Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 47 (2026), No. 6, pp. 2391–2400
DOI : 10.47974/JIOS-2175

Stochastic modeling of a retrial queue with server vacations and customer impatience in a multi-level environment

C. T. Dora Pravina *
G. Hemavathi †
Department of Mathematics
Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology
Avadi
Chennai 600062
Tamil Nadu
India

Sukumaran Damodaran §
Department of Mathematics
Vel Tech High Tech Dr. Rangarajan Dr. Sakunthala Engineering College
Avadi
Chennai 600062
Tamil Nadu
India

Abstract

This paper analyzes a single-server retrial queue with vacations and customer impatience in a multi-level stochastic environment. Customer arrivals follow a Poisson process whose rate depends on the environment level and receive immediate exponential service if the server is idle. Otherwise, the customer joins an orbit and is queued as per the FCFS discipline, and retries for service follows an exponential distribution. Whenever the system becomes empty, the server takes a vacation during which arrivals enter the orbit but are not served, and orbiting customers may abandon due to impatience. After vacation, the server stays idle until service is triggered by a new arrival or a retrial.

Subject Classification: *60K25, 60J27, 90B22, 90B25.*

Keywords: *Single-server retrial queue, Vacation policy, Customer impatience, Multi-level stochastic environment.*

* *E-mail:* **tdorapravinac@veltech.edu.in (Corresponding Author)**
† *E-mail:* **hemagovidh@gmail.com**
§ *E-mail:* **Sukumaran.d@velhightech.com**

1. Introduction

Retrial queueing systems are a class of queueing models in which customers who find the server busy upon arrival do not wait in a conventional queue but instead leave the service area temporarily and retry for service after a random period of time. In these systems, customers (or requests) unable to receive immediate service are directed to an orbit, from where they make repeated attempts to access the server. Retrial queues have wide-ranging applications in real-world systems such as call centers, cellular networks, telecommunication systems, computer networks, manufacturing systems, and customer support services. Vacation queues from another important class of models in which the server becomes unavailable for certain periods due to maintenance, energy-saving modes, or scheduled breaks. Since both retrial behaviour and vacation policies strongly influence congestion and performance, combining these ideas offers a more realistic, flexible, and practical service systems. In developing such advanced models, insights from stochastic processes, optimization techniques, and system performance analysis become essential. For instance, Nain et al. [16] modeled signal error variation in an aquatic robot sensor using heavy-tailed distributions, identified the Weibull distribution as best via AIC, BIC, and K–S tests, and introduced PRDP and TRDP to measure extreme errors. Kale et al. [17] analyzed security and privacy issues in edge computing, discussing encryption, RBAC/ABAC, secure protocols, and evaluating DES and 3DES with privacy-preserving techniques. Stefanov [18] presented theoretical foundations of discrete and continuous Markov processes, including transition matrices and stationary distributions. Sivakumar, Kaliappan, and Ramasamy [19] proposed a Fog-Assisted Cloud Model using WDO for threat detection and optimization, achieving 25% latency reduction. A wide range of studies has examined retrial queues under diverse operating conditions. Early research on retrial systems in stochastic environments and server unreliability was carried out by Cordeiro [1] and later extended by Cordeiro and Kharoufeh [2]. Udayabaskaran and Pravina [3] analyzed transient behaviour of MM1 queues exposed to disasters. While Liu and Yu [4] further generalized these ideas to multi-server settings under environmental variation. Another important direction concerns vacation-based retrial systems with limited input sources, initially studied, by Li and Yang [5]. Wenhui [6] extended this work to include Bernoulli vacation schedules, and subsequent enhancements involving modified and threshold-based vacation schemes were provided in [7-9]. Further developments include single-server models with working vacations and customer impatience within a multi-phase service structure, as demonstrated by the works of Bouchentouf et al. [10]. Relatedly, Pravina, Kamala, and Sreelakshmi [11] investigated systems subject to disaster, recovery, and repair processes within a multi-phase framework, highlighting renewed interest in resilience-based queueing structures. A Parallel line of research emphasizes strategic customer behavior and balking decisions. Economou, Gómez-Corral, and Kanta [12] provided early insights into optimal balking in queues with vacation features. Guo and Hassin [13-14] developed foundational models describing strategic choices in Markovian vacation queues with homogeneous and heterogeneous customers. Building on these ideas, Zhang [15] examined strategic behaviour specifically in constant retrial queues with single vacations. This study focuses on a single-server retrial queueing system that incorporates vacation periods and customer impatience within a multi-level stochastic environment. Although retrial queues, vacation mechanisms, and impatience phenomena have been extensively studied individually, their combined influence under dynamic environmental conditions remains unexplored. To address this gap, the present work develops a comprehensive analytical model that integrates these factors and analyses their impact on system performance.

