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Chance constrained programming problem with shifted exponential random variables

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Abstract

A solution technique is developed for Chance Constrained Programming (CCP) problems where the coefficients appearing in the objective function and constraints are assumed to follow a two-parameter exponential distribution.

$$f(x) = \frac{1}{\lambda} e^{-\frac{1}{\lambda}(x-\mu)}, x\mu$$

is presented. Here the parameters μ and λ are called location and scale parameters of the exponential distribution. We call the location parameter μ as the minimum guarantee time and, λ is the mean time after the guarantee time is observed. This distribution is quite familiar in real life situations where the random variables X has a shift i.e., $X\mu$. For example in inventory systems the demand X is a random variable X satisfying a minimum quantity as the demand may not start with zero. So we aim to present a complete deterministic equivalent of the CCP where the coefficients follow a two parameter exponential distribution. Also, the CCP problem is derived in a mixed environment i.e., the coefficients are random whose parameters are fuzzy numbers. In both cases, the problem is converted to its deterministic equivalent. Here the Fuzzy Probability theory due to Buckley [6] is used. Also, a real-life situation is presented. The method is justified by numerical examples.

Subject Classification (2020): *62F15, 90C15, 62N05.*

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1. Introduction

A significant amount of work is available in the literature in the field of CCP, and some of the major references in this context are discussed as follows. The CCP model was first developed by Charnes and Cooper [1]. Three models with various objective functions are developed. They are,

(i) A model that optimizes the objective function's expected value.
(ii) A model that maximizes the probability of meeting a desired target value of the objective function.
(iii) The E-model, which minimizes the generalized mean square of the objective function.

Miller and Wagner [2] have considered the restriction on the joint probability of a multivariate random event of CCP problems. They assumed two models as follows

(i) A model that incorporates random right-hand side constants in linear constraints, but is known to lose concavity.
(ii) A model where the coefficients of the linear programming problems follow a multivariate normal distribution.

Iwamura and Liu [4] reformulated the chance-constrained programming model in a fuzzy environment and designed a fuzzy simulation-based genetic algorithm to compute feasible solutions. Nemirovski and Shapiro [5] studied a CCP model with linear constraints and independent random variables. They developed a large deviation approximation, known as the Bernstein approximation, to simplify the chance constraints. Biswal, Biswal, and Li [7] addressed probabilistic linear programming with a single-parameter exponential distribution and developed a transformation technique to produce its deterministic counterpart. Dash, Panda, and Nanda [8] considered a solution method for the CCP model where the right-hand side constraint's parameters are assumed to follow different fuzzy probability distributions. Nanda, Panda, and Dash [9, 10] formulated a methodology for FCCP, treating the coefficients of the objective and constraint expressions as fuzzy normal random quantities. Nanda, Panda, and Dash [11] also considered an FCCP structure in which fuzzy constraints appear in two different inequality forms. Katagiri, Kato, and Tanaka [12] introduced a multi-objective LPP where the coefficients are FRV. The authors developed a decision-making methodology that draws upon principles from both stochastic programming and possibilistic programming. According to Barik [13], the right-hand side coefficients in a linearly constrained probabilistic fuzzy goal programming model can be represented by a Pareto distribution whose mean and variance are known. Pantazis, Fele, and Margellos [14] developed a model for investigating the probabilistic feasibility of randomized solutions of two distinct classes of uncertain multi-objective optimization programs. Here the cost function depends on uncertainty. Mammarella, Grammatico, and Dabbene [15] invented a sample-based method for obtaining a basic and computational approximation of CCP. This method permits the approximating set's complexity to be controlled. Sakawa, Kato, and Katagiri [16] analyzed a multi-objective linear programming framework with random coefficients in both the objective and constraint functions. As a combination of stochastic and fuzzy approaches, an interactive method is derived which updated the reference membership levels and provided the decision maker with a satisfactory resolution. Sakawa and Matsui [17] examined a bi-level linear programming problem with fuzzy randomness and employed α-level sets to derive an α-stochastic two-level LPP. Panda and Dash [18] considered a model which solves fuzzy nonlinear CCP problems by using the definitions of the mean and variance for a fuzzy random variable. Ranarahu, Dash, and Acharya [19] considered a bilevel multi-objective problem containing multiple goals at each level. By applying α-cut techniques to remove fuzziness and chance constraints to handle randomness, they transformed the problem into a single-objective nonlinear model through fuzzy programming. A method for addressing probabilistic programming problems with quadratic, multi-objective functions was presented by Ranarahu, Dash, and Acharya [20]. Dash, Panda, and Panda [21] introduced the B-S (Black and Scholes) option pricing model (OPM) where every decision is taken under uncertainty due to randomness and fuzziness. Ranarahu and Dash [22] developed a Two-Stage Stochastic (TSS) programming in a fuzzy environment. Due to the existence of fuzzy random variables and multiple objective functions, they developed this model by using fuzzy probability theory due to Buckley [6]. Several recent studies published in Taru journals have also addressed optimization, fuzzy modeling, and multi-objective decision-making problems in related domains [23, 24, 25, 26].

Here a CCP model is introduced in which the parameters appearing in both the objective functions and the constraints follow a two-parameter exponential distribution. This optimization problem is solved by converting it to its deterministic equivalent. The second step involves modeling the right-hand side of the constraint as a fuzzy value within the CCP formulation. A solution methodology is presented using Buckley [6] fuzzy probability technique. Also, some real-life situations are provided. This problem is interesting because of the two parameter exponential distribution with unknown shifts. For example in most of the chance constrained linear programming problems the coefficients in the objective function and in the constraints are more than a threshold value. The novelty of this research is the formulation of a fuzzy chance-constrained model that incorporates fuzzy and unknown scale and location parameters. The contribution of our work is to handle uncertainty in both scale and location parameters, a new approach is proposed to find the deterministic equivalent of the fuzzy CCP.

