

Adaptive chaos control via neural networks : From microorganism-driven climate to cancer growth

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Abstract

This research work investigates how to control the chaos in dissipative dynamical systems. To this aim, we have analyzed the novel systems: one is related to the variation in temperature caused by micro-organisms, and the other corresponds to the proliferation of cancer cells via interaction with healthy host cells and immune effector cells introduced in Kumar et al. [23]. To control the chaos between these two systems, we have developed the anti-synchronization technique based on neural networks which is capable to manage unknown nonlinearities by approximating it with the time. Further, the effectiveness of the purposed scheme was assessed through numerical simulation and confirmed that anti-synchronisation conditions were rapidly achieved even with the presence of perturbation. This study emphasizes the impact for controlling the chaos by using neural network methods and to obtain the stability in dissipative biological and environmental systems.

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1. Introduction

Anti-synchronization of chaotic systems has become a prominent concern in non-linear dynamics, with possible uses in biological and ecological processes as well as secure communications. Anti-synchronization forces connected chaotic subsystems into precise inverse-phase trajectories (e.g., $x_1(t) + x_2(t) \rightarrow 0$), in contrast to traditional synchronization, which causes two systems to evolve similarly. Recently, using suitable feedback control, chaotic oscillators mostly dissimilar are forced to manage opposite-sign locking as done by Chen et al. [1] and Shahverdiev [2]. Anti-synchronization control has been used in extremely complicated biological models, such as Lotka–Volterra predator–prey models or neural networks, revealing strong anti-phase rhythms that can be great for safe signal processing in biochemical or brain circuits, as illustrated by Chaudhary et al. [3] in 2023 and Meng and Wang [4].

In view of dissipative dynamics, anti-synchronization is an emerging area, since the revolutionary work by Pecora and Carroll [5] and it has been widely

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used in controlling several nonlinear systems. An active generalized backstepping controller that anti-synchronized two distinct hyperchaotic systems (Lorenz and Lü) with quick error decay was presented by Sahab and Ziabari [6]. Similarly, robustly chaotic oscillators, despite unknown parameters, were anti-synchronized by using sliding-mode control as Sliding-mode controllers impose global anti-synchronization in multi-wing chaotic circuits by taking advantage of their insensitivity to parameter fluctuations described in works of Jawaada, Noorani, and Al-Sawalha [7] and in 2012 by Vaidyanathan and Sampat [8].

Both integer-order and fractional Lorenz systems with unknown parameters may be anti-synchronized using a unified finite-time controller, pointing out that greater fractional order may accelerate up convergence given by Chen et al. [1]. As a whole, these investigation using appropriate adaptive, sliding-mode, or active control techniques and demonstrate that chaotic states can be driven into exact opposite-phase motion as described by Huang and Cao [9], Al-Sawalha and Noorani [10] and Shahzad and Haris [11].

In the Recent work of Ahn [12] and Vaidyanathan [13], Neural-network control has also been used to address anti-synchronization of chaotic systems. In particular, radial basis function are used by Fang et al. [14] and Yang, Guo, and Wang [15] to optimize the error and capture the nonlinearities in the presence of parametric uncertainty by including RBFNNs into sliding-mode or backstepping control laws. Gu, Sun, and Fu [16] described an innovative technique that synchronizes chaotic systems in the presence of ambiguity, disorder, and adjustable time delay using a sliding-mode neural network. In order to prevent the "explosion of complexity" in Lyapunov designs, advanced systems integrate RBFNNs with dynamic-surface/backstepping techniques. These controllers accomplish asymptotic antisynchronization with easier tuning and proven stability by employing a filtered RBFNN approximation of the unknown dynamics as done by Khan, Bashir, and Malik [17] and earlier by Fang et al. [18]. In all cases, Lyapunov arguments ensure that the drive-response states satisfy $x_{\text{slave}}(t) \approx -x_{\text{master}}(t)$ as $t \rightarrow \infty$, with the RBF weights adapting in real time to compensate modeling errors.

Despite these advances, significant gaps remain for anti-synchronization research. In particular, application-driven chaotic models, those arising in climate-microorganism dynamics or tumour-immune-host interactions, have rarely been used in the anti-synchronization literature. Similarly, there is still much to explore about the really strong anti-synchronization of substantially non-identical chaotic systems under extreme uncertainty. The majority of RBFNN-based controllers now in use assume comparable system architectures; expanding them to extremely varied (such as heterodimensional or heterogeneous) drive-response pairings. This can be seen in the work by Ahn [19], Li, Zheng, and Wu [20], Boubakir and Labiod [21], and Li et al. [22]. New RBFNN control structures and observer-based techniques designed for real-world nonlinear oscillators will probably be needed to close these gaps and guaranty anti-phase convergence even in the presence of significant model discrepancies and disturbances.

