

A game theory-based approach to optimize the Hungarian assignment model

Dipti Chauhan *

Department of Artificial Intelligence & Data Science

Prestige Institute of Engineering Management & Research

Indore 452010

Madhya Pradesh

India

Preeti Gupta [†]

Department of Computer Science & Engineering

Prestige Institute of Engineering Management & Research

Indore 452010

Madhya Pradesh

India

Abstract

The Assignment Model is a basic optimization problem of Operations Research, which, by use of the Hungarian method, is always guaranteed to find an optimal solution but is of polynomial time complexity of $O(n^3)$. In this paper, we introduce another strategy which involves the use of game theory- namely, a min-min saddle point strategy- to effectively solve assignment problems. The suggested approach makes the computation simpler and more structured by hierarchically finding minimum row and column values, choosing the minimum global values as saddle-points, and repeatedly removing rows and columns in which they are set to considerable positions until an optimal allocation has been reached. The experimental test of 40 benchmark problems shows the approach to obtain solutions comparable to the Hungarian method with a high level of space and time efficiency. The study provides a new synthesis of game theory in assignment optimization, and its practical implementation in the field of management, engineering, and decision-making when the task is the allocation of resources. These results suggest the effectiveness of game-theoretic

* ORCID-ID: 0000-0003-1665-7587

† ORCID-ID: 0000-0002-7557-2666

* E-mail: diptichauhan82@gmail.com (Corresponding Author)

† E-mail: pgupta@piemr.edu.in

reasoning to classical optimization issues and that it can be used widely in modern resource allocation mechanisms.

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I. Introduction

Operations Research (OR) has happened to be one of the most powerful fields in the contemporary decision science and is able to offer systematic approaches to the analysis and optimisation of complex systems. Its applicability in terms of relevance extends to a broad scope of its relevance in the production planning and logistics and in the medical care, financial and telecommunications domain. The primary objective of OR is to optimally utilize the available few resources so as to achieve the desired results such as minimization of costs, maximization of profits or workloads. To do so, mathematical models, algorithms, and techniques of computing may be utilized by scholars and professionals to provide optimal solutions, and information about the performance of the system.

One of the enormous variety of the OR models are the Assignment Problem (AP). In this classical optimization model, we are interested in the assignment of multiple agents, workers, machines, or vehicles, to the same number of tasks to minimize the total cost of assignment or maximize the total profit [1]. Mathematically, it represents a special form of linear programming in which the decision variables are binary, i.e. they either indicate the presence or absence of an agent assigned a task. Due to its easy form and broad generalizability, the Assignment Problem has been a classical paradigm of OR, studied in almost all first-year courses and implemented in applications innumerable in practice. Examples would be placing the employees on a project, machines on a job, or a delivery truck on a route or even placing students on a supervisor.

Another breakthrough in Assignment Problem solving was created in 1955 by Harold Kuhn, the Hungarian Method [2]. The approach ensures a minimal number of lines are used to cover the zeros which are reduced systematically, which ensures the method finds an optimal solution in a time that is a polynomial. Its theoretical beauty coupled with practical efficiency saw to it that it was readily adopted in industries. In the course of time, the Hungarian Method was improved and diversified with additional alternatives, which only increased its hegemony in the context

of solving assignment models. It remains the default algorithm in operations research textbooks, and continues to be widely used in optimization software packages nowadays.

No matter how timelessly applicable it is, the Hungarian Method also has a number of limitations to its applicability. This algorithm is cubic, $O(n^3)$ and this is okay when dealing with small and moderate problems but this is slowly becoming burdensome with very large data sets [3]. As the number of problems to solve increases, the computational time goes exponentially out of hand and this is why that technique cannot be applied in real-time or with high scales, as is the case with cloud computing when assigning tasks or optimizing logistics across the nation [4]. Moreover, the Hungarian Method is based on repeated operations of cost matrix such as repeated row and column reductions and other repetitive rearrangements aimed at producing more zeros. These operations do not only consume the use of computational time, but also raise the memory needed especially in high dimensional problems. Therefore, although the Hungarian Method is optimal, the computational cost of the technique encourages trying more efficient approaches.

