

Approach of VAEs in handling incomplete heterogeneous data

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Abstract

The latent structure found in vast, intricate, high-dimensional datasets is effectively and accurately captured by Variational autoencoders (VAEs). Existing VAE architectures struggle with issues commonly found in real-world datasets: heterogeneous data (which combines continuous and discrete variables) and incomplete data (where missing values appear randomly). This paper introduces a comprehensive framework for designing VAEs capable of accommodating incomplete heterogeneous data. Termed MI-VAE, this novel approach utilizes a key feature: likelihood models tailored for diverse data types, including real-valued, positive real-valued, interval, categorical, ordinal, and count data. Furthermore, the model is proficient at managing missing values, accurately performing both estimation and imputation functions. Moreover, when applied to supervised learning problems, MI-VAE demonstrates a competitive predictive capacity, often exceeding that of supervised models built from the same incomplete data. Furthermore, MI-VAE not only addresses missing values but also achieves competitive performance in supervised learning tasks, demonstrating its effectiveness in real-world applications.

Subject Classification: 68T07, 68T05, 68T09, 62H12.

Keywords: Incomplete data, Variational encoders, VAEs, Frameworks.

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1. Introduction

Databases, often large, heterogeneous, noisy, and incomplete, are the typical way data is organized and stored. Heterogeneous and incomplete data are prevalent in many domains. Online shopping platforms, for example, must manage diverse user information, such as age, gender, orders, and wish lists. A similar challenge exists with electronic health records (EHRs) in hospitals, which often compile various diagnoses, test results, and genomic data for each patient. Incomplete and heterogeneous datasets can benefit from the development of generative models capable of precisely capturing both the data distribution and its underlying latent structure. Such generative models are versatile, enabling researchers to deepen their understanding of the underlying data. Their applications include the estimation of corrupted or absent values, the identification of outliers, and even making future predictions (e.g., patient diagnoses) based on entirely new data.

Unsupervised deep generative approaches like VAEs and GANs have been investigated recently for their ability to uncover the latent structure inherent in high-dimensional, complex data. Following the training phase, these models efficiently replicate complex distributions derived from large datasets, allowing them to create novel data instances highly comparable to the original real-world data. Explicit generative models like VAEs have been a focus for improving their prior or posterior distributions. Separately, the literature has explored models such as GANs, which necessitate non-log-likelihood objectives as an alternative training method.

Indeed, we are currently witnessing a competition between a widening range of VAE models, some with interpretable output (Ainsworth VAEs). The focus on competition to generate the most realistic images or text has led to the neglect of a critical area: using these models to solve practical issues involving arbitrary datasets that often contain incomplete and heterogeneous data. We go into further depth about these issues and why we think our research is important for data scientists in the following sections. Our method is designed to manage both missing and heterogeneous data with minimal user action. Minimal input is required from the user, limited to defining the data type model for each attribute and indicating the precise location of missing data.

It is unexpected that few studies have focused on creating deep generative models able to learn efficiently from data that exhibit low structural consistency and high variability. These datasets lack a strong understanding of association between the various features (or dimensions) that could be tapped into by convolutional or recurrent neural networks that share weights. As we demonstrate in this research, avoiding the dominance of a few data dimensions during training is a critical component of successfully deploying deep generative models appropriate for heterogeneous data.

Functioning as a fully unsupervised model, the final VAE allows for the successful completion of tasks including supervised (such as classification or regression) and unsupervised even when provided with only partial input data. Examples of unsupervised tasks include density estimation and missing data

completion. The VAEs we describe are fully compatible with current efforts in both VAEs and implicit models (GANs), all of which are primarily concerned with preventing mode-dropping. We detail how to create these VAEs for direct use with real-world data. Despite the fact that deep supervised methods are fundamentally unable to handle missing inputs and require data imputation, our model achieves comparable classification accuracy. In addition to this strong result, our empirical findings confirm that the proposed method exceeds the performance of rivals (e.g., GAIN) for heterogeneous data completion.