In this study, we present two chaotic subsystems-one representing climate fluctuations driven by microbial activity, and the other capturing aggressive tumour-immune dynamics and then design a neural-network-based adaptive controller to enforce anti-synchronization between them. To couple these systems, we employ an adaptive neural controller that is trained online for managing unknown nonlinearities and adjust control gains so that the two system states evolve in opposite phase (i.e., anti-synchronized). In our design, the neural network continually tunes itself so that the tumor--immune and microbial-climate subsystems settle into opposite trajectories, achieving the intended anti-phase coordination and by using Lyapunov stability theory, we derive sufficient conditions that assure the minimization of the error.

The efficiency of the proposed method is ensured by simulation through MatLab. Starting from different initial conditions, the climate and tumour states are driven into tightly coupled but opposite trajectories in a fraction of the time required by classical active control schemes. In these tests the adaptive controller consistently outperforms conventional methods: whereas a simple active control would need exact model cancellation (and suffers high cost and sensitivity to unknown parameters as seen in Li, Zheng, and Wu [20] and Boubakir and Labiod [21], our neural scheme learns and compensates for model errors on the fly. The result is fast, robust anti-synchronization under parameter variations and noise, demonstrating improved convergence speed, robustness, and adaptability. Overall, this work advances intelligent chaos control by uniting disparate complex dynamics under adaptive anti-synchronization.

2. Mathematical Modeling Framework

The Mathematical model governed by the differential equations given in Kumar et al. [23], based on the compartmental diagram Figure 1, which demonstrates the interaction between tumour cells, host cells, and immune effector cells. The encoded model is presented with the help of state variables as

- $p(t)$: tumor cells,
- $q(t)$: healthy host cells,
- $r(t)$: immune effector cells.

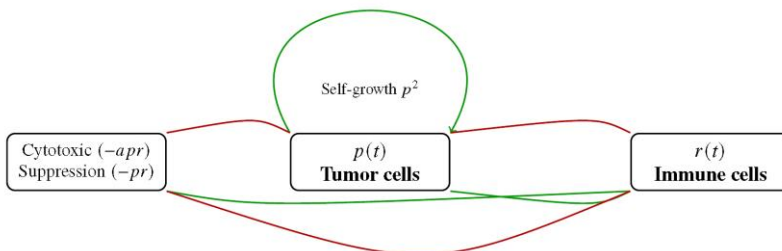


Figure 1
Compartmental diagram of the tumour-immune interaction model.

To capture the essential biological mechanisms, the following assumptions are imposed:

- (H1) Tumor cells exhibit nonlinear growth, including potential super-growth effects under specific conditions.
- (H2) Interactions between immune and tumor cells reduce host cell vitality.
- (H3) Both host and immune cells actively participate in killing tumor cells.

With these assumptions, the governing equations are:

$$\begin{aligned}\frac{dp}{dt} &= -apr - q + p^2, \\ \frac{dq}{dt} &= -pr + c, \\ \frac{dr}{dt} &= pq - br,\end{aligned}\tag{1}$$

with initial conditions $p(0) = p_0, q(0) = q_0, r(0) = r_0$.

This formulation describes a competitive system in which the nonlinear self-replication of tumor cells is counterbalanced by host and immune responses. For certain parameter regimes, the system exhibits chaotic dynamics, indicating unpredictable tumor evolution in the absence of therapeutic control.

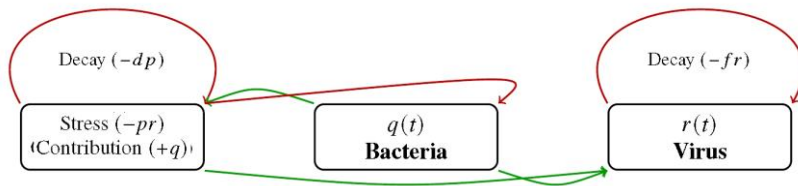


Figure 2
Compartmental diagram of the microorganism--climate interaction model.

Based on compartmental diagram Figure 2, the climate-microorganism model was given in Kumar et al. [23] to illustrate the feedback interactions among environmental temperature, bacterial populations, and viral populations and the corresponding state variables are:

- $p(t)$: environmental temperature,
- $q(t)$: bacterial population,
- $r(t)$: viral population.