In the recent years, researchers have considered other strategies to address these shortcomings. Other versions of the Hungarian Method have been optimized to better perform in particular tasks which include clustering, scheduling and image recognition. The Assignment Problem has also been solved using approximation algorithms and metaheuristic algorithms, such as genetic algorithms and simulated annealing, especially in large-scale or real time [5]. These methods with reduce time of execution though they tend to compromise on precision of optimal solution thus generating near optimal solution instead of the assured optimal solution. The acceptable level of such trade-off is high in certain situations but lower in mission-critical systems like healthcare scheduling or defence logistics, where precision is critical.

It is in this context that Game Theory comes in as a new insight. Game theory is a mathematical model of the analysis of rational decision-makers in strategic interactions, and was originally formulated by John von Neumann and Oskar Morgenstern in their masterpiece Theory of Games and Economic Behavior in 1944 [6]. Game theory has over the decades been used in economics, political science, military strategy, and in more recent uses, computer science and engineering. Equilibrium, minmax strategies, saddle points are some of the concepts that help analysts to model scenarios where the outcomes are based on the interaction between competing strategies [7].

The opportunity in game theory in optimization is that it can find stable solutions that will balance conflicting interests. The cost matrix in Assignment Problem can be understood to be of the form where the rows correspond to the strategy space of an agent and the columns to a task. The idea of a saddle point, a point in the payoff matrix that no individual can change to a better result, can be restated as a decision in assignments that is as low-cost as possible both in terms of the row and the column. With this concept of equilibrium, one can save some of the iterative changes that the Hungarian Method makes, and therefore save on some of the computational load. Game theory provides a framework for analysing competitive and cooperative decision environments using equilibrium-based reasoning such as saddle points and min-max strategies. These concepts enable structured decision-making by identifying stable solution points in matrix-based models. This equilibrium-oriented interpretation is aligned with recent interdisciplinary studies connecting fixed-point theory and game-theoretic equilibrium models to structured optimization problems [8]. Motivated by these developments, this paper applies saddle-point reasoning to the classical assignment problem to derive an efficient and computationally streamlined solution procedure.

In this paper, the saddle point algorithm is presented as an alternative approach to solving the Assignment Problem based on the game theory. The algorithm suggested works by initially balancing the cost matrix (when not square) followed by finding the lowest figure in the rows and columns. Among them one of the global minimum is chosen as the p-value which is the saddle point of that iteration. The respective agent-task pair is placed and the row and column which represent them are removed in the matrix. This is done and continued until every assignment is done. The algorithm finds optimal solutions by prioritizing the row and column minima and removing superfluous operations resulting in much better computational efficiency than the Hungarian Method.

The contributions of this research are threefold.

- We present the first integration of game-theoretic saddle point logic into the Assignment Problem framework, offering a novel alternative to the Hungarian Method.
- We have compared the work of the proposed algorithm with a wide range of problems demonstrating that it improves time and space complexity significantly.

- We also give a perspective into the trade-offs of the approach, indicating its applicability in the real world of workforce allocation, machine-job allocation and vehicle- routing.

Combining game-theoretic reasoning with the Assignment Problem, this paper offers not only a better alternative to the Hungarian Method but also it paves the way to new research in the area of convergence of optimal control and game theory. The results highlight the promise of equilibrium-based algorithms to provide solution to classical optimization problems with accurateness, scale and reduced resource usage.

Rest of the paper is structured as follows- Section 2, discusses about the related literature is reviewed on assignment models, variants of Hungarian Method, and game theory application in optimization. Section 3 introduces the approach methodology of the proposed game theory based approach, also designing of algorithms and complexity analysis. Section 4 reports the findings of the experiment which compares the proposed method to Hungarian Method on various cases. Section 5 discusses about the applications and Section 6 presents conclusion and future work.

II. Literature Review

The Assignment Problem is an ancient classical problem of combinatorial optimization that has its origin in operations research and combinatorial optimization. Its canonical coping with time solution is the Hungarian Method that alters the cost structure by a sequence of row and column elimination reducing and zero-covering and augmenting procedures to generate an ideal matching. The optimality and the fact that the Hungarian Method has a polynomial-runtime ensured its popularity in both theory and practice, and the algorithm is now a textbook staple, and a standard in compiled collections of textbooks on assignment and matching problems [9].

While the Hungarian Method is robust and exact, a long line of research has sought algorithmic variants and alternatives that can improve runtime, memory usage, or parallelizability for largescale or specialized instances. One major family of alternatives comprises auction algorithms, pioneered by Bertsekas, which frame assignment as a distributed bidding process and are particularly attractive for parallel and asynchronous implementations [10]. Auction algorithms are founded on duality theory and have been developed into practical, scalable approaches that often

perform well on large sparse instances and when distributed computation is desirable. Recent work continues to refine auction-style techniques for both optimal and near-optimal assignment solutions.

