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A cost optimisation study on managing a single Weibull ameliorating and constant deteriorating inventory system with prepayment option

Deepika Gopi *
J. Viswanath†
Department of Mathematics
Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology
Avadi
Chennai 600062
Tamil Nadu
India

Abstract

Handling inventory post-maturity presents challenges for supply chain revenue. This study investigates advance payment options through finite installments to improve inventory costs. It assesses the effects of partial payments on inventory management and livestock sales via the EGQ model, while considering item deterioration over time. The methodology includes screening livestock to discard weaker items before maturation for sale. Retailers can opt for pre-payments in instalments which, when analyzed using the gradient method, suggest that fewer installment options may lead to cost reductions. The process begins with receiving young items, followed by a screening phase to improve sale quality. Deterioration risks are considered, along with the financial strategy of paying in advance before restocking. The total cost for the system is derived analytically and optimized through gradient techniques, focusing on maximum inventory levels post-management and purchase quantities. The findings indicate that minimizing pre-installment payments can further decrease costs, and managing feeding expenses effectively can lessen overall expenditures. The research provides managerial insights and demonstrates practical parameter applications.

Subject Classification: *90C30.*

Keywords: *EGQ model, Prepayment, Deterioration, Screening, Multi-variable optimization.*

* *E-mail:* **deepikaresearch1273@gmail.com (Corresponding Author)**
† *E-mail:* **mathsviswa@gmail.com**

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1. Introduction

Live broilers, ducks, and chicks are rapidly grown through an intensive amelioration process to enhance their market size and value, particularly during festive seasons. Investors focus on transforming day-old chicks into profitable products within weeks through proper nutrition and medical care. However, challenges such as illness, predators, and health hazards pose risks to poultry livestock, leading to mortality that can impact profitability, with potential for entire flocks to be lost during disease outbreaks. Achieving a balance in livestock inventory management involves optimizing system costs by assessing health, age, and growth rates while considering environmental factors to determine selling periods and prices. Early sales can lead to lower income, whereas delaying sales risks livestock deterioration. Market demand directly influences selling prices, which can fluctuate due to customer behavior, feeding costs, labor availability, regulatory policies, and climate changes. Effective contingency planning and tailored insurance can mitigate risks and losses. Additionally, the study identifies a gap in existing literature regarding the advanced pre-payment option for purchasing deteriorating items, particularly those with Weibull distribution characteristics. This involves a model relevant to various industries, promoting cost optimization through prepayment schemes in competitive markets.

The article provides an overview of its structure: Section 2 presents an extensive literature review, Section 3 outlines the notations and assumptions, Section 4 describes the mathematical model and optimization algorithm, Section 5 discusses numerical verification and sensitivity of cost functions and parameters, and Section 6 concludes with future scope.

2. Review of literature

Managing inventory of growing items, like poultry and livestock, needs special modeling because weight gain, death, and quality change over time. One of the first Economic Order Quantity (EOQ) models made for growing items was introduced by Rezaei [4]. His model took into account how the value and size of things like poultry change over time during the inventory cycle and gave answers for the best order quantity and the longest growing cycle. Zhang et al. [6] looked at how environmental policies affect feeding time, order quantity, and emissions in a carbon-constrained environment for growing inventory systems. They made a model that showed how these policies affect these things. They discovered that changing the number of orders was a better way to cut emissions than changing the times they were fed. Sebatjane and Adetunji [9] came up with EOQ models for items that grow but don't always have perfect quality. They looked at screening processes and growth functions like logistic and linear forms. Another model by the same authors that year included incremental quantity discounts. It showed that ordering a lot of items could lower the cost of buying them, but it could also raise the risk of holding and deterioration. Nobil, Sedigh, and Cárdenas-Barrón [8] developed a general EOQ model for poultry

that considered feeding, hygiene-related setup times, and fully backlogged shortages. They used non-linear programming to cut down on the total cost. FIFO/LIFO issuing policies and partial selling were also covered by the models. Giri, Jalan, and Chaudhuri [1] provided fundamental techniques for deteriorating inventory systems by analyzing EOQ models with ramp-type demand, Weibull deterioration, and full backordering. An integrated production-inventory model that takes partial backordering, time discounting, and Weibull-based amelioration and deterioration into account was presented by Law and Wee [2]. Also, may refer Hatibaruah and Saha [13]. Arunadevi and Umamaheswari [15].