The sections of this paper are organized in the following way: an introduction and a summary of the key references are provided in Section 1. In Section 2 some preliminary definitions are stated. The complete formulation of the problem is considered in Section 3. Finally, a study of the Fuzzy Chance Constrained Programming Problem with exponential distribution with two fuzzy parameters has been stated in Section 4. Section 5 presents the concluding remarks.

2. Basic Preliminaries

Definition 2.1: (Probability density function). Let Z denote a random variable possessing the density function G over the probability space(Ω, B, P). If G is absolutely continuous, meaning that there is a nonnegative function $g(x)$ such that for each real number y, we have, then Y is said to be continuous.

$$G(y) = \int_{-\infty}^{y} g(t)dt.$$

The function is known as the random variable Y's PDF.

Definition 2.2: (Fuzzy Number [6]). A fuzzy number $\tilde{b}$ is described as a fuzzy set on the real axis ℝ that fulfills two basic properties: it attains a maximum grade of membership equal to one, and its support forms a convex set.

Let $\mu_{\tilde{b}}: \mathbb{R} \to [0,1]$ be its membership function. Then:

1. There is an interval $I \subseteq \mathbb{R}$, which may shrink to a single point, such that
$$\mu_{\tilde{b}}(x) = 1 \text{ for every } x \in I.$$
2. The function $\mu_{\tilde{b}}(x)$ is continuous except for a finite number of jump points.

Definition 2.3: (α-cut of a fuzzy number). Given a fuzzy number $\tilde{a}$ with membership function $\mu_{\tilde{a}}(x)$, the α-cut for $\alpha \in [0,1]$ is defined as

$$\tilde{a}[\alpha] = \{ x \in \mathbb{R} | \mu_{\tilde{a}}(x) \geq \alpha \}.$$

For any pair $0 \leq \alpha_1 \leq \alpha_2 \leq 1$, the nesting property

$$\tilde{a}[\alpha_1] \supseteq \tilde{a}[\alpha_2]$$

always holds. Moreover, each cut generates a compact interval expressed as

$$\tilde{a}[\alpha] = [a_*(\alpha), a^*(\alpha)],$$

where the endpoints satisfy:

$$a_*(\alpha) \text{ increases with } \alpha, a^*(\alpha) \text{ decreases with } \alpha, \quad a_*(1) \leq a^*(1).$$

Definition 2.4: (Inequality due to Buckley [6]) A partial order can be defined between a fuzzy number and real numbers. Let $\tilde{a} = \langle a_1, a_2, a_3 \rangle$ be a triangular fuzzy number and let r and s be two real values. Then the relation

$$r \leq \tilde{a} \leq s$$

holds whenever

$$a_1 \geq r \text{ and } a_3 \leq s.$$

Definition 2.5: (Inequality due to Nanda and Kar [3]). Between two fuzzy numbers, a partial order relation can be defined. Nanda and Kar introduced a partial order $\preceq$ between two fuzzy numbers $\tilde{\imath}$ and $\tilde{\jmath}$, based on their α-cuts. Let $\tilde{\imath}[\alpha]$ and $\tilde{\jmath}[\alpha]$ denote the α-cuts of $\tilde{\imath}$ and $\tilde{\jmath}$, respectively. Then,

$$\tilde{\imath} \succeq \tilde{\jmath} \Leftrightarrow i(\alpha) \geq j(\alpha) \text{ for all } \alpha \in [0,1],$$
$$\text{where } i(\alpha) \in \tilde{\imath}[\alpha] \text{ and } j(\alpha) \in \tilde{\jmath}[\alpha].$$

Equivalently, if

$$\tilde{\imath}[\alpha] = [i_*(\alpha), i^*(\alpha)] \text{ and } \tilde{\jmath}[\alpha] = [j_*(\alpha), j^*(\alpha)],$$

then

$$\tilde{\imath} \succeq \tilde{\jmath} \text{ iff } i_*(\alpha) \geq j^*(\alpha),$$

and

$$\tilde{\imath} \preceq \tilde{\jmath} \text{ iff } i^*(\alpha) \leq j_*(\alpha),$$

for every $\alpha \in [0,1]$. This type of partial order has the advantage of requiring less computational effort.

Formulation

The general model of CCP [1] is given by

$$\begin{aligned} P(1) \quad & Optimize \ \textstyle\sum_{j=1}^{n} c_j x_j \\ & Subject\ \ to\ P(\textstyle\sum_{j=1}^{n} a_{ij} x_j \leq b_i) \geq 1 - \alpha_i, i = 1,2, \dots, k, \\ & \textstyle\sum_{j=1}^{n} b_{ij} x_j \geq h_i, i = k+1, k+2, \dots, m, \\ & x_j \geq 0, h_i \in R, i = 1,2,3, \dots, m, \\ & j = 1,2,3, \dots, n, 0 \leq \alpha_i \leq 1. \end{aligned} \tag{3.1}$$

where at least one of a_{ij}, c_j and b_i can be treated as random variables.

Many authors have considered the coefficients of the above model which follow different probability distributions and obtained their deterministic equivalent.

In this paper, we have tried to derive the deterministic equivalent of P(1) by considering the coefficients a_{ij} and c_j as random variables which follow a two parameter exponential distribution. The functional form of the density for a two-parameter exponential distribution is:

$$f(x) = \frac{1}{\lambda} e^{-\frac{1}{\lambda}(x-\mu)}, x > \mu, \lambda > 0.$$

In this distribution, μ serves as the location parameter, while λ functions as the scale parameter. The above distribution is helpful in reliability and life testing experiments. We call the location parameter μ as the minimum guarantee time and λ is the mean life time after the minimum guarantee time.

It may be noted the distribution used by Biswal, Biswal, and Li [7] is different from our model. Where they have assumed a_{ij} and c_j as independent random variables following one parameter exponential distribution with mean λ.