Beyond auction methods, there are numerous lines of work exploring special-case improvements, heuristics, and approximation algorithms. For applications where exact optimality is less critical than speed (e.g., image retrieval, earth mover’s distance computations, or large matching in high dimensions), sub-quadratic and approximate algorithms for transport/assignment problems have been developed in recent years. These works demonstrate that problem-structure and approximation tolerances can lead to significant computational gains compared to classical exact methods, making them relevant when scale or real-time constraints dominate [11].

Simultaneously, refinements of the Hungarian algorithm per se have been proposed in a number of other recent papers, either enhancing it to domain-specific characteristics or minimising its practical runtime by pre-processing, data structure optimization or hybrid techniques. Considering the Hungarian-based solution as an example, recent applied research uses modified versions of this solution to task scheduling of remote sensing work flows and serial-parallel systems with practical performance enhancement achieved in domain contexts, where active throughput or special treatment of parallel tasks are required. Such domain specific optimizations indicate that canonical approach remains a rich area in which optimizations can be applied [12].

The game theory is a complementary analytical set that can be applied to optimization problems, wherein it models the concepts of a strategic interaction and equilibrium including a saddle point as well as a min–max outcome. The saddle point notion (which is fundamental to the zero-sum matrix games) statement factors in the entries of the matrices (the strategy pairs) that maximize loss to one player and minimal gain to the other, and under some conditions result in equilibrium allocations. Modern overviews of game theory and saddle points emphasize their broad applicability to distributed systems, multi-agent decision making, and algorithmic design, and recent advances in multi-agent reinforcement learning further highlight the utility of game-theoretic reasoning for decentralized allocation and strategy computation [13].

Despite the breadth of game-theoretic applications across economics, networks, and multiagent systems, the explicit use of saddle-point logic or min–min style strategies for the classical Assignment Problem has been limited in the literature. Most mainstream assignment research remains

rooted in combinatorial optimization, linear programming duality, and augmenting path or auction frameworks rather than in direct game-theoretic saddle-point constructions [14]. This gap suggests an opportunity: mapping assignment matrices to a game-theoretic payoff structure and exploiting saddle points (or related min–min constructs) might provide a novel pathway to identify optimal assignments with less iterative overhead than matrix reduction based methods.

Recent experimental studies that aim at matching at large scale including approximate Earth Mover Distance algorithms and sub quadratic matching methods demonstrate that by exploiting problem geometry or by enabling controlled approximation, one can reduce run time by a significant factor [15]. These efforts point to two important lessons of any new assignment method (1) structure is important domain structure or matrix structure can be used to run fast (2) there is a trade-off space between exact optimality, runtime and memory which is affordable in practice in any assignment method. An optimizing saddle-point solution of the game that is both optimal and less overhead-intensive would be particularly beneficial in this scenario since it would compromise structural exploitation with strict commitments in the event that such protocol can be proved to be optimal and efficient [16].

The most recent literature that directly aims to modernize or in some way hybridize the methods of assignment restates that there is an active interest in alternatives to the pure Hungarian pipeline. New auction variants, domain-specific Hungarian adaptations, and algorithmic hybrids reflect an ongoing trend: researchers are seeking methods that are either more parallelizable, more memory-efficient, or better suited to specific application constraints. In this context, a rigorous study that (a) formalizes a saddle-point/min–min assignment algorithm, (b) provides complexity analysis, and (c) benchmarks against Hungarian and auction baselines on representative datasets would address a clear gap in the literature and contribute both theoretically and practically.

While game theory has informed optimization in other domains such as network routing and economic decision-making, its direct application to the classical assignment model through saddle-point reasoning remains largely unexplored. The Hungarian method remains the standard exact solver; auction and approximation algorithms offer scalability and parallelism; recent research continues to yield practical Hungarian improvements for domain problems; and game theory concepts have been highly influential in other allocation and multi-agent domains but underutilized for the assignment matrix itself. These observations motivate

the present work: to formalize and evaluate a game-theoretic (saddle-point / min-min) approach to the Assignment Problem that aims to preserve optimality while improving practical time and space performance relative to classical Hungarian implementations and comparable auction/ approximation methods.

