}}\int_0^\infty u^{n+1}e^{-u}du-\frac{a\theta^2}{n!(1+\theta)^{n+2}}\int_0^\infty u^{n+1}e^{-u}du$$
$$+\frac{2a\theta^2}{n!(1+2\theta)^{n+2}}+\frac{2a\theta^3}{n!(1+2\theta)^{n+3}}\int_0^\infty v^{n+2}e^{-v}dv$$

$$=\frac{\theta^2(n+1)}{(1+\theta)^{n+2}}-\frac{a\theta^2(n+1)}{(1+\theta)^{n+2}}+\frac{2a\theta^2(n+1)}{(1+2\theta)^{n+2}}+\frac{2a\theta^3(n+1)(n+2)}{(1+2\theta)^{n+3}}$$

The distribution becomes the Poisson-Ailamujia Distribution (Hassan et al., 2020) when the transmutation parameter a = 0. Possible shapes of the PMF of the Poisson Quadratic Transmuted Ailamujia Distribution (P-QTAD) (shown in Figure 2) for different values of the parameters θ and a below reveal that the distribution is unimodal and can be used to model positively skewed observations.

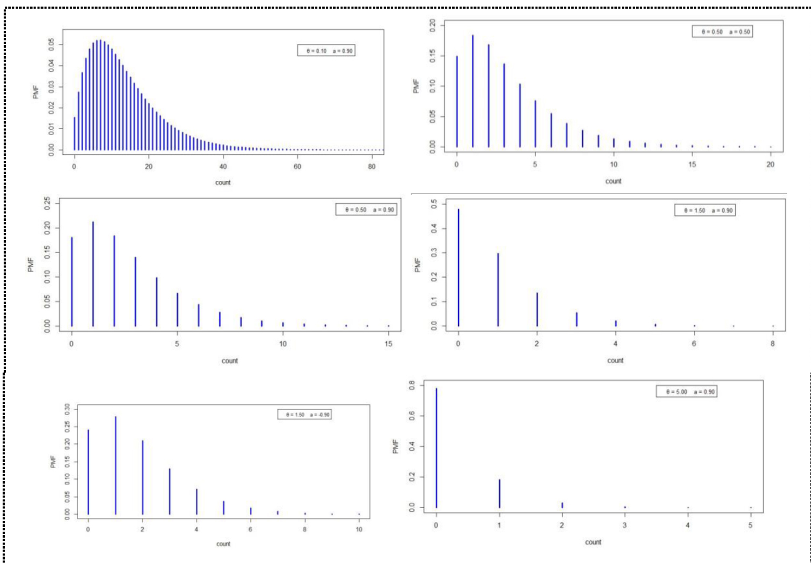

Figure 2

Shapes of the PMF of the Poisson QTAD

3.3 *The CDF of the Poisson QTAD*

Proposition 2: If a discrete random variable *N* has the Poisson QTAD, the CDF is obtained as:

$$F(n)=1+\frac{(a-1)(2\theta+\theta n+1)}{(1+\theta)^{n+2}}-\frac{a\theta^2(n^2+7n+10)+2a\theta(n+3)+a}{(1+2\theta)^{n+3}} \tag{8}$$

Proof:

$$\begin{aligned}
F(n)&=P(N\le n)\\
&=\sum_{n=0}^{n}P_n=\sum_{n=0}^{n}\left(\frac{\theta^2(n+1)}{(1+\theta)^{n+2}}-\frac{a\theta^2(n+1)}{(1+\theta)^{n+2}}+\frac{2a\theta^2(n+1)}{(1+2\theta)^{n+2}}+\frac{2a\theta^3(n+1)(n+2)}{(1+2\theta)^{n+3}}\right)\\
&=\sum_{n=0}^{n}\frac{\theta^2(n+1)}{(1+\theta)^{n+2}}-\sum_{n=0}^{n}\frac{a\theta^2(n+1)}{(1+\theta)^{n+2}}+\sum_{n=0}^{n}\frac{2a\theta^2(n+1)}{(1+2\theta)^{n+2}}+\sum_{n=0}^{n}\frac{2a\theta^3(n+1)(n+2)}{(1+2\theta)^{n+3}}\\
&=\frac{\theta^2}{(1+\theta)^2}\sum_{n=0}^{n}\frac{(n+1)}{(1+\theta)^n}-\frac{a\theta^2}{(1+\theta)^2}\sum_{n=0}^{n}\frac{(n+1)}{(1+\theta)^n}+\frac{2a\theta^2}{(1+2\theta)^2}\sum_{n=0}^{n}\frac{(n+1)}{(1+2\theta)^n}+\frac{2a\theta^3}{(1+2\theta)^3}\sum_{n=0}^{n}\frac{(n+1)(n+2)}{(1+2\theta)^n}\\
&=1+\frac{(a-1)(2\theta+\theta n+1)}{(1+\theta)^{n+2}}-\frac{a\theta^2(n^2+7n+10)+2a\theta(n+3)+a}{(1+2\theta)^{n+3}}
\end{aligned}$$