In order to derive the deterministic equivalent of P(1), let us consider the i^{th} probability constraint of P(1), which is given by

$$Prob\ \ (\textstyle\sum a_{i1} x_1 \leq b_i) \geq 1 - \alpha_i,$$
$$for\ \ i = 1.2, \dots, m,$$

where, $\alpha_i \in R$ and $0 \leq \alpha_i \leq 1$ are real numbers for $i = 1, \dots, m$ and a_{ij}, c_i are assumed to follow a two parameter exponential distribution. So the DF(density function) of a_{ij} provided by

$$f(a_{ij}) = , \begin{cases} \frac{1}{\lambda_{ij}} e^{-\frac{1}{\lambda_{ij}}(a_{ij} - \mu_{ij})} & , a_{ij} > \mu_{ij} \\ 0, & \text{otherwise}, \forall\, i = 1, \dots, m, j = 1, \dots, n \end{cases} \tag{3.2}$$

Further define,

$$\begin{aligned} Y_i &= a_{i1}x_1 + a_{i2}x_2 + \dots + a_{in}x_n \\ &= z_{i1} + z_{i2} + \dots + z_{in}, \\ &\text{where } z_{ij} = a_{ij}x_j, j = 1, 2, 3, \dots, n, (Say) \\ &\text{for any } i = 1,2,3, \dots, m. \end{aligned} \tag{3.3}$$

The distribution of z_{ij} is described by the following probability density function:

$$f(z_{ij}) = \begin{cases} \frac{1}{\lambda_{ij}x_j} e^{-\frac{1}{\lambda_{ij}x_j}(z_{ij} - \mu_{ij}x_j)} & , z_{ij} > \mu_{ij}x_j \\ 0, & \text{otherwise}, \forall\, i = 1, \dots, m, j = 1, \dots, n \end{cases} \tag{3.4}$$

First we will derive the deterministic equivalent of P(1) by considering n=2 decision variables, so, we have,

$$y_i = z_{i1} + z_{i2}, \ \ where \ \ z_{i1} \geq \mu_{i1}x_1 \ \ and \ \ z_{i2} \geq \mu_{i2}x_2.$$

Next we will derive the probability distribution of y_i, as follows,

For this let us consider the transformation

$$y_{i1} = z_{i1}, y_i = z_{i1} + z_{i2}.$$

The inverse of the above transformation is obtained as

$$z_{i1} = y_{i1} \text{ and } z_{i2} = y_i - y_{i1}.$$

So the jacobi J of the transformation can be found out as

$$J = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}.$$

$|J| = determinat(J) = 1.$

Now the density of y_i can be computed as

$$\begin{aligned} f_{y_i}(y_i) &= \int_{\mu_{i1}x}^{y_i - \mu_{i2}x_2} \frac{1}{\lambda_{i1}x_1\lambda_{i2}x_2} e^{-\frac{1}{\lambda_{i1}x_1}(y_{i1} - \mu_{i1}x_1)} e^{-\frac{1}{\lambda_{i2}x_2}(y_i - \mu_{i2}x_2)} |j|, dy_{i1} \\ &= \frac{1}{\lambda_{i2}x_2 - \lambda_{i1}x_1} \left[e^{\frac{\mu_{i1}x_1 + \mu_{i2}x_2 - y_i}{\lambda_{i2}x_2}} - e^{\frac{\mu_{i1}x_1 + \mu_{i2}x_2 - y_i}{\lambda_{i1}x_1}} \right], y_i \geq \mu_{i1}x_1 + \mu_{i2}x_2. \end{aligned} \tag{3.5}$$

At this point, the subsequent theorem has been concluded.

Theorem 3.1: For n=2, If a_{i1},a_{i2} are two exponential random variables having density given in (3.5) with known means, then for some coefficients x_1, x_2 the probability distribution of $Y_i = a_{i1}x_1 + a_{i2}x_2$ is represented by the pdf $f_{y_i}(y_i) = \frac{1}{\lambda_{i2}x_2 - \lambda_{i1}x_1} \left[e^{\frac{\mu_{i1}x_1 + \mu_{i2}x_2 - y_2}{\lambda_{i2}x_2}} - e^{\frac{\mu_{i1}x_1 + \mu_{i2}x_2 - y_2}{\lambda_{i1}x_1}} \right]$, where $y_i \mu_{i1}x_1 + \mu_{i2}x_2$

$\forall i = 1, 2, 3, \ldots, m$ and $j = 1, 2, 3, \ldots, n$.

Next we try to derive the deterministic equivalent of P(1) as follows.

For n=2 consider $(P(z_{i1} + z_{i2} \leq b_i))$. This can be evaluated as

$$\int_{\mu_{i1}x_1+\mu_{i2}x_2}^{b_i} f_{y_i}(y_i) = \frac{1}{\lambda_{i2}x_2-\lambda_{i1}x_1}\int_{\mu_{i1}x_1+\mu_{i2}x_2}^{b_i}\left[e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-y_2}{\lambda_{i2}x_2}} - e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-y_2}{\lambda_{i1}x_1}}\right]$$

$$= \frac{\lambda_{i2}x_2}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}\left[1 - e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i2}x_2}}\right] - \frac{\lambda_{i1}x_1}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}\left[1 - e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i1}x_1}}\right]$$

$$\Rightarrow g_{y_i}(y_i) = 1 + \frac{\lambda_{i1}x_1}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i1}x_1}} - \frac{\lambda_{i2}x_2}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i2}x_2}}.$$

Hence from P(1) the i^{th} constraint for n=2 no of decision variables is given by

$$(P(z_{i1} + z_{i2} \leq b_i)) \geq 1 - \alpha_i$$

$$\Rightarrow g_{y_i}(y_i) = 1 + \frac{\lambda_{i1}x_1}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i1}x_1}} - \frac{\lambda_{i2}x_2}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i2}x_2}} \geq 1 - \alpha_i. \tag{3.6}$$

Now, the next theorem presents the deterministic equivalent of $P(1)$.