The subsequent discussion covers: 2nd Portion presents describes the system model. The 3rd Portion formulates the state probabilities through a set of equations. The 4th Portion derives the exact steady-state probabilities. The 5th Portion presents the system's performance metrics. The 6th Portion provides numerical experiments that validate the analytical findings, while the 7th Portion concludes the work with a synthesis of the steady-state analysis.

2. Model Description

We consider a single-server retrial queueing system with vacation and customer impatience operating under a multi-level stochastic environment with $\mathcal{R}$ distinct levels. Customers arrive according to a Poisson process with rate λ_e, where $e \in \{1,2,\dots,R\}$ represents the environment level. If an arriving customer finds the server idle, they immediately receive service, and the service time follows an exponential distribution with rate μ_e. Otherwise, the customer joins an orbit and waits according to a first-come, first-served discipline. Customers in the orbit retry for service follows an exponentially distributed rate with ν. Whenever the system becomes empty, the server takes a vacation with rate ξ. During this vacation period, the environment operates at level 0, where customers arrive at rate λ_0; however, they are not served immediately and instead enter the orbit. Orbiting customers may abandon the system owing to prolonged waiting, leaving without receiving service due to impatience at a rate ω. Once the vacation ends, the server remains idle until service is initiated either by a new arrival or by a retrial customer from the orbit.

3. Governing Equations

Let $\mathcal{E}, \mathcal{Q}$, and $\mathcal{O}$ represent the environment level, states of the server, and number of customers in orbit. The server is represented state 0 when it is idle, and when it is busy, it is set to 1. When the server is in the vacation, it is said to be in state 2 and the environment to be at level 0. The three-dimensional stochastic process $(\mathcal{E}, \mathcal{Q}, \mathcal{O})$defines a Continuous-time Markov process (CTMP) with a discrete state space is defined by

$$\boldsymbol{\Omega} = \{(\mathbf{0}, \mathbf{2}, \boldsymbol{x})\} \cup \{(\boldsymbol{e}, \boldsymbol{s}, \boldsymbol{x}) |\ \boldsymbol{e} = \mathbf{1}, \mathbf{2}, \dots, \boldsymbol{R};\ \boldsymbol{s} = \mathbf{0}, \mathbf{1};\ \boldsymbol{x} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \dots.\}.$$

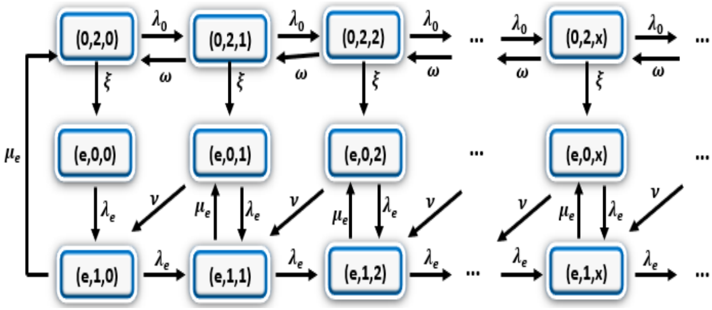

Figure 1
Transition rate diagram

The representation of the transition rate diagram is shown in Figure 1.

We define the steady-state probability function

$$\pi(e,s,x) = Pr[\mathcal{E}=e,\mathcal{Q}=s,\mathcal{O}=x \mid \mathcal{E}=e,\mathcal{Q}=0,\mathcal{O}=0],\ \ (e,s,x)\in\Omega.$$