The governing equations are:

$$\begin{aligned}\frac{dp}{dt} &= -dp + q, \\ \frac{dq}{dt} &= \varepsilon q - pr, \\ \frac{dr}{dt} &= dpq - fr,\end{aligned}\tag{2}$$

with starting states $p(0) = p_0, q(0) = q_0, r(0) = r_0$.

2.1 Chaotic Dynamics and Relevance

We have observed in Kumar et al. [23], both systems exhibits chaotic behaviour on the basis of their Lyapunov characteristics exponents. This coupling forms the basis of the control and anti-synchronization strategy developed in later sections.

3. Neural Network–Based Anti-Synchronization Between the Systems

Consider System (2) as the drive system and System (1) as the response system. We rewrite them in the form:

$$\begin{aligned}\dot{p}_1 &= q_1 - dp_1, \\ \dot{q}_1 &= \varepsilon q_1 - p_1 r_1, \\ \dot{r}_1 &= dp_1 q_1 - fr_1,\end{aligned}\tag{3}$$

and

$$\begin{aligned}\dot{p}_2 &= -ap_2 r_2 - q_2 + p_2^2 + u_1(t), \\ \dot{q}_2 &= -p_2 r_2 + c + u_2(t), \\ \dot{r}_2 &= p_2 q_2 - br_2 + u_3(t),\end{aligned}\tag{4}$$

where $u_i(t)$ ($i = 1, 2, 3$) are control inputs to be designed.

Define the anti-synchronization errors:

$$e_1 = p_2 + p_1, e_2 = q_2 + q_1, e_3 = r_2 + r_1.\tag{5}$$

Adding (3) to (4), the error system becomes:

$$\begin{aligned}\dot{e}_1 &= -ap_2 r_2 + p_2^2 - q_2 - dp_1 + q_1 + u_1(t), \\ \dot{e}_2 &= -p_2 r_2 + c + \varepsilon q_1 - p_1 r_1 + u_2(t), \\ \dot{e}_3 &= p_2 q_2 - br_2 + dp_1 q_1 - fr_1 + u_3(t).\end{aligned}\tag{6}$$

3.1 Neural Network–Based Adaptive Controller for Anti-Synchronization

The nonlinear terms in the anti-synchronization error dynamics (see Eq. (6)) are generally uncertain. We approximate them with Radial Basis Function Neural Networks (RBFNNs) described by Thorat et al. [24] as:

$$\varphi_i(e) \approx \widehat{W}_i^T \Phi(e), i = 1, 2, 3,\tag{7}$$

where $\Phi(e) \in \mathbb{R}^m$ is the vector of Gaussian basis functions and $\widehat{W}_i$ are adaptive weight estimates.

3.2 Stability Analysis for Anti-Synchronization

The control inputs are designed has root in Al-Azzawi and Al-Hayali [25]:

$$u_i(t) = -k_i e_i - \widehat{W}_i^T \Phi(e), k_i > 0,\tag{8}$$

where k_i are linear feedback gains together with the adaptive law

$$\dot{\widehat{W}}_i = \Gamma_i \Phi(e) e_i - \sigma_i \widehat{W}_i,\tag{9}$$

achieves anti-synchronization under the assumptions:

A1. The RBF feature vector $\Phi(e) \in \mathbb{R}^m$ is uniformly bounded: $\|\Phi(e)\| \leq \overline{\Phi}$.

A2. There exist ideal weights $W_i^* \in \mathbb{R}^m$ and bounded residual

approximation errors $\Delta_i(e, t)$ such that

$$\varphi_i(e, t) = W_i^{*T} \Phi(e) + \Delta_i(e, t), |\Delta_i(e, t)| \leq \bar{\Delta}_i.$$

A3. The signals $e(t)$ and $\widehat{W}_i(t)$ are continuous and differentiable.

Here $\Gamma_i > 0$ are adaptation gains and $\sigma_i \geq 0$ are small leakage terms.

Using the equation (8) into the system (6), we obtain

$$\dot{e}_i = -k_i e_i + (W_i^* - \widehat{W}_i)^T \Phi(e) + \Delta_i(e, t), i = 1, 2, 3. \quad (10)$$

In compact vector form:

$$\dot{e} = -Ke + \mathcal{W}^* \Phi(e) - \widehat{\mathcal{W}} \Phi(e) + \Delta(e, t),$$

where $K = \text{diag}(k_1, k_2, k_3)$, $\widehat{\mathcal{W}} = [\widehat{W}_1, \widehat{W}_2, \widehat{W}_3]^T$, $\mathcal{W}^* = [W_1^*, W_2^*, W_3^*]^T$, and $\Delta = [\Delta_1, \Delta_2, \Delta_3]^T$.

