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A study of subject allocation to faculties in colleges as a quadratic assignment problem

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Abstract

The Quadratic Assignment Problem (QAP) is an optimization problem that deals with the assignment of a set of facilities to an equal number of locations while minimizing the overall cost, based on pairwise distance and flow data. Traditionally applied in fields such as facility layout and scheduling, QAP provides a structured approach to solving complex

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allocation problems. This paper presents an innovative study that models the subject allocation process in educational institutions as a QAP which would ensure an efficient and balanced assignment of subjects to faculty members and optimize the results. First, we establish the required flow and distance matrices by incorporating faculty experience and the prerequisite dependencies among some chosen subjects. Then, we apply heuristic methods to solve the formulated problem and derive an optimal assignment of subjects to faculty members. The proposed approach aims to provide a systematic and optimized framework for subject allocation, offering valuable insights into resource management in educational institutions.

Subject Classification: *90B80.*

Keywords: *Assignment, Subject allocation, Greedy algorithm, Optimization.*

Introduction

Assignment problems are one of the most liked methods among the Operations research models among students, Teaching faculty as well as researchers, mainly because of their efficiency and simplicity. [11] Wikipedia gives a simple definition to assignment problems as, "The problem instance of an assignment problem has some agents and some predefined tasks. Each of the agent can be assigned to any of the task and in doing so, there will be some associated cost that depends on the specific assignment. The basic problem objective is to assign an agent to every task and at most one task to each agent, in such a way that the total cost of the entire assignment schedule is minimum."

So, we have a set of tasks and a set of agents and we need to assign a task to an agent in such a way that the total cost of the all the tasks is minimized. This is, by definition, a one dimensional problem.

When we increase the dimension to two, we move into a Quadratic Assignment Problem (QAP). [12] Wikipedia again gives a nice and simple definition" There are a set of n facilities and a set of n locations. For each pair of locations, a distance is specified and for each pair of facilities a weight or flow is specified (e.g., the amount of supplies transported between the two facilities). The objective of the problem is that we have to assign every facility to different locations, minimizing the sum total of the distances multiplied by the corresponding flows. [13], [14], [15], and [16] are the papers which have given us more insight in the recent works and various alternatives in modeling which have kind of helped us narrow down to a plausible model.

In the general form, a QAP has n facilities which need to be assign to n locations as in an assignment problem but here we input two types of matrices , a flow matrix giving flow from a facility I to a facility J and a distance matrix giving a distance from a location u to a location v. We seek a minimal cost by this kind of assignment. The facility-location assignments are actually represented in the form a permutation where pi is the assigned facility for the facility i. This is a NP-hard problem defined mathematically by,

$$\min_{\pi \epsilon \emptyset} f(\pi) = \sum \sum f_{ij}\, d_{\pi(i)\pi(j)}\,, \quad \emptyset \text{ is the set of all such permutations}$$

So, in its general form, the QAP is very difficult to solve and interpret.

[1] is the paper which has actually initiated the discussion and interest in the study of a QAP. They have given a beautiful insight into what is QAP and discuss all the basics to everyone, from a student to a researcher. [2] is a journal which has given insights into how problem description and complexities arising out of it can be addressed.

[3] and [4] are two papers which give us a broader view of the QAP from a researcher's point of view as they help us connect with the related problems. [8], [9] and [10] are the articles which have given us the necessary knowledge to start this paper as they discuss, each in their own way, means of solving QAP. [5] is a nice article by professors of Cornell university in which they discuss method to solve and obtain costs for a permutation and we have used their idea in obtaining the solutions in the paper.

[6] and [7] are the inspirational articles based on which we have the Heuristic methods.

Inspired by the above, we decided to approach the QAP from the teaching domain perspective. Because, as teaching members of colleges, we know that in any college, assigning a paper/subject to a Faculty has been a very difficult task as it involves a huge number of factors. From our past experience, we can also add that there is not a single semester which goes by in which the teaching members are satisfied and happy with this aspect.

We consider Professors as Locations and Subjects as Facilities. The distance matrix gives the average number of failed students when a faculty had been assigned a particular subject. The flow matrix gives the pairwise difficulty level between the subjects. If we make these assumptions, the problem of assigning subjects to faculties fits quite nicely into a QAP and the objective is to reduce the average number of failed students which is actually a universal objective in the teaching domain.

We were very relieved to have fit our problem into the QAP structure. Then comes the more difficult aspect of how to find solution as the general form of a QAP is a NP-Hard problem.