4. Mathematical properties of the poisson qtad

4.1 *Probability Generating Function*

Given $\pi(\lambda)$ as the mixing distribution of a mixed Poisson distribution for a discrete random variable ***N*** with the Poisson QTAD, Willmot (1986) defined the PGF of ***N*** as:

$$\begin{aligned}
P_n(z)&=\int_0^\infty e^{\lambda(z-1)}\,\pi(\lambda)d\lambda\\
&=\int_0^\infty e^{\lambda(z-1)}(\theta^2\lambda e^{-\theta\lambda}-a\theta^2\lambda e^{-\theta\lambda}+2a\theta^2\lambda e^{-2\theta\lambda}+2a\theta^3\lambda^2 e^{-2\theta\lambda})d\lambda\\
&=\theta^2\int_0^\infty(\lambda e^{-(1+\theta-z)\lambda})d\lambda-a\theta^2\int_0^\infty(\lambda e^{-(1+\theta-z)\lambda})d\lambda+2a\theta^2\int_0^\infty(\lambda e^{-(1+2\theta-z)\lambda})d\lambda\\
&\quad+2a\theta^3\int_0^\infty(\lambda^2 e^{-(1+2\theta-z)\lambda})d\lambda\\
&=\theta^2\int_0^\infty(\lambda e^{-(1+\theta-z)\lambda})d\lambda-a\theta^2\int_0^\infty(\lambda e^{-(1+\theta-z)\lambda})d\lambda+2a\theta^2\int_0^\infty(\lambda e^{-(1+2\theta-z)\lambda})d\lambda\\
&\quad+2a\theta^3\int_0^\infty(\lambda^2 e^{-(1+2\theta-z)\lambda})d\lambda
\end{aligned}$$

Therefore,

$$P_n(z) = \frac{\theta^2}{(1+\theta-z)^2} - \frac{a\theta^2}{(1+\theta-z)^2} + \frac{2a\theta^2}{(1+2\theta-z)^2} + \frac{4a\theta^3}{(1+2\theta-z)^3} \tag{9}$$

4.2 *Moment Generating Function*

The moment generating function is obtained by replacing z with e^t from the PGF. Hence, the MGF is defined as:

$$M_N(t) = \frac{\theta^2}{(1+\theta-e^t)^2} - \frac{a\theta^2}{(1+\theta-e^t)^2} + \frac{2a\theta^2}{(1+2\theta-e^t)^2} + \frac{4a\theta^3}{(1+2\theta-e^t)^3} \tag{10}$$

4.3 *Moments*

The first four raw moments are for the Poisson QTAD are obtained using the MGF as:

$$E(N) = \frac{8-3a}{4\theta} \tag{11}$$

$$E(N^2) = \frac{(8-3a)\theta - 15a + 24}{4\theta^2} \tag{12}$$

$$E(N^3) = \frac{(8-3a)\theta^2 + (72-45a)\theta - 75a + 96}{4\theta^3} \tag{13}$$

$$E(N^4) = \frac{(8-3a)\theta^3 + (168-105a)\theta^2 + (576-450a)\theta - 420a + 480}{4\theta^4} \tag{14}$$

