Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 47 (2026), No. 7, pp. 2601–2606
DOI : 10.47974/JIOS-2152

A new variant of trapezoidal technique for simulation of nonlinear equations

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Abstract

Nonlinear equations arise in many fields of science and engineering. Solving these equations numerically is often challenging due to their complex nature. Finding roots of nonlinear equations is challenging in numerical analysis. Analytical solutions to nonlinear equations are often intractable necessitating the use of numerical methods. In this paper, we proposed a newly iterative technique for simulation of nonlinear equations based on trapezoidal rule of integration. Theoretical analysis and numerical experiments demonstrate and effectiveness of newly proposed iterative technique for solving a wide range of nonlinear problems. When working with certain higher order functions, the suggested approach greatly improves the trapezoidal rule and gets outcomes the error value problem, even when solving for a single interval. This provides grounds for confidence in the proposed method and provides the prospect for further refinement in future work.

Subject Classification: *65A05, 65D05, 65D30, 65D32.*

Keywords: *Numerical simulation, Nonlinear equations, Error, Trapezoidal rule.*

Introduction

Integration is the process of calculating the area that a function has drawn on a graph. $I = \int_a^b f(x)\,dx$ is the sum or total of $f(x)$ on the interval $[a, b]$. These days, it is crucial since computers can integrate data in analytical ways and link analytical models to computer processors. [1]. Numerous approaches,

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including the Euler-Maclaurin formula, adaptive quadrature, Gaussian integration, and quadrature methods, can be used to perform numerical integration to compute functions that are challenging to integrate. The value of the definite integral is also included, which is determined by integrating from a to b after altering the function y using an interpolation formula. Using this method, we can get a quadrature formula with known numerical values [2]. However, many researchers have already completed large-scale studies aimed at modelling and expanding the various numerical integration domains for various purposes [3]. Concepcion Ausin investigated more complex numerical integration methods and compared multiple providers of numerical integration. The most common method for solving various integration difficulties is numerical integration because some integration issues cannot be resolved analytically [4]. For unequal data spaces, numerical integration problems are approached and solved using a variety of methods. Numerical integration is a process of approximately computing an integral using a numerical technique [5-6]. Rajinder Thukral solves nonlinear equations efficiently by using Simpson's technique. Jayakumar extended Simpson's Newton technique to solve nonlinear equations with cubic convergence. The simplest and probably oldest technique for numerical quadrature, the (composite) trapezoidal rule, is currently being criticized as being insufficient to produce high accuracy quadrature. Although summation and mathematical integration are closely connected, symbol of integration is a stylized capital "S" to emphasize such connection [7]. The scientific community has been interested in using numerical methods to solve engineering problems. This interest has been encouraged by its efficiency and the advancement of high-speed computations carried out on processors of the twenty-first century. Trefethen and Weideman have also critically analyzed the trapezoidal rule from a mathematical and historical standpoint, highlighting its connections to computational methods utilized in several scientific computing domains. It was discovered to be related to methods for calculating the eigen values of operators and matrices, as well as to sophisticated concepts, special functions, integral equations, and rational approximation [8]. Hamzah and Shiker discuss a new line search algorithm modified of Spectral gradient projection method (SGPM) called (D.A), combining the modified spectral gradient method and projection method for solving nonlinear equations systems. This approach can be more efficient because it requires less CPU time, less function evaluation, and fewer iterations [9]. Inderjeet and Rashmi Bhardwaj presented modified iterative approaches to tackle the nonlinear equations with improved accuracy and robustness of the technique, and the proposed approaches are better than the existing techniques like Bisection, Newton Raphson, Secant technique, Fixed– Point iteration etc. [10-17].

Proposed Methodology:

The existing Trapezoidal rule is $I = \frac{b-a}{2}\,[f(a) + f(b) \ldots (1)$

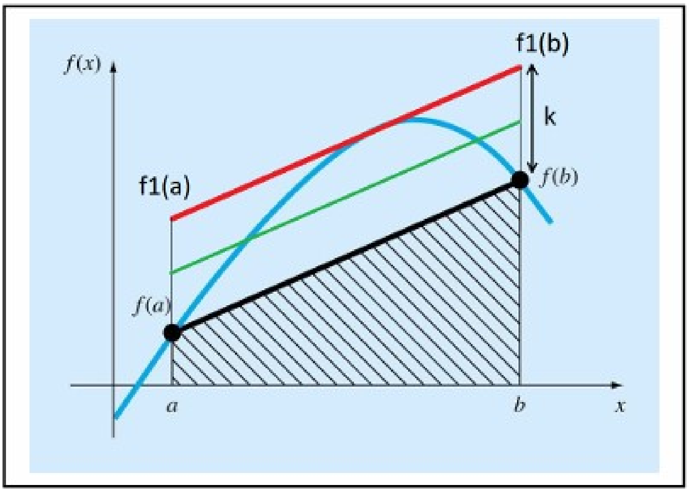

Figure 1
Trapezoidal rule and proposed rule

In the proposed methodology $f(a)$ replaced by the $\frac{f(a)+f_1(a)}{2}$, and $f(b)$ replaced by the $\frac{f(b)+f_1(b)}{2}$