In the next section, we derive the balance equations:

$$\lambda_e\pi(e,0,0) = \xi\pi(0,2,0), e=1,2,\ldots,R \tag{7}$$

$$(\lambda_e+\nu)\pi(e,0,x) = \mu_e\pi(e,1,x)+\xi\pi(0,2,x), e=1,2,\ldots,R, x\geq 1 \tag{8}$$

$$(\lambda_e+\mu_e)\pi(e,1,0) = \lambda_e\pi(e,0,0)+\nu\pi(e,0,1), e=1,2,\ldots,R \tag{9}$$

$$(\lambda_e+\mu_e)\pi(e,1,x) = \lambda_e\pi(e,0,x)+\lambda_e\pi(e,1,x-1) \\ +\nu\pi(e,o,x+1), x\geq 1 \tag{10}$$

$$(\lambda_0+\xi)\pi(0,2,x) = \mu_e\pi(e,1,0)+\omega\pi(0,2,1) \tag{11}$$

$$(\lambda_0+\omega+\xi)\pi(0,2,x) = \lambda_0\pi(0,2,x-1)+\omega\pi(0,2,x+1), x\geq 1 \tag{12}$$

Define the PGF's

$\mathcal{A}_{e,s}(z)=\sum_{x=0}^{\infty}\pi(e,s,x)z^x$, $\mathcal{A}_{e,s}(z)=\sum_{x=0}^{\infty}\pi(0,2,x)z^x$, $s=0,1$, $e=1,2,\ldots,R$

Multiplying equations (8), (10), and (12) by $z^x$and summing over x, give that

$$(\lambda_e+\nu)\mathcal{A}_{e,0}(z)-\nu\pi(e,0,0) = \mu_e\mathcal{A}_{e,1}(z)-\mu_e\pi(e,1,0)+\xi\mathcal{A}_{0,2}(z) \tag{13}$$

$$\left(\lambda_e+\frac{\nu}{z}\right)\mathcal{A}_{e,0}(z)-(\lambda_e+\mu_e-\lambda_e z)\mathcal{A}_{e,1}(z) = \frac{\nu}{z}\pi(e,0,0) \tag{14}$$

$$[\lambda_0 z^2-(\lambda_0+\xi+\omega)z+\omega]\mathcal{A}_{0,2}(z) = \frac{\omega(1-z)\lambda_e}{\xi}\pi(e,0,0) \quad -\mu_e z\,\pi(e,1,0)$$
(15)

From (15), after simplification we obtain,

$$\mathcal{A}_{0,2}(z) = \frac{\omega(1-z)\lambda_e\pi(e,0,0)-\mu_e z\xi\pi(e,1,0)}{\xi[\lambda_0 z^2-(\lambda_0+\xi+\omega)z+\omega]} \tag{16}$$

The zeros of the denominators az^2-bz+c of equation (16) is given by

$$\alpha,\beta = \frac{(\lambda_0+\xi+\omega)\pm\sqrt{(\lambda_0+\xi+\omega)^2-4\lambda_0\omega}}{2\lambda_0} \tag{17}$$

It can be easily shown that $b^2 - 4ac > 0$ and hence α and β real and distinctly positive.

Furthermore $\alpha < 1 < \beta$ we can write (16) as follows

$$\mathcal{A}_{0,2}(z) = \frac{\omega(1-z)\lambda_e\pi(e,0,0) - \mu_e z\xi\pi(e,1,0)}{\xi[\lambda_0(z-\alpha)(z-\beta)]} \tag{19}$$

Using the analyticity of $\mathcal{A}_{0,2}(z)$ for $|z| < 1$, we impose the condition that the denominator of equation (19) must vanish at the root $z = \alpha$, and by equating the numerator to zero at this point, we obtain

$$\pi(e,1,0) = \frac{\omega(1-\alpha)\lambda_e\pi(e,0,0)}{\mu_e\alpha\xi} \tag{20}$$

Using (20) in (19) and simplifying, we get

$$\boldsymbol{\mathcal{A}_{0,2}(z) = \frac{\omega\lambda_e\pi(e,0,0)}{\alpha\xi\lambda_0(\beta - z)}} \tag{21}$$

Substitute (21) in (13) and from (14), we get

$$\boldsymbol{(\lambda_e + \nu)\mathcal{A}_{e,0}(z) - \mu_e\mathcal{A}_{e,1}(z) = \frac{[\nu\alpha\xi - \omega\lambda_e(1-\alpha)]\lambda_0(\beta - z) + \xi\omega\lambda_e}{\alpha\xi\lambda_0(\beta - z)}\pi(e,0,0)} \tag{22}$$

$$(\lambda_e z + \nu)\mathcal{A}_{e,0}(z) - (\lambda_e + \mu_e - \lambda_e z)z\mathcal{A}_{e,1}(z) = \nu\pi(e,0,0) \tag{23}$$