To analyze stability, consider the standard composite Lyapunov function

$$V(e, \widetilde{W}) = \frac{1}{2} e^T e + \sum_{i=1}^3 \frac{1}{2} \widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i, \quad (11)$$

where $\widetilde{W}_i = \widehat{W}_i - W_i^*$ is the weight estimation error. The first term penalizes the anti-synchronization error e , while the second term penalizes weight estimation error.

Differentiating (11) along the trajectories of (10) and using the adaptive law (9), we get

$$\dot{V} = e^T \dot{e} + \sum_{i=1}^3 \frac{1}{2} \widetilde{W}_i^T \Gamma_i^{-1} \dot{\widetilde{W}}_i.$$

Since W_i^* are constant, $\dot{\widetilde{W}}_i = \dot{\widehat{W}}_i$ so

$$\begin{aligned} \dot{V} = e^T (-Ke + \sum_{i=1}^3 (W_i^* - \widehat{W}_i)^T \Phi(e) e_i + \Delta(e, t)) + \\ \sum_{i=1}^3 \widetilde{W}_i^T \Gamma_i^{-1} (\Gamma_i \Phi(e) e_i - \sigma_i \widehat{W}_i). \end{aligned}$$

For each i , observe that

$$e_i (W_i^* - \widehat{W}_i)^T \Phi(e) + \widetilde{W}_i^T \Gamma_i^{-1} \Gamma_i \Phi(e) e_i = -e_i \widetilde{W}_i^T \Phi(e) + e_i \widetilde{W}_i^T \Phi(e) = 0.$$

Thus, the nonlinear NN approximation terms cancel exactly. Therefore,

$$\dot{V} = -e^T Ke + e^T \Delta(e, t) - \sum_{i=1}^3 \widetilde{W}_i^T \Gamma_i^{-1} \sigma_i \widehat{W}_i.$$

Expand the last term:

$$\widetilde{W}_i^T \Gamma_i^{-1} \sigma_i \widehat{W}_i = \sigma_i \widetilde{W}_i^T \Gamma_i^{-1} (\widetilde{W}_i + W_i^*).$$

So

$$-\widetilde{W}_i^T \Gamma_i^{-1} \sigma_i \widehat{W}_i = -\sigma_i \widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i - \sigma_i \widetilde{W}_i^T \Gamma_i^{-1} W_i^*.$$

Using Young's inequality, for any $\varepsilon > 0$,

$$-\sigma_i \widetilde{W}_i^T \Gamma_i^{-1} W_i^* \leq \frac{\sigma_i}{2} (-\widetilde{W}_i^T \Gamma_i^{-1} \widetilde{W}_i) + \frac{\sigma_i}{2} (W_i^*)^T \Gamma_i^{-1} W_i^*.$$

By Assumption A2, $\|\Delta(e, t)\| \leq \bar{\Delta}$. Then

$$e^T \Delta(e, t) \leq \|e\| \bar{\Delta} \leq \frac{k_{\min}}{2} \|e\|^2 + \frac{\bar{\Delta}^2}{2k_{\min}},$$

using $ab \leq \frac{\alpha}{2} a^2 + \frac{1}{2\alpha} b^2$ with $\alpha = k_{\min}$.

Substituting these bounds, we obtain

$$\dot{V} \leq -\frac{k_{\min}}{2} \|e\|^2 - \sum_{i=1}^3 \frac{\sigma_i}{2} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i + \frac{\bar{\Delta}^2}{2k_{\min}} + \sum_{i=1}^3 \frac{\sigma_i}{2} (W_i^*)^T \Gamma_i^{-1} W_i^*, \quad (12)$$

where $k_{\min} = \min\{k_1, k_2, k_3\} > 0$.

Implications: Define the constant

$$C := \frac{\bar{\Delta}^2}{2k_{\min}} + \sum_{i=1}^3 \frac{\sigma_i}{2} (W_i^*)^T \Gamma_i^{-1} W_i^* \geq 0.$$

Then (12) becomes

$$\dot{V} \leq -\frac{k_{\min}}{2} \|e\|^2 - \sum_{i=1}^3 \frac{\sigma_i}{2} \tilde{W}_i^T \Gamma_i^{-1} \tilde{W}_i + C.$$

This inequality guarantees that all closed-loop signals remain bounded. Specifically, $V(t)$ cannot diverge because $\dot{V}$ becomes negative whenever $\|(e, \tilde{W})\|$ grows large. Thus $e(t)$ and $\tilde{W}_i(t)$ are uniformly bounded for all $t \geq 0$.