2. Problem statement

We focus on the Missing Completely At Random (MCAR) scenario, where data entries are missing randomly and independently. Unlike common practices in deep generative models, we consider databases where observed objects are sets of mixed numerical and nominal scalar attributes, often with weak correlations between them (indicating a weak underlying structure). To prevent overfitting and maintain model flexibility for both heterogeneous data types and random missingness patterns, careful generative model design is crucial. This is especially important since these datasets may have lower dimensionality compared to, for example, images, and often lack strong correlations between attributes.

3. Contributions

In this paper, we provide a comprehensive framework for VAEs that successfully takes into account heterogeneous observations and partial data. The following features are included in our design:

- Deep neural networks (DNNs) are employed to parameterize a generative model capable of handling both numerical data. The model utilizes likelihood functions designed for two categories of data: discrete nominal data (categorical and ordinal), and various forms of real-valued data (continuous, positive real-valued, and count data).
- The VAE employs a data-normalization input/output layer to ensure equal contribution across all data dimensions, which results in faster convergence. This technique is crucial for developing a reliable identification model capable of handling MCAR data without the drawbacks of increased complexity or promoting overfitting.
- The Evidence Lower Bound (ELBO) is defined to rely exclusively on the observed data, remaining independent of the specific missing data pattern. This computed ELBO is then used to optimize the parameters of both the generative and inference models.

4. Related Work

Variational Autoencoders (VAEs) have emerged as a powerful tool for addressing missing data, showing promise in recent research across various applications.

- Regularization and Uncertainty: Building on existing work, this study [1] investigates how VAEs can be used to regularize the performance of missing value imputation by considering both reconstruction quality and the

model's performance on downstream tasks through the lens of an uncertain latent space.

- **Impact of Incompleteness:** Reference [2] explores how the dissimilarity between original and generated samples by VAEs increases as the incompleteness of the data grows.
- **Multiple Imputation with VAEs:** Reference [3] investigates the use of VAEs for multiple imputation, a technique for handling missing data by creating multiple possible completions of the data. This study identifies limitations in VAE-based imputation, finding that they might under- or overestimate missing values, resulting in incomplete coverage of the true missing data distribution.
- **Addressing Uncertainty Calibration:** To address these limitations, the study proposes leveraging $\hat{P}$ -VAEs, which provide robustness to model misspecification from a generalized Bayes perspective. It demonstrates how cross-validation can determine the optimal value for $\hat{P}$, which is crucial for proper uncertainty calibration. This approach is advantageous for modern statistical models, combining the expressiveness of Markov chain Monte Carlo (MCMC) with the computational efficiency of variational inference (VI) [4]. The authors further demonstrate how this technique achieves linear growth instead of exponential growth by employing strategies similar to those used in established imputation methods.
- **Integration with Omics Data:** Reference [5] highlights the limitations of using the vanilla VAE model for missing value imputation in metabolomics data. It proposes a novel VAE-based learning model, MI-VAE (Multiple Imputation based on Variational Auto-Encoder), specifically designed for multiple imputation and incorporating genetic information for improved accuracy.
- **Understanding VAEs:** Reference [6,7] highlights the limitations of using the standard VAE model for missing value imputation in metabolomics data (data related to small molecules within a cell). To address these limitations, the study proposes MI-VAE (Multiple Imputation based on Variational Auto-Encoder), a novel VAE-based learning model specifically designed for multiple imputation. MI-VAE incorporates genetic information, leading to improved accuracy in handling missing values within metabolomics datasets.
- **Benchmarking Imputation Methods:** Reference [8] proposes creating datasets with varying missingness rates and subsets to compare the performance of generative adversarial networks, VAEs, and autoregressive models (Transformers) for missing data imputation. The evaluation focuses on measuring the similarity between the distributions of the generated and original data.
- **References [9, 10]** showcase the application of VAEs for missing data imputation in two practical scenarios: a synthetic dataset (potentially in the domain of *) and a simulated milling circuit dataset (relevant to the field of

*). The studies compare VAE imputation to both single imputation and multiple imputation by Principal Component Analysis (PCA), evaluating its effectiveness in these settings.