Modern inventory systems are increasingly including payment options, particularly in settings with limited money or lengthy lead times. This line of inquiry was started by Taleizadeh et al. [3] who presented an EOQ model that allowed payment division before delivery and included numerous partial prepayments and backordering. Using a case study of a petrol station, Taleizadeh [5] further improved this with a model for evaporating items that integrated partial backorders and payment installments. Tavakoli and Taleizadeh [7] created EOQ models with conditional discounts to enhance financial management under full advance payment and different shortage scenarios, including no shortage, full backordering, and partially lost sales. With an emphasis on degrading goods sold in mixed quality, Taleizadeh, Tavassoli, and Bhattacharya [10] suggested a hybrid payment structure that combines inspection procedures, repeated prepayments, and trade credit. Inspections lowered decay rates in the inspection-dependent deterioration model proposed by Khakzad and Gholamian [11]. One may also refer Viswanath et al. [12], Khan et al. [14]

Limi and Rangarajan [16] incorporates price-dependent demand, preservation technology, and carbon emission control in a sustainable two-warehouse inventory system for non-instantaneous deteriorating items under an inflationary environment. One may refer Limi and Rangarajan [17], Lin et al. [18] for preservation technology, carbon emissions, price-sensitive demand, and a hybrid payment system.

Storage and sale of short-life perishable agricultural products and the profit optimisation are studied by Katariya and Shukla [19] with the consideration of a two-stage pricing policy. The article concludes with a numerical example and with valuable suggestions for the decision-making of production policies. Optimizing retailers' total profit of selling new and reselling old products with a buyback policy is initiated in Dubey and Kumar [20], with the consideration of price and time-dependent demand, and permitted partial backlogs, followed by a numerical example and sensitivity analysis. The optimal total cost of the inventory system in a Trapezoidal Neutrosophic environment is established by Chen [21] projects the concept of a decision support system in an uncertain

environment for minimising the total cost of a two-echelon production-inventory system, which includes the defective production rate by deriving a linear regression equation and solving with a numerical example.

3. Notations and Assumptions

3.1 *Notations*

The following notations are used to solve this model in the amelioration inventory parameters. All notations of the model are listed in Table 1.

Table 1

Notation and definition

Notation	Definition
ω_0	Initial weight of an item (per gram).
ω_1	Targeted weight of an item (per gram).
D	Demand rate per period (per gram).
α	Percentage of good items after screening.
$Q_r(t)$	The inventory order quantity at time t.
p_n	Purchasing cost of the livestock (per gram) per period.
s_n	Cost of screening of livestock per gram per period.
f_n	Feeding cost of the livestock per gram per period.
h_n	Holding the cost of the livestock per gram per period.
S_e	Total setup cost for the one-period inventory cycle.
T_3	Demand period.
C_s	Rate of screening in the initial stage.
θ	Deterioration rate per gram per period.
K	The duration of the buyer's prepayment period.
n	Number of uniformly spaced prepayment slots.
ϵ	Rate of discount for prepaying the purchase price.
$\Im_p$	Interest rate per year.
T	The entire time period of an inventory cycle.
$A(t)$	Weibull amelioration.
TC	Total variable cost of the inventory.
a	Shape parameter
b	Scale parameter
Decision variables	
T_2	Amelioration duration period.
η	Number of initial purchases.

3.2 *Assumptions*

All constraints of this model are as follows: The model is developed for one particular item whose weight/utility increases over time. Let $A(t)$ denote the Weibull amelioration rate, and is defined by $A(t) = abt^{b-1}$, where a and b are

the scale and shape parameters a and b respectively. Overlooks the amelioration of items at the time of screening, because the interval of screening is relatively short. There is a recognised constant in the demand from customers. Shortages are not allowed during the planning horizon. Deterioration is deterministic and occurs consistently over time. The holding cost is considered in both the growth and sales phases. The discounted purchase must be paid in n equal instalments before receiving the items. The pre-payments are in multiple equal sizes.