To this end, we started our study with a minimal number of subjects and professors so that we can have the ever reliable "Brute Force search Method". But we felt that this alone will be insufficient for a good study and so, we have added two Greedy search methods-

Closest Location Heuristic and Lowest Flow Heuristic into the picture. Now we have three methods to solve the proposed QAP of subject allocation.

Then came our input issue- how are we going to get the distance matrix and the Flow matrix?

First we selected four mathematics subjects taught in engineering colleges

1. Mathematics I,

2. Mathematics II,
3. Mathematics III and
4. Probability and Queuing theory

Then, we asked faculty members who had taught all these subjects to compare and give a comparative difficulty level pairwise in teaching them in a scale of 1-5. We also obtained the average number of failed students from previous years. We formed the average of these two aspects and that data is processed into Table 2 and Table 3.

Then we obtained assignments using the three chosen methods. And for each of those assignments/permutations, calculated cost using the method given by [5]. A simple comparison then completed the study.

Methodology:

1. Obtain two input matrices, a flow matrix for different subjects to be allotted and a distance matrix between the Lecturers under consideration. Using a Google form based survey conducted among Professors currently teaching these four courses.
2. Obtain all possible assignments of subjects to the Lecturers using the Brute force search method
3. Obtain assignment of subjects to the Lecturers using Two Greedy search methods.
4. Calculate the costs and compare the assignments.

Methods used to solve:

The solutions for the QAP are given in the form of permutations, as given below in Table 1

Table 1

3	1	5	6	.	.	.	N(Facilities)
1	2	3	4	.	.	.	N(Locations)

To find such permutations we use the Brute force method and two types of Heuristics- Closest Location Heuristic and Lowest Flow Heuristic.

1. Closest Location Heuristic:

   In the closest Location Heuristic, Facilities (Subjects) are arranged in increasing order. First Location is randomly selected and the rest of the Locations (Professors) are selected based on the minimum Distance.
2. Lowest Flow Heuristic:

   In the Lowest Flow Heuristic, Locations (Professors) are arranged in increasing order. First Facility is randomly selected and the rest of the Facilities(Subjects) are selected based on the minimum flow.
3. The method given by [5] to obtain the costs associated with each permutation.

Input matrices:

Flow matrix, Table 2, gives the Pairwise Difficulty level Between Subjects compared as a pair. This is obtained using the survey conducted among currently active professors who had handled these courses.

Table 2

Flow matrix , Pairwise Difficulty level Between Subjects

Flow/Subjects	S1	S2	S3	S4
S1	0	3	4	4
S2	3	0	3	4
S3	4	3	0	3
S4	4	4	3	0

The Distance matrix, Table 3, also obtained from the survey gives the difference in the average number of failed students in the last year for the four professors selected for the study.

Table 3

Distance Matrix difference in the average number of failed students

Distance/Profs	P1	P2	P3	P4
P1	0	5	9	10
P2	5	0	7	8
P3	9	7	0	1
P4	10	8	1	0

Results and Analysis:

Table 4

Brute Force method based Solution with costs

Permutation	Cost	Permutation	Cost	Permutation	Cost	Permutation	Cost
1234	294	2134	290	3214	276	4231	272
1243	292	2143	288	3241	211	4213	211
1324	272	2314	206	3124	206	4312	255
1342	229	2341	155	3142	172	4321	294
1423	229	2413	172	3412	288	4123	155
1432	284	2431	197	3421	255	4132	197

When we employed the Brute force technique to assign the 4 subjects to the 4 professors, we obtained the 4!=24 solutions and we have obtained the cost for each assignment using the method discussed by [5].

Then we employed two greedy Heuristic methods to obtain two solutions.

In the closest Location Heuristic, Facilities(Subjects) are arranged in increasing order. First Location is randomly selected and the rest of the Locations(Professors) are selected based on the minimum Distance.

Table 5

Closest Location Heuristic solution

S1	S2	S3	S4	Permutation	cost
P1	P2	P3	P4	1234	294

In the Lowest Flow Heuristic, Locations(Professors) are arranged in increasing order. First Facility is randomly selected and the rest of the Facilities(Subjects) are selected based on the minimum flow.

Table 6

Lowest Flow Heuristic solution

P1	P2	P3	P4		
S3	S4	S1	S2		
Rearranging				Permutation	Cost
S1	S2	S3	S4	3412	288
P3	P4	P1	P2		

Cost Calculation:

We calculate the costs for every solution obtained, for every one of the three methods, employing the method discussed by [5]. If F is the flow matrix, D is the distance matrix and X is the permutation matrix(solution), then the cost is Cost= $F*(XDX^{-1})$, the dot product of the two matrices. In this paper, F is given by Table 1, D is given by Table 3 and from Table 4, 5, and 6 we get the respective X. One sample cost calculation is displayed in Table 7.