4.4 *Measures of Dispersion*

The moments obtained are used to obtain the variance and dispersion index as:

$$Var(N) = \frac{(32-12a)\theta - 9a^2 - 12a + 32}{16\theta^2} \tag{15}$$

The Dispersion Index is:

$$DI(N) = \frac{9a^2 + 12a\theta + 12a - 32\theta - 32}{4\theta(3a-8)} \tag{16}$$

If the kth moment of a random variable N us denoted with $E(N^k)$, then the measure of skewness and kurtosis of N using (De Jong & Heller, 2008) are respectively defined as:

$$S_k(N) = \frac{E(N^3)-3E(N^2)E(N)+2(E(N))^3}{(Var(N))^{\frac{3}{2}}}$$

$$Kurt(N) = \frac{E(N^4)-4E(N^3)E(N)+6E(N^2)(E(N))^2-3(E(N))^4}{(Var(N))^2}$$

Hence, the skewness and kurtosis for the Poisson QTAD are obtained as:

$$S_k(N) = \frac{2(64-24a)\theta^2+(192-54a^2-72a)\theta-27a^3-54a^2-24a+128)}{((32-12a)\theta-9a^2-12a+32)^{\frac{3}{2}}} \tag{17}$$

$$K_t(N) = \frac{-243a^4-648a^3(\theta+1)-576a^2(\theta^2+6\theta+4)-192a(\theta^3+19\theta^2+30\theta+12)+512(\theta+1)(\theta^2+12\theta+12)}{(9a^2+12a\theta+12a-32\theta-32)^2} \tag{18}$$

For different values of the two parameters (θ and a), table 1 shows simulated Dispersion Index, Skewness, and Kurtosis for the Poisson QTAD.

Table 1

Different statistics of the Poisson QTAD for different parameter combinations

	Dispersion Index			**Skewness**			**Kurtosis**		
	$\theta = 0.1$	$\theta = 1.5$	$\theta = 8.0$	$\theta = 0.1$	$\theta = 1.5$	$\theta = 8.0$	$\theta = 0.1$	$\theta = 1.5$	$\theta = 8.0$
$a = -0.9$	9.30	1.55	1.10	1.18	1.28	1.99	5.23	5.33	7.60
$a = -0.5$	10.41	1.63	1.12	1.22	1.39	2.14	5.31	5.72	8.37
$a = -0.1$	10.97	1.66	1.12	1.37	1.53	2.31	5.81	6.28	9.30
$a = 0.0$	11.00	1.67	1.13	1.42	1.57	2.36	6.00	6.45	9.56
$a = 0.1$	10.97	1.66	1.12	1.47	1.60	2.40	6.24	6.63	9.82
$a = 0.5$	10.13	1.61	1.11	1.68	1.73	2.59	7.51	7.36	10.89
$a = 0.9$	7.56	1.44	1.08	1.61	1.71	2.75	7.72	7.18	11.64

Remarks:

i. For different parameter combinations, the Dispersion Index is greater than 1, implying that the distribution is suitable for any dispersed count data.

ii. For different parameter combinations, the Skewness is positive, indicating that the distribution is suitable for any positive skewed count data.

iii. For different parameter combinations, all values obtained for Kurtosis show that the distribution is suitable for peaked (unimodal) count data.

iv. For fixed θ, the Dispersion Index is symmetry about $a = 0.0$.

v. For fixed θ, both Skewness and Kurtosis increases as a increases.

vi. For fixed a, the Dispersion Index reduces as θ increases.

vii. For fixed a, both Skewness and Kurtosis increases as θ increases.

4.5 *Maximum Likelihood Estimation*

Given $n_1, n_2, \ldots, n_k$ are random samples from the Poisson QTAD with (θ, a), the log-likelihood function for the distribution is obtained below.