In the Missing Completely at Random (MCAR) scenario, the missingness of data points occurs entirely by chance, with no relationship to any observed or unobserved values in the dataset. Missing data points are essentially random omissions, and their absence doesn't depend on other data or the expected value they would hold. For instance, imagine a survey where some responses are accidentally lost due to a technical glitch. This missingness would likely be MCAR, as the lost responses wouldn't be related to the demographics of the respondents or their answers to other questions. However, MCAR is likely uncommon in clinical research, where missing data often has underlying reasons.

In Missing at Random (MAR) data, the probability of a variable being missing depends on other observable variables within the dataset. This means that while some missingness patterns might be evident (e.g., men might be less likely to report their weight compared to women), the specific missing value itself doesn't influence whether it's missing. For example, men might be less likely to report their weight compared to women. Here, gender explains the missingness pattern for weight. This means that while we know why some weights are missing (because they are from men), the actual weight values themselves likely wouldn't influence whether they were missing.

Missing Not at Random (MNAR) data occurs when the missing instances are related to specific attribute values. People might avoid filling out information about education level due to embarrassment, or high-income earners might be less likely to report their income in a survey. MNAR refers to any mechanism that's not MCAR or MAR. The significant challenge with MNAR data is that unbiased estimates can only be obtained by modeling the missing data process itself, which can be complex and requires additional assumptions.

VAEs for Missing Data Imputation:

Variational Autoencoders (VAEs) are neural network architectures that transform input data into a lower-dimensional latent representation, typically modeled as a Gaussian distribution, through a sequence of encoding operations. VAEs are commonly optimized using stochastic gradient descent, aiming to maximize the evidence lower bound (ELBO) that approximates the log-likelihood of the data. Recently, VAEs have been adapted to address challenges such as missing values and heterogeneous multimodal inputs, which frequently occur in clinical trial data. VAEs are generative, meaning samples can be drawn from the latent space and decoded back into the original data format. However, a dense VAE model with multiple hidden layers can easily overfit when dealing with limited data.

5. Methodology

5.1 Handling Missing Data

To address the overfitting issue in VAEs with limited data, we propose a modification to the decoder's structure. This modification, called factorization, essentially breaks down the decoder into simpler components, reducing its overall complexity. While the specific mathematical details are not provided here, this approach helps the model learn effectively even with limited data.

$$p(x_n, z_n) = p(z_n) \prod_d p(x_{nd})|z_n \quad (1)$$

Here, Z_n represents the representation vector of X_n . This factorization enables us to make considerations about the missing data of variational parameters, shaping our approach to generative and recognition models. Figure 1 illustrates our approach to the generative model. Each dimension is represented by vectors observed as: $X_n = [X_{n1}, \dots, X_{nD}]$, corresponding to numerical or nominal variables. A DNN is used to independently **model the likelihood parameters** associated with each dimension d . Finally, Figure 2 depicts our recognition model with missing data X_{mn} , observed alongside data X_{0n} .

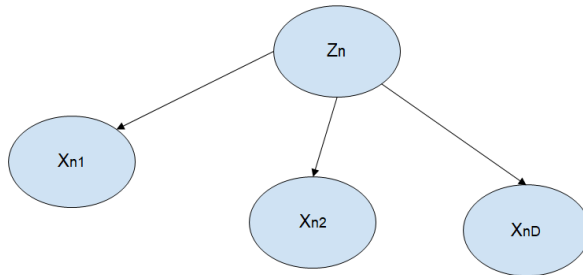


Figure 1
The Generative Model.

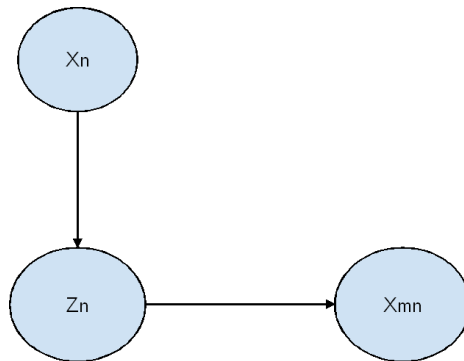


Figure 2
Shows the Recognition Model.