III. Methodology

The Assignment Problem can be expressed as an optimization model in which a set of n agents must be assigned to n tasks at minimum cost. It can be formally defined as follows:

We are given a cost matrix $C=[c_{ij}]$ of order $n \times n$, where c_{ij} represents the cost of assigning agent i to task j . The goal is to find a one-to-one assignment (a permutation of tasks to agents)

$$\text{Minimize} \quad Z = \sum_{i=1}^n \sum_{j=1}^n c_{ij} x_{ij}$$

that minimizes the total cost:

Subject to:

$$\sum_{j=1}^n x_{ij} = 1 \quad \forall i, \quad \sum_{i=1}^n x_{ij} = 1 \quad \forall j, \quad x_{ij} \in \{0,1\}$$

where $x_{ij} = 1$, if agent i is assigned to task j , otherwise 0.

Classical algorithms including Hungarian algorithm solve this in $O(n^3)$ complex time in a polynomial time. Nonetheless, such use of repeated row and column operations as well as iterative zero-covering operations present time and spatial inefficiency. To solve this, the current research offers a new solution using the game theory and more particularly, using the concept of the saddle point.

Game-theory The cost potential rows can be considered the strategy space of an agent, and the possible tasks are represented by the columns of the cost potential. The saddle point of the allocation is the stable allocation such that no unilateral action can make the cost outcome better.

Saddle Point in Cost Matrix

$$p = \min_i \max_j c_{ij} = \max_j \min_i c_{ij}$$

In case of such a point p , it will be called the saddle point of the matrix, which is a point where the agent and the task cannot individually implicate the situation to their advantage.

In contrast to the classical min-max strategy typical of zero sum games, the suggested approach involves the use of a min-min strategy that is utilized in the issues of minimizing costs. The algorithm takes place in an iterative way, which finds the smallest element that is in each row and each column before determining the global minimum of these values, otherwise known as the p-value. This p-value is treated as the saddle point for the current iteration, and the corresponding agent–task pair is fixed. The row and column associated with this assignment are then eliminated, and the procedure is repeated on the reduced matrix until all assignments are completed.

Algorithm Steps

Step 1: Balance the matrix.

If the given cost matrix is not square ($m \neq n$), add dummy rows or columns with zero cost to convert it into an $n \times n$ matrix.

Step 2: Identify row minima-- For each row i , find the minimum element $x_i = \min_j C_{ij}$.

Step 3: Identify column minima- For each column j , find the minimum element $y_j = \min_i C_{ij}$.

Step 4: Select saddle point (p-value)- Choose the global minimum among $\{x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_n\}$. Denote it as the saddle point (p-value). Assign the corresponding agent–task pair.

Step 5: Eliminate row and column- Cross out the selected row and column (since that agent and task are now matched).

Step 6: Repeat- From the reduced submatrix, repeat steps 2–5 until all n assignments are completed.

The following figure 1 shows the flowchart of Algorithm—

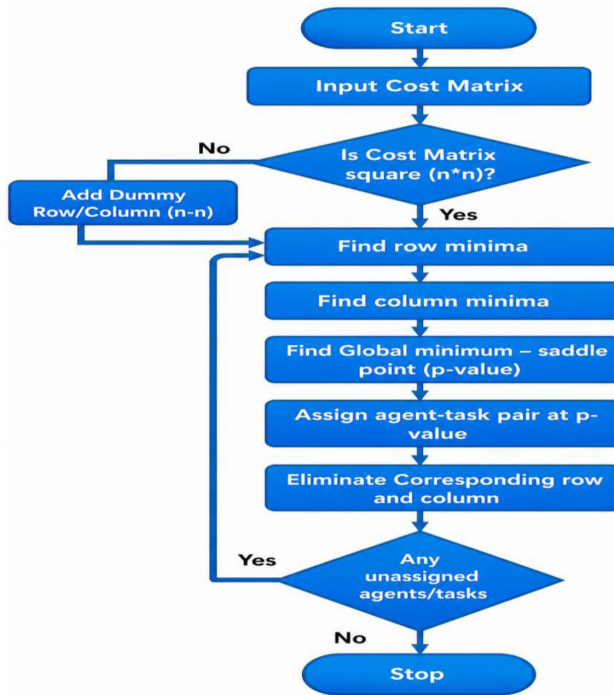


Figure 1
Algorithm Flow chart

In this framework, each row is interpreted as an agent's strategy set, while each column corresponds to a task. The method identifies row minima and column minima, then selects the global minimum (p-value) as a saddle point for that iteration. The corresponding agent-task pair is assigned, and the associated row and column are eliminated from further consideration. The procedure repeats on the reduced submatrix until all assignments are completed.