By cross multiplication rule (22) and (23) yields

$$\mathcal{A}_{e,0}(z) = \frac{\{[\lambda_e z^2 - (\lambda_e + \mu_e)z]\{[\nu\alpha\xi - \omega\lambda_e(1-\alpha)]\lambda_0(\beta - z) + \xi\omega\lambda_e\} + \mu_e\nu\alpha\xi\lambda_0(\beta - z)\}\pi(e,0,0)}{\{(\lambda_e + \nu)[\lambda_e z^2 - (\lambda_e + \mu_e)z] + \mu_e(\lambda_e z + \nu)\}\alpha\xi\lambda_0(\beta - z)} \tag{24}$$

$$\mathcal{A}_{e,1}(z) = \frac{\{(\lambda_e + \nu)\nu\alpha\xi\lambda_0(\beta - z) - (\lambda_e z + \nu)\{[\nu\alpha\xi - \omega\lambda_e(1-\alpha)]\lambda_0(\beta - z) + \xi\omega\lambda_e\}\}\pi(e,0,0)}{\{(\lambda_e + \nu)[\lambda_e z^2 - (\lambda_e + \mu_e)z] + \mu_e(\lambda_e z + \nu)\}\alpha\xi\lambda_0(\beta - z)} \tag{25}$$

Combining equations (21), (24), and (25) and applying the normalization condition

$$\mathcal{A}_{e,0}(1) + \mathcal{A}_{e,1}(1) + \mathcal{A}_{0,2}(1) = 1,$$

We derive that

$$\boldsymbol{\pi(e,0,0) = \frac{(\beta - 1)[\lambda_e^2 + \gamma(\lambda_e - \mu_e)]\alpha\xi\lambda_0}{(\beta - 1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e - \nu\alpha\xi] + \omega\lambda_e[\lambda_e^2 + \nu(\lambda_e - \mu_e) - \xi\mu_e - \lambda_0(1-\alpha)(\lambda_e + \mu_e + \nu)]}} \tag{26}$$

By substituting $z = 1$ into the expressions for $\mathcal{A}_{e,0}(z), \mathcal{A}_{e,1}(z)$, and $\mathcal{A}_{0,2}(z)$, we obtain the probabilities corresponding to the different states of the server.

5. Performance Measures

This portion presents the analytical formulations for key performance metrics of the proposed model.

1. *Stationary probability server idle*

While the server remains idle, the system may have any number of orbiting customers. As a result, the server's idle probability is expressed as follows.

$$\boldsymbol{\mathcal{M}_1 = \sum_{e=1}^{R}\sum_{x=0}^{\infty}\frac{(\beta - 1)(\lambda_e - \mu_e)\lambda_0[\nu\alpha\xi - \omega\lambda_e(1-\alpha)] + \omega\lambda_e[\xi(\lambda_e - \mu_e) - \mu_e\lambda_0(1-\alpha)]}{(\beta - 1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e - \nu\alpha\xi] + \omega\lambda_e[\lambda_e^2 + \nu(\lambda_e - \mu_e) - \xi\mu_e - \lambda_0(1-\alpha)(\lambda_e + \mu_e + \nu)]}}$$

2. *Stationary probability server busy*

When the server is in busy state, the system may have any number of orbiting customers. As a result, the probability of the server being busy mode is expressed as follows.

$$\mathcal{M}_2 = \sum_{e=1}^{R}\sum_{x=0}^{\infty}\frac{(\beta-1)\lambda_e\lambda_0[\omega\lambda_e(1-\alpha)-\nu\alpha\xi]-\omega\lambda_e[(\lambda_e+\nu)(1-\alpha)\lambda_0+\xi\lambda_e]}{(\beta-1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e-\nu\alpha\xi]+\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)-\xi\mu_e-\lambda_0(1-\alpha)(\lambda_e+\mu_e+\nu)]}$$

3. *The server's vacation probability*

While the server undergoes a vacation, there may be an arbitrary number of orbiting customers. Therefore, the server's vacation probability can be formulated as

$$\mathcal{M}_3 = \sum_{x=0}^{\infty}\frac{\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)]}{(\beta-1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e-\nu\alpha\xi]+\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)-\xi\mu_e-\lambda_0(1-\alpha)(\lambda_e+\mu_e+\nu)]}$$