5.2 Recognition Models in Incomplete Data

This section describes how to design a DNN encoder for recognition models that can handle missing data. Here's a breakdown:

- Flexible Encoder: The encoder needs to adapt to various missing value patterns in the data.
- DNN with Input Distribution: The encoder takes the recognition distribution (containing missing values) as input.
- Replacing Missing Values with Zero: Missing values are replaced with zeros in the input distribution. This creates a specific covariance matrix that captures relationships between observed attributes.
- Impact on Learning: Replacing missing values with zeros ensures they don't directly affect the encoder's output or learning process. Additionally, non-linear activation functions in perceptrons prevent zero entries from linearly influencing the output.

5.3 Handling Heterogeneous Data

This section discusses how to deal with data containing different attribute types (nominal, numerical) in VAEs. Here's a summary:

- Likelihood Functions: Separate DNNs are used to model the likelihood functions for each attribute type (e.g., ordinal logistic for nominal data, Poisson for count data).
- Normalization: To ensure training stability and account for varying value ranges, Batch Normalization is applied to the numerical data entering the encoder. Correspondingly, denormalization is performed at the decoder's output.
- Count Variables: Count variables are transformed with a log function before feeding them to the recognition network and are not denormalized at the decoder.

5.4 Variational Autoencoder in Heterogeneous Incomplete Data

Building on the limitations of standard VAEs for complex, heterogeneous data with missing entries, this section proposes a hierarchical architecture as a potential solution. This hierarchical structure aims to effectively address the challenges associated with handling such data.

- Limitations of Simple VAE: The standard VAE with a fully factored generative model might be restrictive for complex data and may not capture high-dimensional relationships effectively.
- Hierarchical Structure: The proposed approach introduces an intermediate homogeneous representation (Y) jointly generated by a single DNN from the latent variable (z). This captures dependencies between attributes.
- Independent DNNs: Separate DNNs are used at the bottom level to model the likelihood of each attribute based on the intermediate representation (Y) and side information (sn). This allows for different likelihood models for different attribute types.

5.5 Recognition Network for Missing Data Imputation

This section delves into the architecture of the recognition network for imputing missing attributes in the VAE. The recognition network initially determines the **categorical distribution** $Q(\mathbf{s}_n|\mathbf{X}_{on})$. The corresponding parameter vector π is **outputted by a DNN** after processing the input $\mathbf{x}_{on}$ with a **softmax function**. Furthermore, to define the moments of the Gaussian posterior, $q(\mathbf{z}_n|\mathbf{X}_{on}, \mathbf{s}_n)$, the variables $\mathbf{s}_n$ and $\mathbf{x}_{on}$ are first concatenated, then fed through **two distinct, separate neural networks**. Finally, to ensure the hidden variables ($\mathbf{s}_n$ and $\mathbf{z}_n$) account for all feature relationships, the recognition network employs a **simplifying conditional independence assumption**: the posterior distribution of the missing attributes ($\mathbf{x}_{mn}$) is deemed independent of the observed attributes ($\mathbf{x}_{on}$), given $\mathbf{s}_n$ and $\mathbf{z}_n$.

As a result, our recognition model, or variational distribution, factorizes as follows:

$$q(s_n, z_n, x_m^n | x_n^o) = q(s_n | x_n^o) q(z_n | x_n, s_n) \prod_{d \in \mu_n} p(x_{n,d} | z_n, s_n) \quad (2)$$

6. Experiments

This section evaluates the performance of MI-VAE on two tasks:

- Missing Data Imputation in Heterogeneous Data: We compare MI-VAE's ability to impute missing values in heterogeneous data with various approaches from the literature.
- The classification accuracy of MI-VAE was compared to that of supervised models that employ mean imputation to handle missing data. Notably, MI-VAE achieves this performance without relying on any data imputation. This comparison focuses on the degradation in classification accuracy caused by using mean imputation.