Theorem 3.2: *If we consider a_{ij}, c_j are RVs follow a shifted exponential distribution with location μ, then it is possible to convert the CCP problem as mentioned in P(1) into a deterministic nonlinear programming problem, which is represented by*

$$\begin{aligned}
&Maximize\ E(z) &&= \textstyle\sum_{j=1}^{n} E[c_j]x_j \\
& &&= \textstyle\sum_{j=1}^{n} (\mu_j + \lambda_j)x_j, j = 1, \ldots, n, \\
&Subject\ to &&1 + \frac{\lambda_{i1}x_1}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i1}x_1}} \\
& &&- \frac{\lambda_{i2}x_2}{(\lambda_{i2}x_2-\lambda_{i1}x_1)}e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i2}x_2}} \geq 1 - \alpha_i i = 1, 2, 3, \ldots, k, \\
& &&\mu_{i1}x_1 + \mu_{i2}x_2 \leq b_i i = k + 1, k + 2, \ldots, m, \\
& &&\lambda_{i2}x_2 \geq \lambda_{i1}x_1 \\
& &&x_1, x_2 \geq 0 i = k + 1, k + 2, \ldots, m.
\end{aligned}$$

Following the procedure adopted earlier, the following Remark (3.1) can be proved by using theorem-1 and theorem-2. We also can prove the deterministic equivalent of P(1) for n = 3, 4, 5,... and some number of decision variables. However we have mentioned the deterministic equivalent of P(1) for n=3, as follows.

Remark 3.1: The i^{th} constraint of CCP P(1) for n=3 number of decision variables is given by

$$\Rightarrow g_{y_i}(y_i) = 1 - \frac{\lambda_{i2}x_2\lambda_{i3}x_3}{(\lambda_{i3}x_3-\lambda_{i2}x_2(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i3}x_3}} + \frac{(\lambda_{i2}x_2)^2}{(\lambda_{i3}x_3-\lambda_{i2}x_2)(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i2}x_2}} + \frac{\lambda_{i1}x1\lambda_{i3}x3}{(\lambda_{i3}x_3-\lambda_{i1}x1)(\lambda_{i2}x2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i3}x_3}} - \frac{(\lambda_{i1}x_1)^2}{(\lambda_{i3}x_3-\lambda_{i1}x_1)(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_1x_1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i1}x_1}} \geq 1-\alpha_i. \tag{3.7}$$

We omit the proof as it remains same as the above procedure. Using the same technique above one can derive the i^{th} probability of P(1) for $n3$.

Finally we are in a position to write the deterministic equivalent of P(1) for n=2 and 3 respectively.

3.1 *Deterministic model for case n=2*

$$\begin{aligned} &Maximize \quad Z = (\mu_1+\lambda_1)x_1 + (\mu_2+\lambda_2)x_2 \\ &Subject\ to \quad 1 + \frac{\lambda_{i1}x_1}{(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i1}x_1}} - \frac{\lambda_{i2}x_2}{(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2-b_i}{\lambda_{i2}x_2}} \geq 1-\alpha_i, \\ &\mu_{i1}x_1+\mu_{i2}x_2 \leq b_2, \\ &\lambda_{i2}x_2 \geq \lambda_{i1}x_1, \\ &x_1, x_2 \geq 0. \end{aligned}$$

3.2 *Deterministic model for case n=3*

$$\begin{aligned} &Maximize\ Z = (\mu_1+\lambda_1)x_1 + (\mu_2+\lambda_2)x_2 + (\mu_3+\lambda_3)x_3 \\ &Subjectto\ 1 - \frac{\lambda_{i2}x_2\lambda_{i3}x_3}{(\lambda_{i3}x_3-\lambda_{i2}x_2(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i3}x_3}} \\ &+ \frac{(\lambda_{i2}x_2)^2}{(\lambda_{i3}x_3-\lambda_{i2}x_2)(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x_1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i2}x_2}} \\ &+ \frac{\lambda_{i1}x1\lambda_{i3}x3}{(\lambda_{i3}x_3-\lambda_{i1}x1)(\lambda_{i2}x2-\lambda_{i1}x_1)} e^{\frac{\mu_{i1}x1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i3}x_3}} \\ &- \frac{(\lambda_{i1}x_1)^2}{(\lambda_{i3}x_3-\lambda_{i1}x_1)(\lambda_{i2}x_2-\lambda_{i1}x_1)} e^{\frac{\mu_1x_1+\mu_{i2}x_2+\mu_{i3}x_3-b_i}{\lambda_{i1}x_1}} \geq 1-\alpha_i, \\ &\mu_{i1}x_1+\mu_{i2}x_2+\mu_{i3}x_3 \leq b_3, \\ &\lambda_{i3}x_3 \geq \lambda_{i2}x_2, \\ &\lambda_{i2}x_2 \geq \lambda_{i1}x_1, \\ &x_1, x_2, x_3 \geq 0. \end{aligned}$$

Some numerical examples according to the above models are given below.

Example (For 2-Variables)

$$\begin{aligned}&Maximize \quad Z = (c_1)x_1 + (c_2)x_2\\ &Subject\ to \quad Prob(a_{11}x_1 + a_{12}x_2 \le 0.2) \ge 0.8\\ &\qquad\qquad x_1, x_2 \ge 0.\end{aligned}$$

Solution:

Here $E(a_{11}) = 0.25, E(a_{12}) = 10,\ \mu_1 = 1$ and $\mu_2 = 0.5$ and $E(c_1) = \mu_1 + \lambda_1 = 1 + 0.25 = 1.25$.
$E(c_2) = \mu_2 + \lambda_2 = 0.5 + 10 = 10.5$.
Then the above problem converts to its deterministic equivalent as below.

$$\begin{aligned}&Maximize \quad Z = (1.25)x_1 + (10.5)x_2\\ &Subject\ to \quad 1 + \frac{0.25x_1}{(10x_2-0.25x_1)}e^{\frac{1x_1+0.5x_2-0.2}{0.25x_1}} - \frac{10x_2}{10x_2-0.25x_1}e^{\frac{1x_1+0.5x_2-0.2}{10x_2}} \le 0.2,\\ &\qquad 10x_2 0.25x_1,\\ &\qquad 1x_1 + 0.5x2 \le 0.2,\\ &\qquad x_1 \ge 0,\\ &\qquad x_2 \ge 0.\end{aligned}$$