Convergence: We analyze the convergence of the anti-synchronization error $e(t)$ under the proposed adaptive control scheme:

Case 1: Ideal conditions ($\bar{\Delta} = 0$ and $\sigma_i = 0$).

In this case, the derivative of the Lyapunov function reduces to

$$\dot{V} = -e^T K e \leq -k_{\min} \|e\|^2,$$

where $k_{\min}$ is the minimum eigenvalue of the positive definite gain matrix K . This expression is *negative definite*, implying that $V(t)$ decreases monotonically and exponentially converges.

Case 2: Nonzero bounded disturbances ($\bar{\Delta} > 0$).

Here, $\dot{V}(t) = -e^T K e + \Delta(e, t)$ is uniformly continuous because $e(t)$ and $\Delta(e, t)$ are bounded. Moreover, the integral of $-e^T K e$ is finite due to L_2 -boundedness of $e(t)$. Therefore, by Barbalat's lemma [26],

$$\|e(t)\| \rightarrow 0 \text{ as } t \rightarrow \infty,$$

provided that $\Delta(e, t) \rightarrow 0$ when $e \rightarrow 0$.

If $\Delta(e, t)$ is persistent (non-vanishing), the error cannot converge exactly to zero. In this case, the closed-loop system achieves *practical stability*, and $e(t)$ converges to a residual set with radius proportional to $\bar{\Delta}/k_{\min}$:

$$\limsup_{t \rightarrow \infty} \|e(t)\| = \mathcal{O}\left(\frac{\bar{\Delta}}{k_{\min}}\right).$$

This provides a quantitative measure of the steady-state tracking error under bounded disturbances.

Thus, under the NN-based adaptive controller, the error system gets converge towards zero, if $\Delta(e, t) \rightarrow 0$ near the origin, or to a small residual neighborhood otherwise. In the ideal case $\bar{\Delta} = 0$, exact anti-synchronization with exponential convergence is achieved base on Roy and Khan [27].

4. Validation of the Neural Network–Based Adaptive Controller

In this section, the efficiency of the proposed Radial Basis Function Neural Network (RBFNN)–driven adaptive controller is validated for the *anti-synchronization* of two nonlinear systems: the microorganism-induced climate dynamics (drive system) and the three-population cancer progression dynamics (response system). Numerical simulations were conducted in MATLAB employing the ode45 routine with stringent accuracy requirements, namely, a relative tolerance of 10^{-7} and an absolute tolerance of 10^{-9} .

To ensure fairness in performance comparison with the previously discussed active control strategy, the same set of parameters was employed:

$$d = 36, \varepsilon = 13, f = 4, a = 1, b = 2, c = 27.$$

The systems were initiated with distinct initial conditions:

$$p_1(0) = 50, q_1(0) = 10, r_1(0) = 10, p_2(0) = 3, q_2(0) = 2, r_2(0) = 1.$$

For anti-synchronization, the error variables are defined as

$$e_1 = p_2 + p_1, e_2 = q_2 + q_1, e_3 = r_2 + r_1.$$

This yields the following initial errors:

$$e_1(0) = 53, e_2(0) = 12, e_3(0) = 11,$$

providing a rigorous test bed for the adaptive controller. In order to attain robust anti-synchronization, in spite of uncertain parameters and nonlinearity, we aim to move the error trajectories asymptotically to zero. Hence the states of the response system track the negative of the states of the drive system.

4.1 Neural Network Controller Configuration

An adaptive Radial Basis Function Neural network (RBFNN) is deployed to counter and account for the unknown nonlinearities in the error dynamics synchronization.

The Gaussian spread parameter is considered as $\sigma = 0.5$ for acceptable error and steady estimation of the controller inside the domain. Suppose $k_1 = 20$, $k_2 = 1.4$, and $k_3 = 1.4$, are taken as feedback gains to control oscillations, inadequate damping and error.

By using a positive definite symmetric matrix, the adaptive update laws are formulated as

$$\Gamma_1 = \Gamma_2 = \Gamma_3 = 0.5I_9,$$

where, I_9 denoting the 9×9 identity matrix. From this formulation, we can get stable adaptation for all channels in the networks. To control the growth of unboundedness of the parameter variations, we have included a σ -modification term in the adaptive law. This process, with the Lyapunov stability theory, gives a uniformly ultimately bound for the tracking error, and helps to estimate the neural weight. Therefore, we can get a strong stability condition for the error when the system's parameters are uncertain.