4. *Stationary rate of the server being engaged with an arriving customer*

When the server is occupied with an arriving customer, the orbit may contain an arbitrary number of customers. Hence, the stationary probability at which the server is serving the arriving customers is given by

$$\mathcal{M}_4 = \sum_{e=1}^{R}\sum_{x=0}^{\infty}\frac{(\beta-1)(\lambda_e-\mu_e)\lambda_0[\nu\alpha\xi-\omega\lambda_e(1-\alpha)]+\omega\lambda_e[\xi(\lambda_e-\mu_e)-\mu_e\lambda_0(1-\alpha)]}{(\beta-1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e-\nu\alpha\xi]+\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)-\xi\mu_e-\lambda_0(1-\alpha)(\lambda_e+\mu_e+\nu)]}\lambda_e.$$

5. *Stationary rate of the server being engaged with a retrial customer*

When the server is unavailable (busy) with retry customer, the orbit can still contain an arbitrary number of customers. Accordingly, the stationary rate of server being occupied with a retry customer is given by

$$\mathcal{M}_5 = \sum_{e=1}^{R}\sum_{x=0}^{\infty}\frac{(\beta-1)(\lambda_e-\mu_e)\lambda_0[\nu\alpha\xi-\omega\lambda_e(1-\alpha)]+\omega\lambda_e[\xi(\lambda_e-\mu_e)-\mu_e\lambda_0(1-\alpha)]}{(\beta-1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e-\nu\alpha\xi]+\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)-\xi\mu_e-\lambda_0(1-\alpha)(\lambda_e+\mu_e+\nu)]}\nu.$$

6. Stationary rate that the vacation of server is completed

The server returns from vacation with rate ξ from state $(0,2,x)$. Therefore, the stationary rate at which the server becomes active to provide service is given by

$$\mathcal{M}_6 = \sum_{x=0}^{\infty}\frac{\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)]\xi}{(\beta-1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e-\nu\alpha\xi]+\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)-\xi\mu_e-\lambda_0(1-\alpha)(\lambda_e+\mu_e+\nu)]}$$

7. *Mean number of customers leave the system with an impatience during the vacation period*

Let $\mathcal{M}_7$ be the average number of customers in the orbit who leave the system due to impatience during the server's vacation. When the server is on vacation, impatient customers may abandon the system as they cannot receive immediate service. This measure reflects the average abandonment rete in the vacation state.

$$\mathcal{M}_7 = \sum_{x=0}^{\infty}\frac{\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)]\omega}{(\beta-1)\lambda_0\mu_e[(1-\alpha)\omega\lambda_e-\nu\alpha\xi]+\omega\lambda_e[\lambda_e^2+\nu(\lambda_e-\mu_e)-\xi\mu_e-\lambda_0(1-\alpha)(\lambda_e+\mu_e+\nu)]}$$

8. *Average orbit size*

If $\mathcal{M}_8$ denoted the average orbit size, then its expression is given by

$$\boldsymbol{\mathcal{M}_8 = \sum_{x=0}^{\infty}\left[\sum_{e=1}^{R} x[\pi(e,0,x)+\pi(e,1,x)] + x\pi(0,2,x)\right]}$$

9. *Average system size*

If $\mathcal{M}_9$ denotes the average system size, then its expressions is given by

$$\boldsymbol{\mathcal{M}_9 = \sum_{x=0}^{\infty}\left[\sum_{e=1}^{R} x[\pi(e,0,x)+\pi(e,1,x-1)] + x\pi(0,2,x)\right]}$$

6. Numerical Illustration

This section, shows the illustration of the performance of our model numerically. We fix $R = 5$, and the other parameters λ_e, μ_e and $\lambda_0 = 1$ as follows:

Fixed parameters

$\lambda_1 = 1.1$	$\lambda_2 = 1.2$	$\lambda_3 = 1.4$	$\lambda_4 = 1.6$	$\lambda_5 = 1.8$
$\mu_1 = 2.0$	$\mu_2 = 2.2$	$\mu_3 = 2.4$	$\mu_4 = 2.6$	$\mu_5 = 2.8$

The behaviour of the performance metrics varies as a function of the control variables ν, ξ, and ω.