$Z = 0.8482311, x_1 = 0.7410489, x_2 = 0.4410304$

Example (For 3-Variables)

$$\begin{aligned}&Maximize \quad Z = c_1x_1 + c_2x_2 + c_3x_3\\ &Subject\ to \quad Prob(a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \le 50) \ge 0.9.\end{aligned}$$

Solution:

Here $E(a_{11}) = 0.25, E(a_{12}) = 0.33, E(a_{13}) = 0.5,\ \mu_1 = 20$ and $\mu_2 = 25,\ \mu_3 = 26$ and $E(c_1) = \mu_1 + \lambda_1 = 20 + 0.25 = 20.25$.
$E(c_2) = \mu_2 + \lambda_2 = 25 + 0.33 = 25.33$,
$E(c_3) = \mu_3 + \lambda_3 = 26 + 0.5 = 26.58$.
Then the above problem converts to its deterministic equivalent as below.

$$\begin{aligned}&Maximize \quad Z = 20.25x1 + 25.33x2 + 26.5x3\\ &Subject\ to \quad 1 - \frac{0.33x_2 0.5x_3}{(0.5x_3-0.33x_2(0.33x_2-0.25x_1)}e^{\frac{20x_1+25x_2+26x_3-50}{0.5x_3}}\\ &\qquad + \frac{(0.33x_2)^2}{(0.5x_3-0.33x_2)(0.33x_2-0.25x_1)}e^{\frac{20x_1+25x_2+26x_3-50}{0.33x_2}}\\ &\qquad + \frac{0.25x_1 0.5x_3}{(0.5x_3-0.25x_1)(0.33x2-0.25x_1)}e^{\frac{20x1+25x_2+26x_3-50}{0.5x_3}}\\ &\qquad - \frac{(0.25x_1)^2}{(0.5x_3-0.25x_1)(0.33x_2-0.25x_1)}e^{\frac{20x_1+25x_2+26x_3-50}{0.25x_1}} \le 0.1,\\ &\qquad 0.5x_3 > 0.33x_2,\end{aligned}$$

$$0.33x_2 > 0.25x_1,$$
$$20x_1 + 25x_2 + 26x_3 <= 50,$$
$$x_1 \geq 0,$$
$$x_2 \geq 0,$$
$$x_3 \geq 0.$$

Solve the above problem by using Lingo-18.

$\Rightarrow Z = 0.0001135749, x_1 = 0.00000000736054, x_2 = 0.00000207038, x_3 = 0.000002301251.$

We have given a real life situation below.

Example 1: A pharmaceutical company produces two types of antibiotics (M_1, M_2) from leaves of two types of medicinal plants (Plant-A, Plant-B). Since the quality of the leaves of medicinal plants is not exactly known, to produce antibiotics the number of leaves is random. To produce one unit of antibiotic M_1, it requires a_{11} units of plant-A and a_{21} units of plant-B. To produce one unit of antibiotic M_2, it requires a_{12} units of plant-A and a_{22} units of plant-B. We may see that $a_{11}, a_{12}, a_{21}, a_{22}$ are follow a two parameter exponential distribution, where $a_{11}\mu_{11}$, $a_{12}\mu_{12}$, $a_{21}\mu_{21}$, $a_{22}\mu_{22}$. It may be noted that $\mu_{11}, \mu_{12}, \mu_{21}, \mu_{22}$ are minimum requirement of units for producing the antibiotics. Where $\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}$ are an average requirement of units after the minimum requirements for producing antibiotics. The demand for antibiotics depends upon various conditions, so the profit is random. Here the profit also follows two parameter exponential distribution. From market information, the unit profits for M_1 and M_2 are given by the random variables c_1 and c_2, which follow two-parameter exponential distributions with $c_1 > \mu_1$ and $c_2 > \mu_2$. Where μ_1 and μ_2 are minimum profit per unit of M_1 and M_2 respectively. Monthly maximum b_1 units of plant-A and b_2 units of plant-B are available. Find several units of M_1 and M_2 to produce monthly, so that the company will get maximum profit. The following table produces basic data. The structure of the problem along with the required data is presented in Table 1.

Table 1

Basic data

Plants	M_1	M_2	Maximum availability
Plant A	a_{11}	a_{12}	b_1
Plant B	a_{21}	a_{22}	b_2

$$\begin{aligned} &Maximize && c_1x_1 + c_2x_2 \\ &Subject\ to && P(a_{11}x_1 + a_{12}x_2 \leq b_1) \geq 1 - \alpha_i, \\ & && P(a_{21}x_1 + a_{22}x_2 \leq b_2) \geq 1 - \alpha_i, \\ & && x_1, x_2 \geq 0. \end{aligned}$$

Let us consider a problem according to the above.

$$\begin{aligned}
\mathit{Maximize}\quad & Z = c_1x_1 + c_2x_2 \\
& \quad = (\mu_1 + \lambda_1)x_1 + (\mu_2 + \lambda_2)x_2 \\
\mathit{Subject\ to}\ & 1 + \frac{\lambda_{11}x_1}{(\lambda_{12}x_2 - \lambda_{11}x_1)} e^{\frac{\mu_{11}x_1 + \mu_{12}x_2 - b_1}{\lambda_{11}x_1}} - \frac{\lambda_{12}x_2}{(\lambda_{12}x_2 - \lambda_{11}x_1)} e^{\frac{\mu_{11}x_1 + \mu_{12}x_2 - b_1}{\lambda_{12}x_2}} \geq 1 - \alpha_1, \\
& 1 + \frac{\lambda_{21}x_1}{(\lambda_{22}x_2 - \lambda_{21}x_1)} e^{\frac{\mu_{21}x_1 + \mu_{22}x_2 - b_2}{\lambda_{21}x_1}} - \frac{\lambda_{22}x_2}{(\lambda_{22}x_2 - \lambda_{21}x_1)} e^{\frac{\mu_{21}x_1 + \mu_{22}x_2 - b_2}{\lambda_{22}x_2}} \geq 1 - \alpha_2, \\
& \mu_{11}x_1 + \mu_{12}x_2 \leq b_1, \\
& \mu_{21}x_1 + \mu_{22}x_2 \leq b_2, \\
& \lambda_{12}x_2\lambda_{11}x_1, \\
& \lambda_{22}x_2\lambda_{21}x_1, \\
& x_1, x_2 \geq 0.
\end{aligned}$$

Let here $\mu_1 = 3, \lambda_1 = 2$ and $\mu_2 = 2.5, \lambda_2 = 1.5$.