4.2 Simulation Outcomes

Figure 3 illustrates the time evolution of the drive system states (p_1, q_1, r_1) together with the negative response states $(-p_2, -q_2, -r_2)$ under the proposed adaptive control law. It is clearly observed that, after a short transient, the trajectories of the response system approach the inverted dynamics of the drive system, thereby validating successful anti-synchronization.

Specifically, the state variable $p_1(t)$ tracks $-p_2(t)$, confirming antisymmetric alignment of the first subsystem. Likewise, $q_1(t)$ and $-q_2(t)$ converge closely, highlighting the controller's effectiveness in regulating the intermediate dynamics. Finally, the third state $r_1(t)$ versus $-r_2(t)$ exhibits strong anti-synchronization, thereby achieving complete inverse-state correspondence between the two coupled systems.

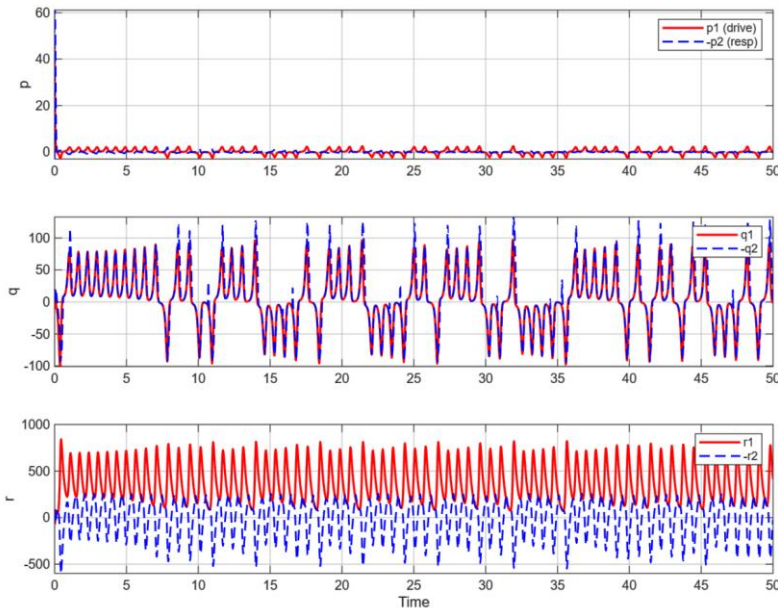


Figure 3

Anti-synchronization of states of the systems (p_1, q_1, r_1) and $(-p_2, -q_2, -r_2)$ under the adaptive controller.

4.3 Error Dynamics under Anti-Synchronization

Figure 4 presents the temporal evolution of the anti-synchronization errors $e_1 = p_1 + p_2$, $e_2 = q_1 + q_2$, and $e_3 = r_1 + r_2$. It is evident that all error trajectories decay rapidly to zero after a short transient period, which confirms the effectiveness of the proposed adaptive controller in achieving complete anti-synchronization.

The vanishing of e_1 indicates that the state $p_2(t)$ evolves as the inverted counterpart of $p_1(t)$, while the convergence of e_2 and e_3 to zero guarantees that

$q_2(t) \approx -q_1(t)$ and $r_2(t) \approx -r_1(t)$, respectively. This validates that the designed control framework enforces a stable Anti-Synchronization regime between the drive and response systems.

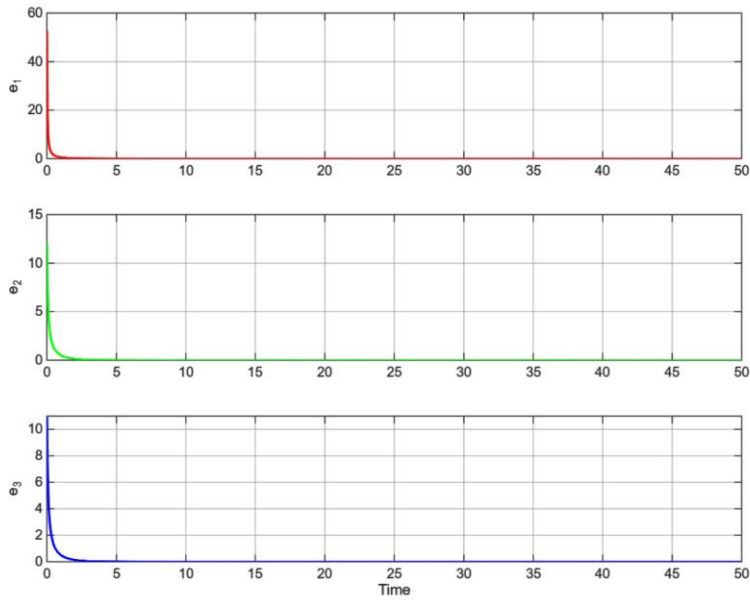


Figure 4
Time evolution of Anti-Synchronization errors e_1 , e_2 , and e_3 , showing convergence to zero under the adaptive control scheme.