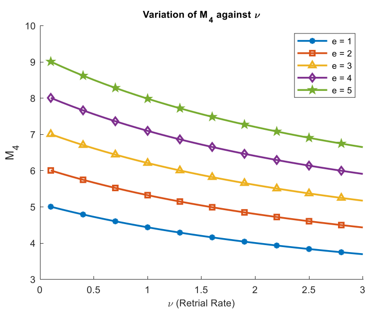

Figure 2
Variation of M_4 against ν

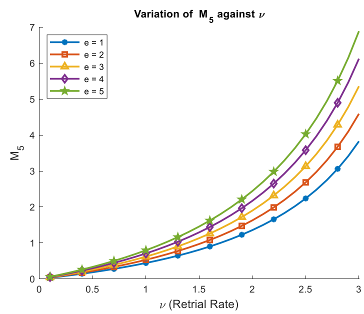

Figure 3
Variation of M_5 against ν

Fixing $\xi = 2, \omega = 0.5$, we vary ν from 0.1 to 3.0. from Figure 2, we observe that as the retrial rate ν increases, orbiting customers compete more for the server, reducing its availability for newly arriving customers. Hence, a higher ν generally leads to a decrease in $\mathcal{M}_4$.

Fixing $\xi = 2, \omega = 0.5$, we vary ν from 0.1 to 3.0. from Figure 3, we observe that as the retrial rate ν increases, orbiting customers attempt to access the server more frequently, leading to a higher chance of successful retrials. Consequently, the stationary rate $\mathcal{M}_5$ of the server being engaged with retrial customers increases.

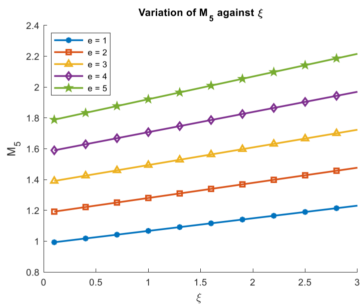

Figure 4
Variation of M_5 against ξ

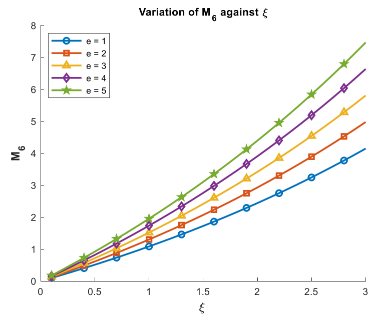

Figure 5
Variation of M_6 against ξ

Fixing $\upsilon = 1, \omega = 0.5$, we vary ξ from 0.1 to 3.0. from Figure 4, we observe that as the vacation return rate ξ increases, the server returns to service more frequently, thereby increasing its availability for retrial customers. Consequently, the stationary rate $\mathcal{M}_5$ generally increases with ξ.

Fixing $\upsilon = 1, \omega = 0.5$, we vary ξ from 0.1 to 3.0. from Figure 5, we observe that as the vacation return rate ξ increases, the server returns from vacation more frequently, thereby increasing the rate at which it becomes active to provide service. Consequently, the stationary rate $\mathcal{M}_6$ generally increases with ξ.

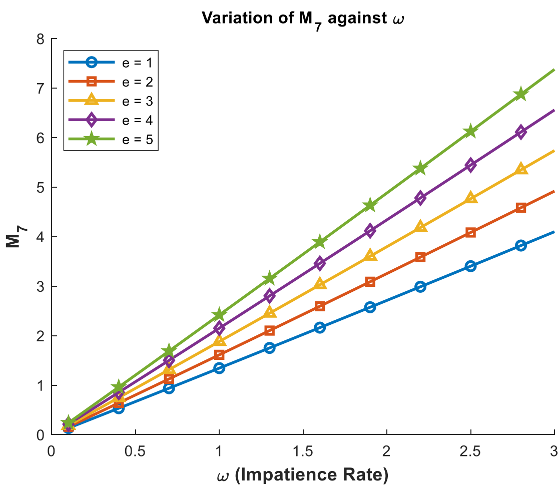

Figure 6
Variation of M_7 against ω

Fixing $\upsilon = 1, \xi\ = 2$, we vary ω from 0.1 to 3.0. from Figure 6, we observe that as the impatience rate ω increases, more customers leave the system during the server vacation, thereby increasing the average abandonment rate. Consequently, the stationary rate $\mathcal{M}_7$ generally increase with ω.

7. Conclusion

This work presents an analytical model for a single-server retrial queue with vacation and customer impatience under a multi-level stochastic environment. The analysis provides key performance measures reflecting the impact of retrials, vacations, and impatience on system behavior. Future research may extend this model to multi-server settings, priority-based services, or optimize retrial and vacation parameters to enhance efficiency and reduce abandonment.

References

[1] J. D. Cordeiro Jr., "Unreliable Retrial Queues in a Random Environment," *M.S. thesis, Air Force Institute of Technology* (2007).