$\lambda_{11} = 0.2, \lambda_{12} = 0.4, \mu_{11} = 3, \mu_{12} = 4, b_1 = 3, \alpha_1 = 0.2.$

$\lambda_{21} = 0.33, \lambda_{22} = 0.5, \mu_{21} = 5, \mu_2 = 7, b_2 = 4, \alpha_2 = 0.3.$

Now the above problem is converted to the following form.

$$\begin{aligned}
\mathit{Maximize}\quad & Z = 5x_1 + 4x_2 \\
\mathit{Subject\ to}\quad & 1 + \frac{0.2x_1}{(0.4x_2 - 0.2x_1)} e^{\frac{3x_1 + 4x_2 - 3}{0.2x_1}} - \frac{0.4x_2}{(0.4x_2 - 0.2x_1)} e^{\frac{3x_1 + 4x_2 - 3}{0.4x_2}} \geq 0.8, \\
& 1 + \frac{0.33x_1}{(0.5x_2 - 0.33x_1)} e^{\frac{5x_1 + 7x_2 - 4}{0.33x_1}} - \frac{0.5x_2}{(0.5x_2 - 0.33x_1)} e^{\frac{5x_1 + 7x_2 - 4}{0.5x_2}} \geq 0.7, \\
& 3x_1 + 4x_2 \leq 3, \\
& 5x_1 + 7x_2 \leq 4, \\
& 0.4x_2 0.2x_1, \\
& 0.5x_2 0.33x_1, \\
& x_1, x_2 \geq 0.
\end{aligned}$$

Here $Z = 1.9707012, x_1 = 0.1104858, x_2 = 0.3545708.$

Also the above model is developed in mixed environment i.e. the right hand side of CCP problem i.e. b_i is treated as a fuzzy random variable presented in the next section.

4. Fuzzy Exponential Distribution with two parameters ($\tilde{b}_i$ follows a fuzzy exponential distribution)

In P(1) if any of a_{ij}, c_j and b_i follow a fuzzy exponential random variable then it is known as fuzzy chance constraint programming problem. Here it has been considered only the right hand side $\tilde{b}_i$ as fuzzy exponential random variable with location and scale parameters $\tilde{\mu}$ and $\tilde{\lambda}$ which are considered as triangular fuzzy numbers.

4.1 *Problem Formulation:*

When $\tilde{b}_i$ is exponentially distributed with fuzzy parameters $\tilde{\mu}$ and $\tilde{\lambda}$, the problem $P(1)$ takes the form of the following fuzzy chance-constrained programming model.

$$FP(1)\quad Minimize \sum_{j=1}^{n} c_{ij}x_j$$
$$Subject\ to\ \tilde{P}\sum_{j=1}^{n}(a_{ij}x_j \le \tilde{b}_i)\tilde{\beta}_i, a_{ij}, x_j \in R,$$
$$x_j \ge 0, \tag{4.8}$$
$$i = 1, \dots, m,$$
$$j = 1,2, \dots, n.$$

Using Buckley fuzzy probability theory [6] the deterministic equivalent of the i^{th} constraint can be obtained using following theorem

Theorem 4.1: *If $\tilde{b}_i$ is a fuzzy random variable following an exponential distribution with fuzzy parameters $(\tilde{\mu}_i, \tilde{\lambda}_i)$, then the fuzzy chance constraint $\tilde{P}\left(\sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i\right) > \tilde{\beta}_i$ is equivalent to $exp\left(\frac{\mu_i(\alpha)u_i}{\lambda_i(\alpha)}\right) \ge \beta_i(\alpha)$ for all $\alpha \in [0,1]$, where $\tilde{\lambda}_i(\alpha) = [\lambda_{i_å}(\alpha), \lambda_i^\star(\alpha)]$, $\tilde{\mu}_i(\alpha) = [\mu_{i_å}(\alpha), \mu_i^\star(\alpha)]$, and $\tilde{\beta}_i(\alpha)$ $= [\beta_{i_\star}(\alpha), \beta_i^\star(\alpha)]$.*

Proof 4.1: The α-cut of the fuzzy probability $\tilde{P}\left(\sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i\right)$ is given by

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i\right)[\alpha] = \{P(u_i \le b_i) | b_i \in \tilde{b}_i[\alpha]\}, \left(where \sum a_{ij}x_j = u_i\right)$$
$$= \left\{\int_{u_i}^{\infty} \frac{1}{\lambda_i} e^{\frac{-1}{\lambda_i}(b_i-\mu_i)} db_i | \lambda_i \in \tilde{\lambda}_i[\alpha]\ and\ \mu_i \in \tilde{\mu}_i[\alpha]\right\}$$
$$= \left\{e^{\left(\frac{\mu_i-u_i}{\lambda_i}\right)}, \lambda_{i_å}(\alpha) \le \lambda_i \le \lambda_i^\star(\alpha)\ and\ \mu_{i_\star}(\alpha) \le \mu_i \le \mu_i^\star(\alpha)\right\} \tag{4.9}$$
$$= \left[p_i^\star(\alpha), p_{i_å}(\alpha)\right](say).$$

where $p_{i\star}(\alpha)$ and $p_i^\star(\alpha)$ are the solutions of the following two optimization problems.

$$Minimize\quad e^{\frac{\mu_i-u_i}{\lambda_i}}$$
$$\lambda_i^\star(\alpha) \le \lambda_i \le \lambda_{i_\star}(\alpha) \tag{4.10}$$
$$\mu_{i_\star}(\alpha) \le \mu_i \le \mu_i^\star(\alpha)$$

and

$$Maximize\quad e^{\frac{\mu_i-u_i}{\lambda_i}}$$
$$\lambda_i^\star(\alpha) \le \lambda_i \le \lambda_{i_\star}(\alpha) \tag{4.11}$$
$$\mu_{i_\star}(\alpha) \le \mu_i \le \mu_i^\star(\alpha).$$