$$I = \frac{b-a}{4}\left[f(a) + f_1(a) + f_1(b) + f(b)\right] \ldots (2)$$

Let, $k = f_1(a) - f(a) = f_1(b) - f(b)$ because these are parallel lines as shown in figure 1.

$$I = \frac{b-a}{2}\left[\frac{f(a)+f_1(b)}{2}\right] \ldots (3)$$

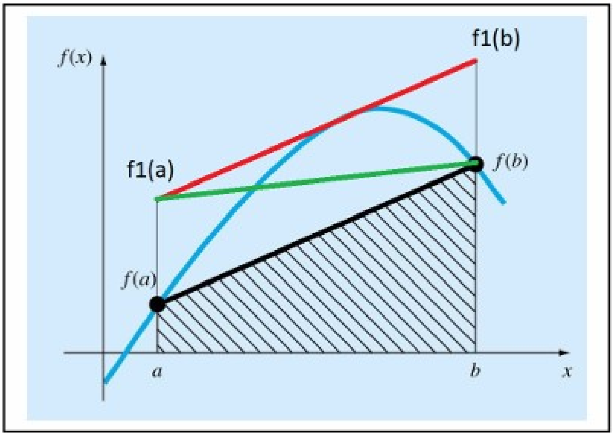

Figure 2
Proposed methodology equivalent under the curve.

Error Analysis

As we know that error formula for trapezoidal rule is

E_T = area under the curve − area under the trapezoidal technique

$$E_T = \frac{b-a}{2}[f(a) + f(b)] - \frac{1}{12}f''(\mu)(\frac{b-a}{2})^3 \ldots (4)$$

$$E_T = \frac{h}{2}[f(a) + f(b)] - \frac{1}{12}f''(\mu)h^3 \ldots (5)$$

Similarly, we calculate the error formula for modified trapezoidal rule from figure 2 as follows:

In another way, from figure 1 if we assume centre line with height $\frac{k}{2}$, then new area will be $\frac{b-a}{2}[f(a) + f(b) + k]$

Error formula for modified trapezoidal technique is

$$E_{T^*} = \frac{1}{12}f''(\mu)h^3 - h\,\frac{k}{2}. . . (6)$$

Numerical Experiments and Comparison of Results

To evaluate performance of the modified approach, numerical simulations were conducted on some benchmark problems involving nonlinear equations. The proposed method was compared with the existing trapezoidal technique.

Problem 1: $f_1(x) = 5 + 3\cos x$ in [0,3]

Problem 2: $f_2(x) = x^5 - 2.25\,x^4 + 1.69\,x^3 - 0.5\,x^2 + 0.06x + 0.0005$ in $[0, 0.5]$

Table 1

Comparison of various iterative schemes

Problem	Number of Intervals	Exact Solution	Trapezoidal Rule	Proposed Rule
1	1	15.43	16.05	15.04
1	2	15.43	15.05	15.35
2	1	1.6405	0.1728	1.5078
2	2	1.6405	1.0688	1.6109

Result Discussions and Conclusion

In this article, we discuss a modified method for numerically simulating and approximation the roots of nonlinear equations. The proposed technique enhances the efficiency of the traditional trapezoidal technique. Theoretical analysis and convergence results were established, showing that the modified method performs better than the traditional trapezoidal method. Numerical experiments on benchmark problems demonstrated the superior performance of the modified technique compared to others established techniques like trapezoidal method as shown in table 1.

In conclusion, the modified approach presented in this paper offers a powerful and efficient numerical algorithm for simulating and solving nonlinear equations. Its performance, and robustness make it a promising approach for tackling a wide range of nonlinear problems in science and engineering.

Author's Contribution

Inderjeet: Methodology, Software, Writing-Original Drafts.

Rashmi Bhardwaj: Conceptualization, Supervision, Investigation, Writing-Review and Editing.

Conflict of Interest

None.

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Received July, 2025