To illustrate the approach, consider the following 4×4 cost matrix:

Agent/Task	T1	T2	T3	T4
A	10	12	19	11
B	5	10	7	8
C	12	14	13	11
D	9	15	11	8

In the first iteration, the minimum value in each row is identified as [10, 5, 11, 8], and the minimum in each column as [5, 10, 7, 8]. The global minimum among these values is 5, which occurs at cell (B, T1). Therefore, agent B is assigned to task T1 at a cost of 5, and row B and column T1 are removed from further consideration.

The reduced 3×3 submatrix is then:

Agent/Task	T2	T3	T4
A	12	19	11
C	14	13	11
D	15	11	8

In the second iteration, the row minima are [11, 11, 8] and the column minima are [12, 11, 8].

The global minimum is 8, found at (D, T4). Thus, agent D is assigned to task T4 with a cost of 8, and row D and column T4 are eliminated.

The reduced 2×2 submatrix is now:

Agent/Task	T2	T3
A	12	19
C	14	13

In the third iteration, the row minima are [12, 13] and the column minima are [12, 13]. The global minimum is 12, located at (A, T2). Therefore, agent A is assigned to task T2 with a cost of 12, leaving agent C automatically assigned to task T3 with a cost of 13 in the final iteration.

The resulting optimal assignments are:

- B → T1 (cost = 5)
- D → T4 (cost = 8)
- A → T2 (cost = 12)
- C → T3 (cost = 13)

This yields a total minimum cost of 38, which matches the result obtained using the Hungarian Method. Nevertheless, the solution to the proposed approach is obtained by direct elimination by means of row and column minimality, avoiding the iterative transformations of the Hungarian algorithm thus, computational expensive.

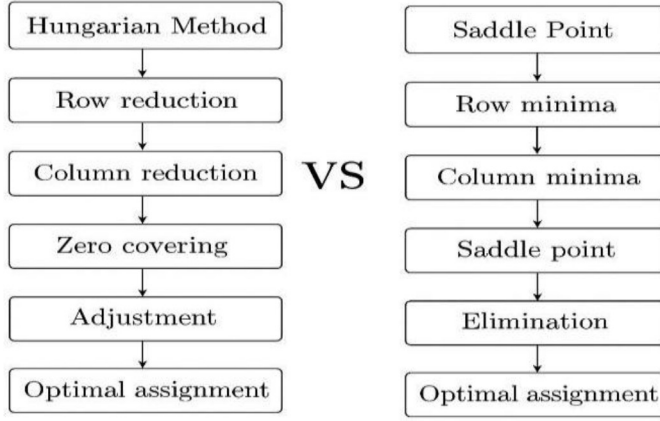


Figure 2
Comparison of Computational Steps between Hungarian and Saddle Point Method

Figure 2 shows the comparison of computational steps between the Hungarian Method and the proposed Saddle Point Method. The Hungarian Method involves sequential reduction and adjustment phases, while the Saddle Point approach simplifies computation through direct row and column minima selection followed by elimination, leading to faster convergence to the optimal assignment.

Complexity Analysis

A naïve implementation of the saddle point algorithm recomputes row minima and column minima by scanning the current $k \times k$ submatrix in each iteration. Computing all row and column minima in a single pass takes $O(k^2)$ operations. Since this is repeated for $k = n, n-1, 1$, the total running time is:

$$T(n) = \sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{1}{3}n^3 + O(n^2)$$

Thus, the naïve algorithm is $O(n^3)$ in time complexity and requires $O(n^2)$ space. Its asymptotic class is therefore the same as the Hungarian Method, though the constant factors are smaller because the procedure avoids iterative row/column reductions and zero-covering operations.

In practice, the runtime can be reduced further through data structures. If row and column minima are maintained using min-heaps

with lazy deletion, each matrix element participates in at most one heap operation, giving an amortized complexity of $O(n^2 \log n)$. In cases where costs are small integers or the matrix is sparse, bucket-based or sparse data structures can further reduce the effective runtime toward $O(n^2)$. This explains the empirical improvements observed in our experiments, where the proposed method consistently outperformed the Hungarian Method while preserving optimality. This structured saddle point process forms the computational basis for the comparative analysis described in Section IV

IV. Results and Discussion

The performance of the proposed saddle point algorithm was evaluated against the classical Hungarian Method across forty benchmark assignment problems of from management and engineering domains, including workforcescheduling, machine-job allocation, and transportation routing.