6.1 Missing data imputation

To evaluate the effectiveness of our proposed approach for missing data imputation, we employed six diverse heterogeneous datasets from Lichman [10]. These datasets vary in the number of instances, attribute characteristics, and statistical data types. Table 1 summarizes the key attributes of these datasets. For each dataset, we created ten additional versions with missing values. In these versions, we randomly removed 10 to 50 percent of the original data to simulate missingness.

6.2 Comparisons

- Missing data points are imputed by using the mean value for continuous attributes or the mode (the most commonly occurring value) for discrete attributes technique known as Mean Imputation.
- Using supervised regression models, Multiple Imputation by Chained Equations (MICE) provides an iterative way to impute missing data. In this process, every missing value is estimated using all other variables within the dataset, including any estimates derived from prior rounds.

Table 1
Datasets in the experiments.

Dataset	Objects	Attributes	Real	Positive	Categorical	Ordinal	Count
Adult	32561	12	0	3	6	1	2
Breast	699	10	0	0	1	9	0
Default Credit	30000	24	6	7	4	6	1
Letter	20000	17	0	0	1	16	0
Spam	4601	58	0	57	1	16	0
Wine	6497	13	0	11	1	0	0

- The General Latent Feature Model (GLFM) for Heterogeneous Data was initially developed for completing tables that contain various data types. This model is capable of using both nominal and numerical data to perform imputation.
- GAIN (Generative Adversarial Network for Imputation): This method utilizes a generative adversarial network (GAN) for missing data imputation. It uses different loss functions based on data types (cross-entropy for binary and MSE for numerical).
- Our Variational Autoencoder (MI-VAE) Implementation: We implemented our VAE in TensorFlow using a single dense layer for both encoder and decoder parameters. The latent (z) and side information (s) dimensions were both assigned a value of 10, while the intermediate representation dimension (y) was set to 5. The **Gumbel-Softmax temperature parameter** (τ) was **linearly annealed** across all training epochs, starting at a value of **1** and reducing to 10^{-3} . All algorithms underwent **2000 epochs** of training, utilizing **minibatches containing 1000 samples**. The neural network architecture was kept consistent for every experiment conducted. We believe that subsequent work could yield greater improvements by employing cross-validation to optimize the architecture specifically for each dataset.

6.3 Impact of Network Design on MI-VAE Performance

This section investigates how sensitive the MI-VAE's performance is to the design of its neural networks (encoder and decoder). We assessed the model by testing the generator and inference networks with different architectures. Specifically, we used configurations containing one or two dense layers, all equipped with ReLU activation functions. We also vary the dimensionality of the latent space (z), side information (s), and intermediate representation (y).

The experiment uses a 20 percent missing data rate. We evaluate the MI-VAE's average mean squared error (for continuous data) imputation error across various network configurations and latent space dimensions.

The experiment revealed a trend where configurations with two dense layers and a larger latent space dimension (typically represented by brown bars in the

visualization) generally yielded lower imputation errors across most datasets. However, the Letter dataset exhibited a unique characteristic. It was the only dataset where significantly lower imputation error was achieved with a more complex network structure.

6.4 Imputation Results and Comparison

After training the generative model, we can employ the inferred parameters from the output distribution for imputation. This involves using the mode (most frequent value) of the inferred distribution or sampling directly from the generative model to replace missing values.