This detailed analysis and simulation evidence robustly validate the NN-based adaptive control technique is an effective and strong method for ant-synchronizing highly nonlinear and biologically motivated chaotic systems.

4.4 Discussion

The simulation results confirm that the NN-based adaptive anti-synchronization method effectively suppresses chaos and forces the response system to track the drive system in finite time. The adaptive neural network compensates for unknown nonlinearities and parameter uncertainties, resulting in superior performance compared to conventional active control. This robustness makes the method suitable for practical scenarios where exact system parameters are not known or may vary over time.

5. Conclusion

This work bridges chaotic dynamics across two disparate domains — climate/biological systems and biomedical tumor models — under a unified intelligent chaos control framework. By anti-synchronizing these two chaotic

models, this work highlights how adaptive neural control can regulate and interlink complex behaviors in ecology and physiology. Our results demonstrate the potential of neural controllers in chaos control: for example, we can design specific protocols to regulate eco-system and improvise climate–biological interplay in systems.

Future work may include fractional-order dynamics, stochastically disturbed systems, or deep artificial intelligence techniques to enhance prediction and adaptability. Exploring such avenues would empower intelligent control methods for chaos management in scientific and engineering systems.

References

- [1] L. Chen, C. Huang, H. Liu, and Y. Xia, "Anti-synchronization of a class of chaotic systems with application to Lorenz system: A unified analysis of the integer order and fractional order," *Mathematics*, vol. 7, no. 6, pp. 559 (2019), doi: 10.3390/math7060559.
- [2] E. Shahverdiev, "Chaos anti-synchronization in multiple time delay power systems," *arXiv preprint*, arXiv:1509.05976 (2015).
- [3] H. Chaudhary, M. Sajid, S. Kaushik, and A. Allahem, "Stability analysis of chaotic generalized Lotka--Volterra system via active compound difference anti-synchronization method," *Mathematical Biosciences and Engineering*, vol. 20, no. 5, pp. 9410-9422 (2023), doi: 10.3934/mbe.2023417.
- [4] J. Meng and X.-Y. Wang, "Robust anti-synchronization of a class of delayed chaotic neural networks," *Chaos*, vol. 17, no. 2, p. 023113 (2007), doi: 10.1063/1.2731306.
- [5] L. M. Pecora and T. L. Carroll, "Synchronization in chaotic systems," *Physical Review Letters*, vol. 64, no. 8, pp. 821-824 (1990), doi: 10.1103/PhysRevLett.64.821.
- [6] A. R. Sahab and M. T. Ziabari, "Anti-synchronization of two different hyperchaotic systems via active generalized backstepping method," *International Journal of Chaos, Control, Modelling and Simulation*, vol. 2, no. 1, pp. 1-9 (2013), doi: 10.5121/ijccms.2013.2101.
- [7] W. Jawaada, M. S. M. Noorani, and M. M. Al-Sawalha, "Anti-synchronization of chaotic systems via adaptive sliding mode control," *Chinese Physics Letters*, vol. 29, no. 12, p. 120505 (2012), doi: 10.1088/0256-307X/29/12/120505.
- [8] S. Vaidyanathan and S. Sampath, "Anti-synchronization of four-wing chaotic systems via sliding mode control," *International Journal of Au-*