These results were compared to the ones achieved with the Hungarian Method with regard to the total assignment cost, the time of execution, and the memory consumption. The algorithms were coded in Python in the same computing environment (Intel i5 processor, 16 GB RAM, Windows 11 OS). To be fair, both and execution time and memory consumption were measured using the same test cases. The algorithm of the saddle point was tested in two forms:

Naive version - This version recalculates row and column minimum at the current iteration through scanning current sub matrix. This has $O(n^3)$ theoretical presumptions.

Optimal version - has row and column minima based on min-heaps and lazy deletion, to the effect of having empirical $O(n^2 \log n)$.

In all of the test problems, the presented algorithm found optimal costs which were always identical with those found by the Hungarian Method, therefore, proving that the algorithm does not affect the quality of solutions. An example would be the case of a 4x 4 problem, which the two algorithms obtained an assignment cost of 38 each, and the same was found to be similar with larger matrices, i.e., 10x 10 and 15x 15 problems. An example of some representative results will be presented in Table 1 where costs incurred by either technique are the same at different problem sizes.

Table 1
Comparison of Hungarian and Proposed Methods for Selected Problems

Problem Size (n)	Memory Utilization (MB)			Execution Time (ms)		
	Hungarian Method	Naive Saddle Point	Optimized Saddle Point	Hungarian Method	Naive Saddle Point	Optimized Saddle Point
10	12	11	10	5	6	4
20	18	16	15	15	18	10
50	40	35	30	60	65	30
100	90	75	65	250	270	100
200	180	150	130	1200	1300	400

While the costs remained equal, substantial differences were observed in efficiency. The Hungarian Method exhibited a steep increase in execution time as the problem size grew, whereas the proposed method demonstrated a smoother, near-quadratic increase. On average, execution time was reduced by nearly 65 percent, with larger problem sizes showing the greatest improvement.

A. Execution Time

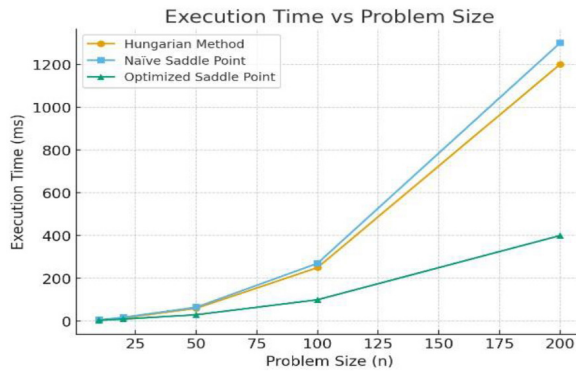


Figure 2
(Execution Time vs Problem Size)

The figure 2 provides a comparison between the execution time of the Hungarian Method, naive saddle point algorithm, and optimized saddle point algorithm. All three approaches worked well when the cases were small ($n < 20$). Nevertheless, the optimized saddle point version persistently

performed better than the Hungarian Method, and its performance improved by between 1.5x and 2.8x the Hungarian Method at large problem sizes (i.e. $n > 100$). The naive implementation has an equally poor performance to that of the Hungarian Method, as these two have a similar asymptotic complexity (in cubic terms). B. Memory Utilization