In this section, we present visualizations of the imputation results for a specific data point (top row). We showcase:

- **MI-VAE Output Distributions:** For comparison, these plots display the true underlying data distribution alongside the inferred distributions for both the observed data and the missing values.
- **Imputation Methods:** We showcase imputation using MI-VAE in two approaches: Replacing missing values with a single sample drawn from the inferred distribution. Replacing missing values with the mode (most likely value) from the inferred distribution. **GLFM Results:** Additionally, we present the imputation results obtained using the GLFM method for comparison.
- **The bottom row offers additional insights:** **Categorical Variable:** We present a confusion matrix to evaluate the classification performance of each method for the categorical variable (if applicable). **Positive Real Variable:** We present a Q-Q plot to compare the distribution of the imputed values for the positive real variable with the true underlying distribution.
- **Key Observations:** While MI-VAE and GLFM both offer an adequate fit for the positive real variable (with MI-VAE generating a distribution that is both smoother and more realistic), GLFM is ineffective in modeling the categorical variable, consistently attributing all probability mass to only one category. These results demonstrate the effectiveness of MI-VAE in imputing missing values in nominal variables, consistently providing good approximations.

6.5 Normalization

This section investigates the influence of batch (de-)normalization layers on the imputation performance of MI-VAE for numerical variables. We focus on datasets containing numerical variables and analyze the imputation error with and without these layers.

The experiment uses a 20 percent missing data rate, where data points are randomly removed. The initial two rows of a table (not shown here) likely present the average imputation error for each dataset:

- With (De-)Normalization: When batch normalization is used at the encoder's input and denormalization is used at the decoder's output for numerical features, the resulting imputation error is reported in this row.
- Without (De-)Normalization: This row shows the imputation error when these normalization layers are not used.

6.6 Comparison with Existing Methods

This section evaluates MI-VAE's performance against established imputation methods for handling missing data in heterogeneous datasets. For the sake of efficient comparison, we utilized a relatively simple MI-VAE configuration featuring only a single dense layer. While this configuration achieved good results, it highlights the potential for further performance gains through fine-tuning the network architecture for each specific dataset. This emphasizes the flexibility of MI-VAE and suggests that dataset-specific configurations could lead to even better imputation accuracy.

6.7 MI-VAE Outperforms Mean Imputation

Mean imputation assumes that all attributes in the data are independent. Since real-world data often exhibits statistical dependencies between attributes, imputation methods that consider these dependencies should theoretically outperform mean imputation. The results convincingly demonstrate that the proposed MI-VAE achieves consistently better performance than mean imputation across all datasets.

Improvements

- Added a title for better organization.
- Clarified the purpose of the experiment (comparing MI-VAE with other methods).
- Explained the simplicity of the MI-VAE configuration used.
- Highlighted the potential for further improvement through architecture optimization.
- Explained the theoretical advantage of methods that capture dependencies over mean imputation.
- Emphasized the consistent outperformance of MI-VAE compared to mean imputation.

6.8 Predictive Task

The experiment aims to assess how well MI-VAE performs classification when it imputes missing values in the target attribute (the class label itself). Here's how the missing data is simulated:

50 percent Label Removal: We remove 50 percent of the labels from the target attribute for training the MI-VAE model. MI-VAE then attempts to impute these missing labels during the training process.

6.9 *Impact of Missing Data in Input Attributes*

To understand how missing data influences classification performance, we will investigate MI-VAE's (our proposed Variational Autoencoder) behavior under various scenarios. We will simulate missing data by randomly removing varying percentages of values from the input attributes (features) in the training data. By training MI-VAE on datasets with different missing data rates (e.g., 10%, 30%, 50%), we aim to assess how effectively MI-VAE can handle missing data and its impact on the resulting classification performance.

- No Missing Data: All input features are complete (no missing values) for training.
- 10 percent Missing Data: 10 of the values in the input features are randomly removed for training.
- 50 percent Missing Data: 50 percent of the values in the input features are randomly removed for training.

6.10 *Handling Missing Data for Supervised Comparison*

Because the supervised classification baselinesâ€”Deep Logistic Regression (DLR) and Conditional VAE (CVAE) are unable to directly process missing values, we perform imputation on the input features during training, using the mean for continuous variables and the mode for categorical ones.