$$P_n = \frac{\theta^2(n+1)}{(1+\theta)^{n+2}} - \frac{a\theta^2(n+1)}{(1+\theta)^{n+2}} + \frac{2a\theta^2(n+1)}{(1+2\theta)^{n+2}} + \frac{2a\theta^3(n+1)(n+2)}{(1+2\theta)^{n+3}}$$

$$\mathcal{L} = \prod_{i=1}^{k} P_{(n_i)} = \prod_{i=1}^{k}\left(\frac{\theta^2(n_i+1)}{(1+\theta)^{n_i+2}} - \frac{a\theta^2(n_i+1)}{(1+\theta)^{n_i+2}} + \frac{2a\theta^2(n_i+1)}{(1+2\theta)^{n_i+2}} + \frac{2a\theta^3(n_i+1)(n_i+2)}{(1+2\theta)^{n_i+3}}\right)$$

$$\ell = \log\mathcal{L} = \sum_{i=1}^{k} \log\left(\frac{\theta^2(n_i+1)}{(1+\theta)^{n_i+2}} - \frac{a\theta^2(n_i+1)}{(1+\theta)^{n_i+2}} + \frac{2a\theta^2(n_i+1)}{(1+2\theta)^{n_i+2}} + \frac{2a\theta^3(n_i+1)(n_i+2)}{(1+2\theta)^{n_i+3}}\right) \tag{19}$$

The maximum likelihood estimators for (θ, a) denoted with $(\hat{\theta}, \hat{a})$ are the solutions of the non-linear equations $\frac{d\ell}{d\theta} = 0$ and $\frac{d\ell}{da} = 0$. Obtaining solutions for the estimators with basic mathematical concepts are quite tasking. In this study, the **MaxLik** function from the R language **(R-Core_ Team, 2020)** is used to obtain the set of numerical solutions for the equation (19) using the Newton Raphson's Algorithm.

5. Applications

The newly proposed distribution is assessed on referred count datasets with unimodal and positively skewed characteristics. The first two datasets have been assessed on the Poisson Ailamujia distribution (Hassan et al., 2020). The first data set contains number of chromatid aberrations (0.2 g chinon 1, 24 hrs) while the second dataset contains mammalian cytogenic dosimetry lesions in rabbit lymphoblast induced by streptonigrin exposure -70µg/kg.

The third dataset consists of the frequency of epileptic seizures. The data has been used to assess the weighted generalized Poisson distribution (Chakraborty, 2010) and on Poisson Transmuted Exponential distribution (Samutwachirawong, 2021). The fourth dataset is the frequency distribution of mistakes in copying groups of random digits as first reported in (Kemp & Kemp, 1965) and later used by (Samutwachirawong, 2021; Sarul & Sahin, 2015; Shanker & Mishra, 2016) on different generalizations of the Poisson distribution. Table 2 shows the descriptive statistics of the four datasets.

Table 2

Descriptive Statistics for Datasets assessed

	Mean	Variance	Dispersion Index	Skewness	Kurtosis	Percentage of Zero
Dataset I	0.548	1.126	2.055	3.122	12.683	67.00
Dataset II	0.553	0.950	1.718	2.356	7.136	66.67
Dataset III	1.544	2.883	1.867	1.311	1.707	35.90
Dataset IV	0.783	1.257	1.257	1.307	0.722	58.33

From table 2, the set of data considered for applications are all dispersed and positively skewed. This informs that assuming the Poisson distribution for such dataset may be misleading.

Table 3

No of Chromatid Aberrations (0.2 g chinon 1, 24 hrs)

No of Aberrations	Obs. Freq.	Expected Frequencies		
		Poisson	P-AIL	P-QTAD
0	268	231.36	246.54	248.49
1	87	126.67	105.97	104.51
2	26	34.68	34.16	33.41
3	9	6.33	9.79	9.75
4	4	0.87	2.63	2.76
5	2	0.09	0.68	0.77
6	1	0.01	0.17	0.22
7	3	0.00	0.04	0.06
Estimates	$\hat{\theta}$	0.5475	3.6530	2.8680
	$\hat{a}$			0.5880
-2 LL		879.03	821.47	817.20
Chi-Square		13474.05	224.13	152.61

Table 4

Mammalian Cytogenic Dosimetry Lesions in rabbit lymphoblast

Class (Exposure)	Obs. Freq.	Expected Frequencies		
		Poisson	P-AIL	P-QTAD
0	200	172.51	184.06	185.33
1	57	95.45	79.78	78.67
2	30	26.41	25.93	25.47
3	7	4.87	7.49	7.53
4	4	0.67	2.03	2.15
5	0	0.07	0.53	0.61
6	2	0.01	0.13	0.17
Estimates	$\hat{\theta}$	0.5533	3.6145	2.9308
	$\hat{a}$			0.5125
-2 LL		646.89	615.79	614.19
Chi-Square		615.44	37.10	29.74