4. Mathematical Model

A mathematical framework of inventory modelling of ameliorating items is more significant, enhancing decision-making and optimizing financial outcomes. For the effective management of fluctuating demands, delay in re-orders and getting retailers to hold on to business, the suppliers' offer of pre-payment and instalment payments is an integral part of inventory management for ameliorating the inventory system. The current model framework is as follows: The business started with the purchase of η biological items with an initial weight of ω_o each. The total initial weight is $Q_0 = \eta\omega_o$ and the item is screened to remove defective items, before the start of the ameliorating process. There is an α percentage of good items remaining, and all other $(1-\alpha)$ the percentage of defective items are removed from the system. It happens at the last stage of the screening procedure at T_1. Therefore, the inventory level at the time T_1 leads to $Q_1 = \alpha\eta\omega_o$. The intensified Weibull amelioration process is assumed, and it is initiated at T_1. The time for amelioration ends at T_2 and demand starts henceforth at a rate D. It leads the inventory to go down and become empty at T_3. As the purchasing cost of the livestock is p_n per gram, the total purchasing cost is assured $p_n\eta\omega_o$ at the time of replenishment of items. Granted, pre-installment payment to attract customers is offered by the supplier, the retailer's advance installment payment is mitigated to ϵ percent, and the remaining purchase cost amount of $(1-\epsilon)p_n\eta\omega_o$ money units are paid in n-equal installments. The system behavior in terms of inventory dynamism and prepayment options is illustrated in Figure 1. The Weibull amelioration $A(t)$ is represented by the following distribution function, which involves two parameters: the scale and shape parameters, denoted as a and b

$$A(t) = abt^{(b-1)}, b > 0 \tag{1}$$

The rate of Demand D for items is considered a constant demand.

Let $Q_r(t)$ be the on-hand inventory at the time $t \geq 0$.

The screening process is initiated at $t = 0$ and prolongs up to time T_1 as the screening rate is C_s and the initial total purchasing quantity is

$Q_0 = \eta\omega_0$, the value of T_1 is derived as

$$T_1 = \frac{Q_0}{C_s}$$

The intensive ameliorating process is initiated at T_1 and proceeds till the time point T_2 of its maturity obtained. Deterioration (livestock may die) of items is inevitable during the period. Since the intensive feeding process is diluted once it starts sale, the dynamic size of ameliorating inventory is captured by the differential equation as follows

$$\frac{dQ_r(t)}{dt} = (A(t) - \theta)Q_r(t), T_1 \leq t \leq T_2 \tag{2}$$

Solving analytically, we get

$$Q_r(t) = Ce^{at^b - \theta t} \tag{3}$$

The condition $Q_r(T_1) = Q_1$ facilitate to get a solution for (2) as follows:

$$Q_r(t) = Q_1 e^{at^b - aT_1^b - \theta(t-T_1)} \tag{4}$$

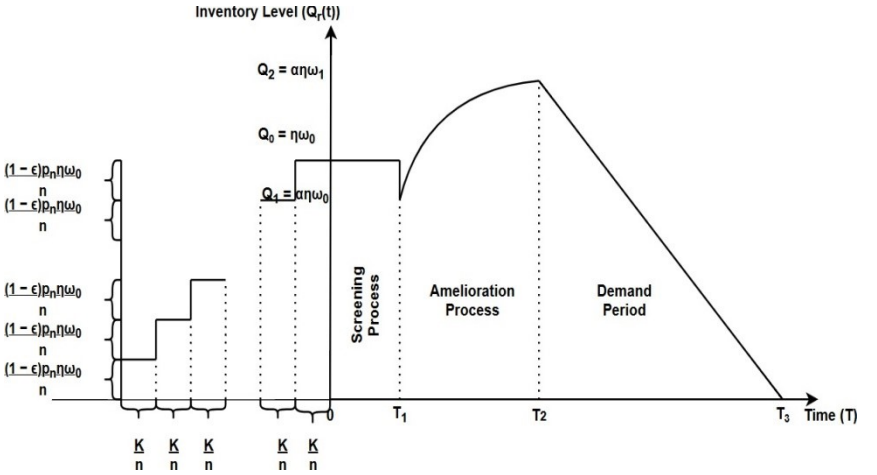


Figure 1
System behaviour

Items in the inventory gradually decreased from T_2 to T_3 owing to demand and deterioration.

Inventory variations in the period interval $[T_2, T_3]$ are as follows:

$$\frac{dQ_r(t)}{dt} = -D - \theta Q_r(t)\ ) \tag{5}$$

solving the above differential equation, applying the condition that $Q_r(T_3) = 0$ leads to the solution

$$Q_r(t) = \frac{D}{\theta}(e^{\theta(T_3 - t) - 1}) \tag{6}$$

The complete cycle time of an inventory is computed by:

$$T = T_1 + T_2 + T_3$$

4.1 *Various Cost Parameters*

The principal cost elements of the system are discussed in this section.