Table 7

Cost calculation for one of the solutions (P=4132) using the Brute Force method

Using the Brute force method, one of the solutions is P=4132				
	0	0	0	1
X	1	0	0	0
	0	0	1	0
	0	1	0	0

XDX'			
8	0	1	10
5	10	9	0
7	1	0	9
0	8	7	5

	F*XDX'
	44
	42
	58
	53
Total Cost	197

The optimal assignment as can be seen from the Brute Force method is illustrated in Figure 1.

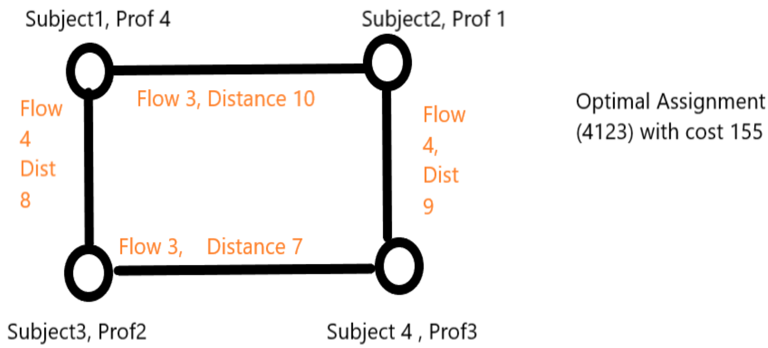

Figure 1
Optimal Assignment using Brute Force method

Discussion:

We have used QAP to study the subject allocation to professors in the engineering institutions. The flow matrix relates the subjects selected for study- it gives the average of the difference in difficulty level between the subjects, taken pairwise. The distance matrix is the difference in the average pass percentage for the courses taught by the selected professors in the last year. So, the cost calculated can be interpreted as the average number of failed students when we assign the subjects to the professors as per the solution obtained and obviously it is a minimization problem from an academic point of view.

Table 8 below gives the descriptive statistics of the solutions, by the “Brute force method”.

Table 8
Descriptive statistics of the Cost by Brute Force Method

Mean	237.5
Standard Error	9.78537803
Median	242
Mode	294
Standard Deviation	47.93836623
Sample Variance	2298.086957
Kurtosis	-1.32333437
Skewness	-0.32011974
Range	139
Minimum	155
Maximum	294
Sum	5700
Count	24

The comparison of the solutions is in Table 8 and the Figures 1 and 2,

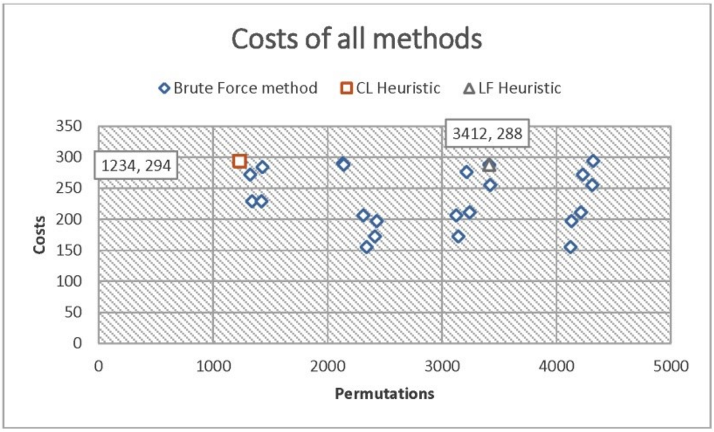

Figure 2
Costs of all permutations

Conclusion

In this study we have introduced a novel way of studying subject allocation to professors in engineering institutions using a basically NP-Hard Quadratic Assigning problem. The data for the study is obtained using Google-form-based survey from the professors who are currently handling these courses to justify the input into the model. We have obtained assignments using Brute Force method and two Heuristics for comparison. We have used a method of solution provided by Cornell University professors for calculating the cost to both the brute force method and the Heuristic methods. Both the Heuristic methods are easy to implement but they do not yield the best one for this data but we can use this as a starting point of analysis for such problems. We are glad to have initiated a path for academicians to approach the subject allocation to professors in a more theoretical way which we hope will definitely provide more trust and satisfaction among the teaching community when they are assigned subjects.

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