[2] J. D. Cordeiro and J. P. Kharoufeh, "The unreliable M/M/1 retrial queue in a random environment," *Stochastic Models*, vol. 28, no. 1, pp. 29–48 (2012).

[3] S. Udayabaskaran and C. D. Pravina, "Transient analysis of an M/M/1 queue in a random environment subject to disasters," *Far East Journal of Mathematical Sciences*, vol. 91, no. 2, pp. 157 (2014).

[4] Z. Liu and S. Yu, "The M/M/C queueing system in a random environment," *Journal of Mathematical Analysis and Applications,* vol. 436, no. 1, pp. 556–567 (2016).

[5] H. Li and T. Yang, "A single-server retrial queue with server vacations and a finite number of input sources," *European Journal of Operational Research*, vol. 85, no. 1, pp. 149–160 (1995).

[6] Z. Wenhui, "Analysis of a single-server retrial queue with FCFS orbit and Bernoulli vacation," *Applied Mathematics and Computation,* vol. 161, no. 2, pp. 353–364 (2005).

[7] K. C. Madan, W. Abu-Dayyeh, and F. Taiyyan, "A two-server queue with Bernoulli schedules and a single vacation policy," *Applied Mathematics and Computation*, vol. 145, no. 1, pp. 59–71 (2003).

[8] M. Haridass and R. Arumuganathan, "Analysis of a single server batch arrival retrial queueing system with modified vacations and N-policy," *RAIRO-Operations Research*, vol. 49, no. 2, pp. 279–296 (2015).

[9] C. Raychaudhuri and A. Jain, "Efficiency of retrial queueing system under n threshold during vacation," *International Journal of Information Technology*, vol. 17, no. 3, pp. 1419–1430 (2025).

[10] A. A. Bouchentouf, A. Guendouzi, M. Houalef, and S. Majid, "Analysis of a single server queue in a multi-phase random environment

with working vacations and customers' impatience," *Operations Research and Decisions*, vol. 32, no. 2, pp. 16–33 (2022).

[11] C. D. Pravina, P. Kamala, and S. Sreelakshmi, "Analysis of a Markovian queue of single server performing in multi-phase subject to disaster, recovery and repair," *Contemporary Mathematics*, pp. 5012–5026 (2024).

[12] A. Economou, A. Gómez-Corral, and S. Kanta, "Optimal balking strategies in single-server queues with general service and vacation times," *Performance Evaluation*, vol. 68, no. 10, pp. 967–982 (2011).

[13] P. Guo and R. Hassin, "Strategic behavior and social optimization in Markovian vacation queues," *Operations Research*, vol. 59, no. 4, pp. 986–997 (2011).

[14] P. Guo and R. Hassin, "Strategic behavior and social optimization in Markovian vacation queues: The case of heterogeneous customers," *European Journal of Operational Research*, vol. 222, no. 2, pp. 278–286 (2012).

[15] Y. Zhang, "Strategic behavior in the constant retrial queue with a single vacation," *RAIRO-Operations Research*, vol. 54, no. 2, pp. 569–583 (2020).

[16] A. Nain, G. M. Upadhyay, N. Bohra, P. K. Soni, and L. Raja, "A mathematical approach to model signal variation in an aquatic robot's environmental sensor using heavy-tailed distributions," *Journal of Interdisciplinary Mathematics,* vol. 28, no. 2, pp. 597–606 (2025), doi: 10.47974/JIM-2103.

[17] S. Kale, J. Hase, S. Deshmukh, S. N. Ajani, P. K. Agrawal, and C. S. Khandelwal, "Ensuring data confidentiality and integrity in edge computing environments: A security and privacy perspective," *Journal of Discrete Mathematical Sciences and Cryptography,* vol. 27, no. 2-A, pp. 421–430 (2024), doi: 10.47974/JDMSC-1898.

[18] S. M. Stefanov, "On the discrete and continuous Markov processes," *Journal of Interdisciplinary Mathematics,* vol. 28, no. 5, pp. 1901–1908 (2025), doi: 10.47974/JIM-2160.

[19] K. Sivakumar, S. Kaliappan, and R. D. Ramasamy, "A lightweight fog assisted cloud model (FACM) for secure environment," *Journal of Information and Optimization Sciences,* vol. 47, no. 2, pp. 777–792 (2026), doi: 10.47974/JIOS-2116.

Received November, 2025