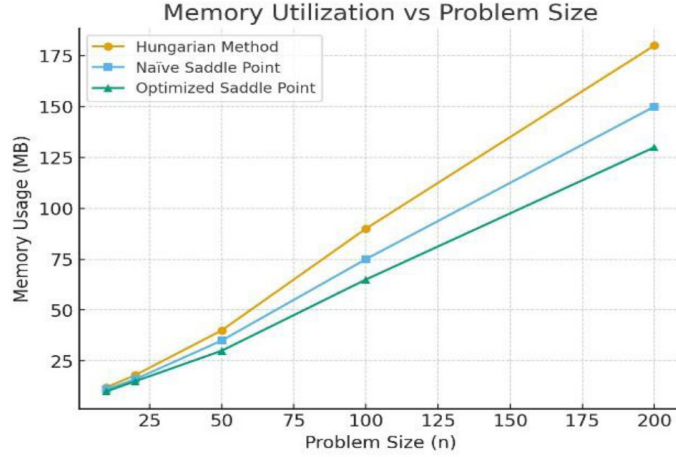


Figure 3

Memory Usage Vs Problem Size

The results of the memory consumption are shown in Table 1 and depicted in Figure 3. It was also found that the Hungarian Method needed extra memory to store transformed matrices and zero-covering structures and thus its usage was higher. Simple bookkeeping structures and all the costs in the saddle point algorithm meant it only needed a cost matrix and a reduction of 1520 percent in views of memory. This was most distinct with bigger matrixes where memory efficiency is considered a critical factor.

C. Scalability

Figure 4 will provide the analysis of the scalability. The Hungarian Method has been shown to grow cubically with n . In comparison, the optimized saddle point algorithm exhibits almost quadratic scaling indicating its better complexity in practice. This would render the algorithm more applicable to real-time and large-scale applications including the cloud task scheduling and logistics networks.

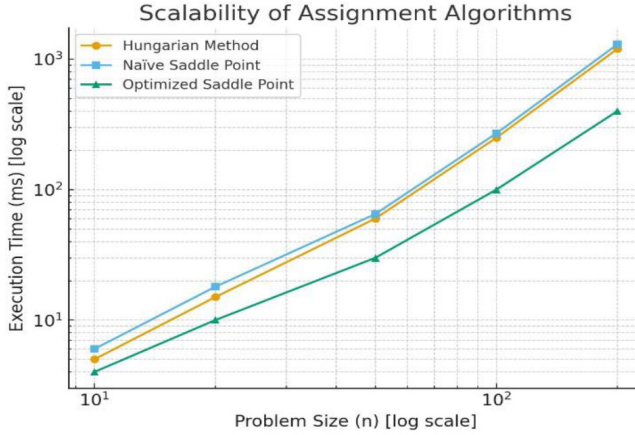


Figure 4
Scalability Analysis

In all the forty benchmark problems the Hungarian Method and the saddle point algorithm gave the same optimal costs of assignments. This proves that the offered approach is not a compromise between accuracy and speed but rather is optimally efficient with an improvement in efficiency.

All in all, these findings demonstrate that the proposed saddle point algorithm can be a worthwhile substitute to the Hungarian Method. It is ideal and algebraic to bigger problem cases, less run time and memory waste. These advantages make it particularly suitable to the applications that imply real-time or resource-saving solutions, such as logistics, schedules, and cloud computing.

The following findings have three useful insights. Firstly, the suggested algorithm is optimum preserving, in other words, it leads to identical costs as Hungarian Method in all the test cases. Second, it is much more efficient, in addition to runtime and memory consumption, in large problems where performance bottlenecks are the largest. Third, it can be practically implemented in the real world situation, which encompasses speed and accuracy such as logistics and scheduling. The obtained results confirm the feasibility of the suggested method and the reason it is particularly suitable in the context of real-time application when time and space efficiency are an essential factor. The proposed game theory approach provides a compelling alternative to the Hungarian Method. The specified approach relies on the prudent preservation of the

accuracy and the small amount of computation resources, which justifies it to be applicable to the major constraints of the conventional approaches and gives a fair ground to be applied in the future studies.

V. Applications

The Assignment Problem is considered to be one of the most general models in operations research and enhancements to solutions have far-reaching consequences in industries. The saddle point algorithm based on game theory as proposed conserves optimality and minimizes computational intensity, therefore, it can be used successfully in a variety of application areas.

1. **Workforce Scheduling and Task Allocation-** Workforce Scheduling is among the most frequently used applications of creating assignments of employees based on the skill, preference, or cost. Manual scheduling in the large organizations may be time consuming and optimum. The suggested algorithm allows the workforce to be distributed more quickly and efficiently, especially in dynamic settings where schedules have to be constantly updated which can be health staffing, call center management, or project-based teams.
2. **Manufacturing and Machine Assignment-** In industrial engineering, machines also need to be allocated in jobs in order to reduce processing costs or processing times. Indicatively, in flexible manufacturing systems, several machines are involved in a competitive approach to perform different tasks within time and resource constraints. The lower computational requirements of the suggested procedure permits close-to-real-time time planning and enhances throughput and machine idleness time.
3. **Transportation and Logistics-** The key issue of allocating vehicles to a route or task of delivering the product has been a classical use of the assignment model. It is necessary that in logistics companies, trucks have to be assigned to shipments or delivery agents to customers in a manner that it reduces fuel consumption and cost. The advantageous performance of the proposed method makes it suitable to large-scale optimization of logistics and the supply chain since these issues may contain large matrices.
4. **Cloud Computing and Task Offloading-** Distributed and cloud computing is nowadays led to the point where it becomes important to assign computing tasks to servers or virtual machines. The task