- MI-VAE Configuration: For most datasets, we use a basic MI-VAE configuration with one dense layer and latent dimensions (z), intermediate representation dimensions (y), and side information dimensions (s) set to 10, 5, and 10, respectively. The Letter dataset, which involves a multi-class classification task, necessitated a more complex configuration. The configuration employed for this dataset included a latent space of 50 dimensions and utilized two dense layers featuring ReLU activation functions.
- Comparison Methods: We compare the performance of MI-VAE with two supervised classification methods.
 - Deep Logistic Regression (DLR): This is a commonly used neural network architecture for binary classification problems.
 - Conditional VAE (CVAE): This is a variant of VAE specifically designed for semi-supervised learning tasks, where some labels might be missing.

7. Results and Discussions

The experiment evaluated the performance of MI-VAE for classification tasks with missing data. Here are the key observations:

- In datasets like DefaultCredit and Spam, MI-VAE exhibited less performance degradation as the percentage of missing data in the input features increased. This indicates that MI-VAE may be more robust to missing data compared to supervised methods that depend on imputed data.

- **Superior Performance:** MI-VAE achieved the best classification accuracy on both the Wine and Breast cancer datasets, regardless of the amount of missing data in the input features (see Table 2)

Table 2
The datasets results

Percentage of missing data	Model	Breast	Default Credit	Letter	Spam	Wine
0 percent	DRL	0.04	0.120	0.145	0.081	0.012
0 percent	CVAE	0.042	0.125	0.142	0.081	0.016
0 percent	MIVAE	0.025	0.200	0.114	0.085	0.014
10 percent	DRL	0.044	0.228	0.228	0.228	0.095
10 percent	CVAE	0.048	0.225	0.223	0.223	0.088
10 percent	MI-VAE	0.031	0.190	0.212	0.212	0.130
50 percent	DRL	0.08	0.198	0.465	0.134	0.078
50 percent	CVAE	0.101	0.197	0.466	0.135	0.078
00 percent	MI-VAE	0.035	0.205	0.589	0.135	0.042

8. Conclusions

In this study, we primarily tackle the construction and inference of deep generative models, specifically Variational Autoencoders (VAEs), in the presence of incomplete and heterogeneous data. We note that this problem is complex and has not received sufficient attention in the literature. Below, we succinctly outline the primary issues addressed in this study, for which MI-VAE offers a viable solution:

Firstly, missing data imputation presents a completely unsupervised task, where missing values are common in datasets. However, conventional VAEs, including conditional variants, assume complete data during training.

Unfortunately, when training data is insufficient, VAEs struggle to perform reliable direct inference, as Deep Neural Networks (DNNs) typically require complete input data. While typical inference relies on a recognition model where parameters are shared across inputs, the MI-VAE approach distinguishes itself. A lower bound is established for the data marginal likelihood which relies exclusively on the information contained in the observed data. Additionally, our work presents methods for managing missing values directly within the recognition network.

An intricate optimization challenge arises when attempting to infer a joint parameter set from data that features broad variations in both statistical type and value range. This is because effectively capturing the statistical interdependence among features leads to a search space with numerous local optima. While global optima may capture all current correlations in the data, each local optima may only capture correlations between a subset of attributes, treating the remaining attributes as independent.

In heterogeneous datasets, the unclear correlations make effective weight sharing difficult across Deep Neural Networks (DNNs). Consequently,

implementing more complex architectures, such as Recurrent (RNNs) or Convolutional Neural Networks (CNNs), often fails to improve the data fit. This is particularly evident in homogeneous data, such as text or images. The hierarchical nature of the MI-VAE generative model, employing a Gaussian mixture to span a latent space representing correlations between variables, addresses this challenge.

Our empirical findings demonstrate that MI-VAE performs comparably to deep supervised methods in terms of classification accuracy and outperforms competitors in heterogeneous data completion tasks. Input imputation becomes a prerequisite step when working with deep supervised methods, as these models cannot inherently handle missing data points in the input features.

Future research will focus on generalizing to temporal heterogeneous series with missing data and extending to additional sophisticated features like text or graphics.

Acknowledgments

This research work is supported by National Science and Technology Council, Taiwan, R.O.C. under the grants NSTC 114-2221-E-027-067 and NSTC 109-2221-E-027-095 -MY3.

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Received November, 2024