As $e^{\frac{\mu_i-u_i}{\lambda_i}}$ is a decreasing function of μ_i and $\lambda_i (u_i \ge \mu_i)$. Hence $e^{\frac{\mu_i-u_i}{\lambda_i}}$ attains its minimum at $\mu_{i_\star}$ *and* $\lambda i^\star$ and maximum at $\mu_i^\star$ *and* $\lambda i_\star$. So

$$\tilde{P}\left(\sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i\right)[\alpha] = \left[e^{\frac{\mu_{i_\star}(\alpha)-u_i}{\lambda_i^\star(\alpha)}}, e^{\frac{\mu_i^\star(\alpha)-u_i}{\lambda_{i_\star}(\alpha)}}\right].$$

Then the deterministic equivalent of the fuzzy chance constraint $\tilde{P}\left(\sum_{j=1}^{n} a_{ij}x_j \le \tilde{b}_i\right) \pm \tilde{\beta}_i$ becomes

$$e^{\frac{\mu_{i_\star}(\alpha)-u_i}{\lambda_i^\star(\alpha)}} \geq \beta_i^\star(\alpha).$$

The deterministic equivalent of FP(1) takes the following form

$$\begin{aligned} DFP(1) \quad & Minimize \ \textstyle\sum_{j=1}^{n} c_{ij}x_j \\ & Subject\ to\ e^{\frac{\mu_{i_\star}(\alpha)-u_i}{\lambda_i^\star(\alpha)}} \geq \tilde{\beta}_i(\alpha), \\ & \textstyle\sum_{j=1}^{n} b_{ij}x_j \geq 0, \\ & x_j \geq 0, \forall \alpha \in [0,1]. \end{aligned} \tag{4.12}$$

$\tilde{\mu}_i, \tilde{\lambda}_i$ are fuzzy numbers which can be chosen by decision maker. This is a linear/nonlinear mathematical programming problem depending upon the deterministic equivalent of the fuzzy chance constraint using the above theorem 4.1. The above model is justified using the following real life example.

Example 2: In the real-life situation given in example 1 the numerical coefficients connected with the objective function as well as the constraints can be taken as fuzzy exponential RVs(Random Variables). But here it has been considered just the constraint's right side of CCP as a fuzzy exponential RV i.e. $\tilde{b}_i$ is considered as a fuzzy exponential RV. Since the maximum availability depends upon the nature of the climate. Hence the maximum availability is not exactly known it may have both types of uncertainty i.e. random as well as fuzzy. From real-life example given in Section 3 the maximum availability $\tilde{b}_i$ is assumed to follow a two parameter exponential distribution. The fuzzy model parameters corresponding to the problem are summarized in Table 2.

Table 2

Basic data

Plants	M_1	M_2	Maximum availability
Plant A	a_{11}	a_{12}	$\tilde{b}_1$
Plant B	a_{21}	a_{22}	$\tilde{b}_2$

$$\begin{aligned} & Maximize \ \ c_1x_1 + c_2x_2 \\ & Subject\ to\ \ P\big(a_{11}x_1 + a_{12}x_2 \leq \tilde{b}_1\big) \geq \tilde{\beta}_i(\alpha), \\ & \qquad P\big(a_{21}x_1 + a_{22}x_2 \leq \tilde{b}_2\big) \geq \tilde{\beta}_i(\alpha), \\ & \qquad x_1, x_2 \geq 0. \end{aligned}$$

Here, $c_1, c_2, a_{11}, a_{12}, a_{21}, a_{22} \in \mathbb{R}$.
Lets consider a problem according to the above.

$$\begin{aligned} & Maximize \ \ 0.5x + 0.3y \\ & Subject\ to\ \tilde{P}((8x+2y) \leq \tilde{b}) \pm \tilde{0}.8, \\ & \qquad \tilde{P}((15x+3y) \leq \tilde{b}) \pm \tilde{0}.55, \\ & \qquad x \geq 0, \\ & \qquad y \geq 0. \end{aligned}$$

Here b_i is an exponential fuzzy random variable with the fuzzy parameters given as λ and μ are triangular fuzzy numbers. $\tilde{\lambda}_1 =< 3/5/7 >$and $\tilde{\mu}_1 =< 10/12/14 >$, $\tilde{\lambda}_2 =< 8/10/12 >$ and $\tilde{\mu}_2 =< 11/13/15 >$. Where $\tilde{0}.8$ is a triangular fuzzy number defined as $\tilde{0}.8 =< 0.7, 0.8, 0.9 >$ and $\tilde{0}.55$ is defined as $\tilde{0}.55 =< 0.4, 0.55, 0.7 >$ Then the above problem can be formulated as follows.

Consequently, by applying DFP(1), the deterministic equivalent of the original problem can be written as follows:

$$
\begin{aligned}
&Maximize\ \ 0.5x + 0.3y\\
&Subjectto\ \ e^{\frac{(10+2\alpha)-(8x+2y)}{(15-2\alpha)}} \geq (0.9 - 0.1\alpha),\\
&\qquad\qquad\quad e^{\frac{(11+2\alpha)-(15x+3y)}{(10-2\alpha)}} \geq (0.7 - 0.15\alpha),\\
&\qquad\qquad\quad x \geq 0,\\
&\qquad\qquad\quad y \geq 0,\\
&\qquad\qquad\quad \alpha = 0.1.
\end{aligned}
$$

By using Lingo-18 the result is as follows:

$$x = 73.11625, y = 414.0123 \text{ and } Z = 160.7618.$$

5. Conclusion

The aim of this work is to develop a solution approach for CCP models where the objective and constraint terms involve coefficients that follow a two-parameter exponential distribution. Here we have derived the method by taking two and three numbers of decision variables. However, similarly, it can be derived for any number of decision variables. In both cases, the problem transformed to its deterministic equivalent and justified by numerical examples. Also we have derived this model in fuzzy environment by taking the right hand side of the constraint of the CCP problem when both location and the scale parameters are unknown. So a proper estimator can be derived for unknown parameter by means of random samples taken from the said distribution.