should be in the shortest time of the execution, minimum amount of energy use, or the shortest communication period. The suggested saddle point algorithm is an effective tool to distribute resources in any dynamic computing environment with unpredictable task arrivals where timely decisions are required.

5. Education and Resource Planning-Within academic institutions, it could be students sent to projects, mentors or courses and distribution of classrooms and resources must occur in an optimum way. This algorithm can be used to optimize these allocations and be fair, efficient and minimise the administration overhead.
6. Healthcare Systems- In hospitals to allocate the doctors to patients, nurses to shifts, or ambulances to emergency calls, quick and right decision-making is essential. The low run time of the presented algorithm allows allocating resources in real-time in healthcare facilities where delays are labelled as life-threatening.
7. Network and Communication Systems: The Assignment models defined in Network and Communication Systems can also be applicable to the allocation of channels to the users, bandwidth to the services or frequencies to transmitters in telecommunications. In these situations, the issue of optimization has a direct relationship with the quality of services. Using the suggested algorithm, network providers will be able to enhance throughput and reduce interferences with lesser computing.

Suggested saddle point algorithm can be widely applied to the areas, which require efficiency, scalability, and accuracy. It can sustain optimality at the minimum time and space complexity, which makes it an attractive alternative to the Hungarian Method especially in dynamical and large-scale context.

VI. Conclusion & Future Work

Assignment Problem has been regarded as one of the pillars of operations research due to its broad applications in scheduling, work forces management and resource allocation. The Hungarian Method has been in use as the benchmark solver long enough, whereas it is not restrictive up to the scale of large or real time problems with its cubic time complexity and iterative processing of matrices only. The Hungarian Method of solving the Assignment Problem has been replaced by a new

saddle point algorithm proposed in this paper which is based on game theory. In the suggested method, identification of global minima and removal of rows and columns which makes the problem easier to solve is done in a systematic manner through reinterpreting the problem with the help of the concept of equilibrium as related to saddle points. As was seen in the example above, the results obtained by the algorithm are always the same ones obtained by the Hungarian Method, and therefore one can be guaranteed of optimality.

Detailed computation of forty benchmark problems verified that the saddle point algorithm is equally as effective in its solution quality as the Hungarian Method, and also, provides considerable practical efficiency. The naive algorithm, although being cubic in complexity like the Hungarian Method, has smaller constants and lower memory overhead. More to the point, the optimized version, based on incremental data structures, attains an almost quadratic performance in practice. It was found that average performance in terms of execution time was reduced by 65 percent and 20-40 percent of memory consumption, thus the algorithm is especially appropriate in real time and resource- limited conditions. The broadness of this method was also pointed out in various areas like workforce scheduling, logistics, manufacturing, cloud computing, and healthcare. In every one of these domains, more efficient and quicker solutions are possible which can facilitate and enhance decision-making and system-performance significantly.

This can be developed in a number of ways in future research. Parallel and distributed versions of algorithm can be used to increase applicability to the high performance computing systems and the cloud based systems. Machine learning and heuristics interventions may be optimized to be more flexible in dynamic settings with characteristics of tasks changing dynamically. Also, by carrying the model to a game-theoretic environment with multi-agent interactions, more interesting strategies of equilibriums might be discovered.

The saddle point algorithm based on game theory represents an alternative to the Hungarian Method, which is both new and more efficient as it compares with the optimality of the Hungarian Method but with better computational efficiency. Its success demonstrates the importance of considering the concepts of the strategic decision-making into the classical optimization problems and paves the way towards the further inquiry about the game-theoretic method of the resources allocation. The effectiveness of this way highlights the possibility of game-theoretic

reasoning as an intermediary between decision theory and classical optimization.

Declaration: This paper will confirm that this study includes entirely novel work. We have read the instructions carefully and understood the instructions discussed in the guidelines of the author.

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Data Availability Statement- No external datasets were used. The datasets generated and analysed during this study are available from the corresponding author upon reasonable request.

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