Purchasing Cost: We ordered a total η items each of weight ω_0. The cost of purchasing a livestock (per gram) is p_n. As the discount rate is ϵ, the purchasing total cost per cycle is

$$P_{nr} = (1-\epsilon)p_n\eta\omega_0 \tag{7}$$

Screening Cost: As it initiates a screening process immediately after the replenishment epoch, there are $(1-\alpha)$ fraction of the amount is rejected. Screening before accepting the replenished items mitigates our system total cost by the way of reducing holding cost and, feeding cost of the purchased livestock. As the cost for screening the purchased livestock (per gram) is s_n, the total cost spending for screening is

$$S_{nr} = s_n\eta\omega_0 \tag{8}$$

Feeding Cost: After completion of the screening process, all the screened good quality livestock are undergone to intensive amelioration process by healthy feeding. As the cost of feeding a livestock(per gram) is f_n, the total cost for feeding $\alpha\eta\omega_0$ quantity of items is derived.

The feeding cost of an inventory is

$$F_{nr} = f_n\left\{\int_{T_1}^{T_2} Q_r(t)dt\right\}$$

$$F_{nr} = f_n\left\{Q_1e^{-aT_1^b}((T_2-T_1) + \frac{aT_2^{b+1}-aT_1^{b+1}}{b+1} - \frac{\theta(T_2(T_2-2T_1)+T_1^2)}{2})\right\} \tag{9}$$

Holding Cost: The holding cost of the items is considered in three segments, namely in the region of screening, ameliorating and demand satisfying regions.therefore,

The holding cost of an inventory is

$$H_{nr} = h_n\left\{T_1Q_0 + \int_{T_1}^{T_2} Q_r(t)dt + \int_{T_2}^{T_3} Q_r(t)dt\right\}$$

$$\begin{aligned} H_{nr} = h_n\{&T_1Q_0 + Q_1e^{-aT_1^b}((T_2-T_1) + \frac{aT_2^{b+1}-aT_1^{b+1}}{b+1} - \frac{\theta(T_2(T_2-2T_1)+T_1^2)}{2}) \\ &+D(\frac{T_3^2}{2} - \frac{T_2}{2}(T_3-T_2) + \frac{\theta(T_3-T_2)^3}{6})\} \end{aligned} \tag{10}$$

Setup Cost: The initial inventory setup cost S_{er} is assigned as S_e.

$$S_{er} = S_e \tag{11}$$

Capital Cost: The Capital cost of the inventory is derived as follows:

$$C_{Cr} = \Im_p \frac{(1-\epsilon)p_n\eta\omega_0K(n-1)}{2n} \tag{12}$$

Deterioration Cost: The cost incurred for the deteriorated items is denoted by d_c per gram, and the total deterioration cost is

$$D_{dr} = d_c\left\{\int_{T_1}^{T_2} Q_r(t)dt + \int_{T_2}^{T_3} Q_r(t)dt\right\}$$

$$\begin{aligned} D_{dr} = d_c\{&Q_1e^{-aT_1^b}((T_2-T_1) + \frac{aT_2^{b+1}-aT_1^{b+1}}{b+1} - \frac{\theta(T_2(T_2-2T_1)+T_1^2)}{2}) \\ &+D(\frac{T_3^2}{2} - \frac{T_2}{2}(T_3-T_2) + \frac{\theta(T_3-T_2)^3}{6})\} \end{aligned} \tag{13}$$

Total Cost: The inventory total cost is integrated by Purchasing cost, Screening cost, Feeding cost, Holding cost, Setup cost, Deterioration cost and Capital cost and derived as follows

Totalcost = Purchasing cost + Screening cost + Feeding cost + Holdingcost + Setupcost + Capitalcost + Deteriorationcost (14)

$$TC = P_{nr} + S_{nr} + F_{nr} + H_{nr} + S_{er} + C_{Cr} + D_{dr}$$

$$TC = (1-\epsilon)p_n\eta\omega_0 + s_n\eta\omega_0$$

$$+f_n\{Q_1e^{-aT_1^b}\left[(T_2-T_1)+\frac{aT_2^{b+1}-aT_1^{b+1}}{b+1}-\frac{\theta\left(T_2(T_2-2T_1)+T_1^2\right)}{2}\right]\}$$

$$+h_n\{T_1Q_0+Q_1e^{-aT_1^b}\left[(T_2-T_1)+\frac{aT_2^{b+1}-aT_1^{b+1}}{b+1}-\frac{\theta\left(T_2(T_2-2T_1)+T_1^2\right)}{2}\right]$$

$$+D\left(\frac{T_3^2}{2}-\frac{T_2}{2}(T_3-T_2)+\frac{\theta(T_3-T_2)^3}{6}\right)\}$$

$$+d_c\{T_1Q_0+Q_1e^{-aT_1^b}\left[(T_2-T_1)+\frac{aT_2^{b+1}-aT_1^{b+1}}{b+1}-\frac{\theta\left(T_2(T_2-2T_1)+T_1^2\right)}{2}\right]$$

$$+D\left(\frac{T_3^2}{2}-\frac{T_2}{2}(T_3-T_2)+\frac{\theta(T_3-T_2)^3}{6}\right)\}$$

$$+S_e+\Im_p\cdot\frac{(1-\epsilon)p_n\eta\omega_0K(n-1)}{2n} \quad (15)$$

The total cost structure is projected in Figure 2.