References

[1] A. Charnes and W. W. Cooper, "Chance-constrained programming," *Management Science*, vol. 6, no. 1, pp. 73-79 (1959).

[2] B. L. Miller and H. M. Wagner, "Chance constrained programming with joint constraints," *Operations Research*, vol. 13, no. 6, pp. 930-945 (1965).

[3] S. Nanda and K. Kar, "Convex fuzzy mappings," *Fuzzy Sets and Systems*, vol. 48, no. 1, pp. 129-132 (1992).

[4] K. Iwamura and B. Liu, “Chance constrained integer programming models for capital budgeting in fuzzy environments,” *Journal of the Operational Research Society*, vol. 49, pp. 854-860 (1998).

[5] A. Nemirovski and A. Shapiro, “Convex approximations of chance constrained programs,” *SIAM Journal on Optimization*, vol. 17, no. 4, pp. 969-996 (2007).

[6] J. J. Buckley, *Fuzzy Probability and Statistics.* Heidelberg: Springer (2006).

[7] M. P. Biswal, N. P. Biswal, and D. Li, “Probabilistic linear programming problems with exponential random variables: A technical note,” *European Journal of Operational Research*, vol. 111, no. 3, pp. 589-597 (1998).

[8] J. K. Dash, G. Panda, and S. Nanda, “Chance constrained programming problem under different fuzzy distributions,” *International Journal of Optimization Theory Methods and Applications*, vol. 1, no. 1, pp. 58-71 (2009).

[9] S. Nanda, G. Panda, and J. K. Dash, “A new solution method for fuzzy chance constrained programming problem,” *Fuzzy Optimization and Decision Making*, vol. 5, no. 4, pp. 355-370 (2006).

[10] S. Nanda, G. Panda, and J. K. Dash, “A new methodology for crisp equivalent of fuzzy chance constrained programming problem,” *Fuzzy Optimization and Decision Making*, vol. 7, pp. 59-74 (2008).

[11] S. Nanda, G. Panda, and J. K. Dash, “Chance constrained programming with fuzzy inequality constraints,” *Opsearch*, vol. 45, no. 1, pp. 34-48 (2008).

[12] H. Katagiri, F. Kato, and K. Tanaka, “Interactive multiobjective fuzzy random linear programming: Maximization of possibility and probability,” *European Journal of Operational Research*, vol. 188, no. 2, pp. 530-539 (2008).

[13] S. K. Barik, “Probabilistic fuzzy goal programming problems involving Pareto distribution: Some additive approaches,” *Fuzzy Information and Engineering*, vol. 7, no. 2, pp. 227-244 (2015).

[14] G. Pantazis, F. Fele, and K. Margellos, “On the probabilistic feasibility of solutions in multi-agent optimization problems under uncertainty,” *European Journal of Control*, vol. 63, pp. 186-195 (2022).

[15] M. Mammarella, S. Grammatico, and F. Dabbene, “Chance-constrained sets approximation: A probabilistic scaling approach,” *Automatica*, vol. 137, p. 110108 (2022).

[16] M. Sakawa, K. Kato, and H. Katagiri, “An interactive fuzzy satisficing method for multiobjective linear programming problems with random variable coefficients through a probability maximization model,” *Fuzzy Sets and Systems*, vol. 146, no. 2, pp. 205-220 (2004).

[17] M. Sakawa and T. Matsui, “Interactive fuzzy programming for fuzzy random two-level linear programming problems through probability maximization with possibility,” *Expert Systems with Applications*, vol. 40, no. 7, pp. 2487-2492 (2013).

[18] G. Panda and J. K. Dash, “Nonlinear fuzzy chance constrained programming problem,” *Opsearch*, vol. 51, pp. 270-279 (2014).

[19] N. Ranarahu, J. K. Dash, and S. Acharya, “Multi-objective bilevel fuzzy probabilistic programming problem,” *Opsearch*, vol. 54, pp. 475-504 (2017).

[20] N. Ranarahu, J. K. Dash, and S. Acharya, “Multi-objective fuzzy quadratic probabilistic programming problem involving fuzzy Cauchy random variable,” *International Journal of Operational Research*, vol. 32, no. 4, pp. 495-513 (2018).

[21] J. K. Dash, S. Panda, and G. B. Panda, “A new method to solve fuzzy stochastic finance problem,” *Journal of Economic Studies*, vol. 49, no. 2, pp. 243-258 (2022).

[22] N. Ranarahu and J. K. Dash, “Computation of multi-objective two-stage fuzzy probabilistic programming problem,” *Soft Computing*, vol. 26, no. 1, pp. 271-282 (2022).

[23] R. Yadav and R. Singh, “A multi objective optimization in wireless sensor network using enhanced NSGA-3 algorithm,” *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 26, no. 7, pp. 1925-1937 (2023), doi: 10.47974/JDMSC-1727.

[24] R. Agarwal, R. P. Chaturvedi, A. Mishra, S. Asthana, and M. Parashar, “An approach for determining the best solution for intutionistic fuzzy transportation problem,” *Journal of Information and Optimization Sciences*, vol. 45, no. 7, pp. 1867-1879 (2024), doi: 10.47974/JIOS-1739.

[25] S. Gautam, A. Khunteta, and D. Ghosh, “Adaptive optimization framework for multimodal software defect prediction using

reinforcement learning," *Journal of Information and Optimization Sciences*, vol. 46, no. 7, pp. 2351-2362 (2025), doi: 10.47974/JIOS-2162.

[26] M. Saleki, M. S. Fallahnezhad, D. Shishebori, and M. A. D. Tafti, "Multi-objective fuzzy optimization under uncertainty for investment portfolio selection in emerging financial markets," *Journal of Information and Optimization Sciences*, vol. 47, no. 3, pp. 849-885 (2026), doi: 10.47974/JIOS-1574.

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