- tomation and Computing, vol. 9, no. 3, pp. 274-279 (2012), doi: 10.1007/s11633-012-0644-2.
- [9] C. Huang and J. Cao, "Active control strategy for synchronization and anti-synchronization of a fractional chaotic financial system," *Physica A: Statistical Mechanics and its Applications*, vol. 473, pp. 262-275 (2017), doi: 10.1016/j.physa.2017.01.009.
 - [10] M. M. Al-Sawalha and M. S. M. Noorani, "Anti-synchronization between two different hyperchaotic systems," *Journal of Uncertain Systems*, vol. 3, no. 3, pp. 192-200 (2009).
 - [11] M. Shahzad and M. Haris, "Synchronization and anti-synchronization of a new hyperchaotic system using nonlinear control," *Asian Journal of Engineering and Technology*, vol. 2, no. 1, pp. 24-34 (2014).
 - [12] C. K. Ahn, "Anti-synchronization of time-delayed chaotic neural networks based on adaptive control," *International Journal of Theoretical Physics*, vol. 48, no. 12, pp. 3498-3506 (2009).
 - [13] S. Vaidyanathan, "Anti-synchronization of the FitzHugh--Nagumo chaotic neuron models via adaptive control method," *International Journal of PharmTech Research*, vol. 8, no. 7, pp. 71-83 (2015).
 - [14] L. Fang, T. Li, Z. Li, and R. Li, "Adaptive terminal sliding mode control for anti-synchronization of uncertain chaotic systems," *Nonlinear Dynamics*, vol. 74, no. 4, pp. 991-1002 (2013).
 - [15] S. Yang, Z. Guo, and J. Wang, "Robust synchronization of multiple memristive neural networks with uncertain parameters via nonlinear coupling," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 45, no. 7, pp. 1077-1086 (2015), doi: 10.1109/TSMC.2014.2363129.
 - [16] Y. Gu, J. Sun, and X. Fu, "A novel robust neural network sliding-mode control method for synchronizing fractional-order chaotic systems in the presence of uncertainty, disturbance and time-varying delay," *Journal of Electrical Engineering and Technology*, vol. 18, no. 2, pp. 1325-1335 (2023), doi: 10.1007/s42835-022-01309-4.
 - [17] M. A. Khan, Z. Bashir, and M. G. Abbas Malik, "A study of the chaotic features of variable-order fractional Liu's system via radial basis neural network," *International Journal of Dynamics and Control*, vol. 13, no. 1, p. 18 (2025), doi: 10.1007/s40435-024-01514-5.
 - [18] L. Fang, T. Li, X. Wang, and X. Gao, "Adaptive synchronization of uncertain chaotic systems via neural network-based dynamic surface control design," in *Proc. International Symposium on Neural Networks (ISNN 2013)*, Lecture Notes in Computer Science (LNCS), vol. 7952,

- Berlin, Germany: Springer, pp. 104-111 (Jul. 2013), doi: 10.1007/978-3-642-39068-5_13.
- [19] C. K. Ahn, "Adaptive H_∞ anti-synchronization for time-delayed chaotic neural networks," *Progress of Theoretical Physics*, vol. 122, no. 6, pp. 1391-1403 (2009), doi: 10.1143/PTP.122.1391.
 - [20] T. Li, Y. Zheng, and Q. Wu, "Adaptive anti-synchronization control of a new chaotic system and its implementation," *Nanjing Xinxing Gongcheng Daxue Xuebao*, vol. 6, no. 4, pp. 306-311 (2014).
 - [21] A. Boubakir and S. Labiod, "Observer-based adaptive neural network control design for projective synchronization of uncertain chaotic systems," *Journal of Vibration and Control*, vol. 29, no. 15-16, pp. 3658-3678 (2023), doi: 10.1177/10775463221149478.
 - [22] Z. Li, H. Yan, H. Zhang, S. X. Yang, and M. Chen, "Novel extended state observer design for uncertain nonlinear systems via refined dynamic event-triggered communication protocol," *IEEE Transactions on Cybernetics*, vol. 53, no. 3, pp. 1856-1867 (2023), doi: 10.1109/TCYB.2022.3203586.
 - [23] P. Kumar, S. Jha, R. Aggarwal, and G. K. Jha, "Effect of climate change on brain tumor," *Applications and Applied Mathematics*, vol. 17, no. 2, pp. 1096-1118 (2022).
 - [24] M. B. Thorat, V. P. Wawage, D. P. Yadav, P. Pushpalatha, and Y. Gandhi, "Mathematical foundations of neural network weight optimization," *Journal of Information and Optimization Sciences*, vol. 47, no. 5-B, pp. 2065-2073 (2026), doi: 10.47974/JIOS-2297.
 - [25] S. F. Al-Azzawi and M. A. Al-Hayali, "Multistability and hidden attractors in a novel simple 5D chaotic Sprott E system without equilibrium points," *Journal of Interdisciplinary Mathematics*, vol. 25, no. 5, pp. 1279-1294 (2022), doi: 10.1080/09720502.2021.1970948.
 - [26] B. Farkas and S. A. Wegner, "Variations on Barblat's lemma," *The American Mathematical Monthly*, vol. 123, no. 8, pp. 825-830 (2016), doi: 10.4169/amer.math.monthly.123.8.825.
 - [27] C. Roy and M. A. Khan, "Targeting synchronization of different-dimensional chaotic systems," *Journal of Information and Optimization Sciences*, vol. 47, no. 7, pp. 2521-2534 (2026), doi: 10.47974/JIOS-1977.

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