Table 5

Frequency distribution of patients with epileptic seizures

No of Aberrations	Obs. Freq.	Expected Frequencies		
		Poisson	P-AIL	P-QTAD
0	126	74.93	111.77	113.30
1	80	115.71	97.40	97.10
2	59	89.34	63.65	62.74
3	42	45.99	36.98	36.33
4	24	17.75	20.14	19.89
5	8	5.48	10.53	10.55
6	5	1.41	5.35	5.47
7	4	0.31	2.66	2.80
8	3	0.06	1.31	1.42
Estimates	$\hat{\theta}$	1.5442	1.295	1.198
	$\hat{a}$			0.203
-2 LL		1272.09	1191.73	1191.45
Chi-Square		256.51	10.18	9.33

Table 6

Frequency distribution of mistakes in copying groups of random digits

Class (Exposure)	Obs. Freq.	Expected Frequencies		
		Poisson	P-AIL	P-QTAD
0	35	27.41	30.98	31.24
1	11	21.47	17.44	17.24
2	8	8.41	7.36	7.24
3	4	2.20	2.76	2.75
4	2	0.43	0.97	1.00
Estimates	$\hat{\theta}$	0.7833	2.553	2.262
	$\hat{a}$			0.308
-2 LL		155.09	148.14	148.02
Chi-Square		14.44	4.60	4.37

From the tables of results (tables 3-6), it can be observed that the Poisson Quadratic Transmuted Ailamujia Distribution (P-QTAD) with the lowest -2LL and Chi-Square values provide the best fit for all datasets when compared with the classical Poisson and the Poisson Ailamujia Distribution (P-AIL) due to (Hassan et al., 2020).

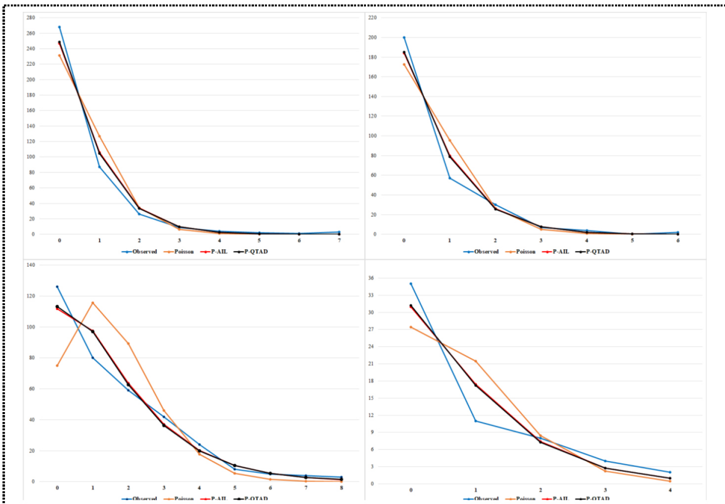

Figure 3

Plots of Observed Frequency and Fitted Values for the datasets assessed

Plots of fitted values and observed values for the three assumed distributions are presented in figure 3. The figure reveals that both P-AIL and P-QTAD give closer fit to the observed frequency than the Poisson distribution.

6. Conclusions

A new count distribution suitable for modelling dispersed and skewed count observations is proposed in this paper. The new distribution is obtained by first expanding the Ailamujia Distribution (Lv et al., 2002) using the quadratic transmutation map (Shaw & Buckley, 2007). The newly obtained continuous distribution is then assumed for the parameter of the classical Poisson distribution. The process of mixed Poisson distribution is then followed in obtaining the new distribution.

Several mathematical properties of the new proposition are obtained and behaviour of major measures of dispersion are obtained by assuming different parameter combinations. Four referred dispersed count data are then assessed by assuming the new proposition and other related discrete distributions. Using maximum likelihood estimation technique for parameter estimation, results obtained show that the new proposition provide better fit for all the datasets considered.

Data Availability

The details of the data used for this study has been discussed inside the article.

Conflicts of Interest

The authors have no conflicts of interest to report.

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