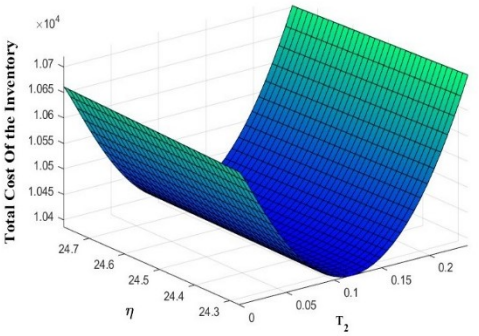


Figure 2
System Total Cost

4.3 *Methodology: Analytic Optimization Procedure*

An analytic approach of optimization of two variables, namely the gradient method, is employed. A simplified form of the system's total cost function is as follows: The methodology for optimizing the cost is presented in an algorithmic form

$$TC = (1-\epsilon)p_n\eta\omega_0 + s_n\eta\omega_0$$

$$+(f_n+h_n+d_c)\{\omega_0\eta\alpha e^{-a\left(\frac{\omega_0\eta}{C_s}\right)^b}\left[\left(T_2-\frac{\omega_0\eta}{C_s}\right)+\frac{aT_2^{b+1}-a\left(\frac{\omega_0\eta}{C_s}\right)^{b+1}}{b+1}\right.$$
$$\left.-\frac{\theta\left[T_2\left(T_2-2\left(\frac{\omega_0\eta}{C_s}\right)\right)+\left(\frac{\omega_0\eta}{C_s}\right)^2\right]}{2}\right]\}$$
$$+(h_n+d_c)\{\frac{(\omega_0\eta)^2}{C_s}+D\left(\frac{T_3^2}{2}-\frac{T_2}{2}(T_3-T_2)+\frac{\theta(T_3-T_2)^3}{6}\right)\}$$
$$+S_e+\Im_p\cdot\frac{(1-\epsilon)p_n\eta\omega_0K(n-1)}{2n} \tag{16}$$

The rate of change of the system's total cost $TC\ w.r.to\ \eta$ is as follows:

$$\frac{dTC}{d\eta}=(1-\epsilon)p_n\omega_0+s_n\omega_0+\Im_p\cdot\frac{(1-\epsilon)p_n\omega_0K(n-1)}{2n}+h_n\left(\frac{2\omega_0\eta}{C_s}\right)$$
$$+(f_n+h_n+d_c)\{\omega_0\alpha e^{-a\left(\frac{\omega_0\eta}{C_s}\right)^b}\left[\left(T_2-\frac{\omega_0\eta}{C_s}\right)+\frac{aT_2^{b+1}-a\left(\frac{\omega_0\eta}{C_s}\right)^{b+1}}{b+1}\right.$$
$$\left.-\frac{\theta}{2}\left(T_2\left(T_2-2\left(\frac{\omega_0\eta}{C_s}\right)\right)+\left(\frac{\omega_0\eta}{C_s}\right)^2\right)\right]$$
$$+\omega_0\eta\alpha[\left(T_2-\frac{\omega_0\eta}{C_s}\right)+\frac{aT_2^{b+1}-a\left(\frac{\omega_0\eta}{C_s}\right)^{b+1}}{b+1}$$
$$-\frac{\theta}{2}\left(T_2\left(T_2-2\left(\frac{\omega_0\eta}{C_s}\right)\right)+\left(\frac{\omega_0\eta}{C_s}\right)^2\right)]$$
$$+\omega_0\eta\alpha e^{-a\left(\frac{\omega_0\eta}{C_s}\right)^b}\left[-\frac{\omega_0}{C_s}-a\left(\frac{\omega_0\eta}{C_s}\right)^b+\theta\left(T_2\left(\frac{\omega_0}{C_s}\right)-\left(\frac{\omega_0\eta}{C_s}\right)\right)\right]\} \tag{17}$$

Rate of change of the system's total cost $TC\ w.r.to\ T_2$ is calculated:

$$\frac{\partial TC}{\partial T_2}=(f_n+h_n+d_c)\cdot\omega_0\eta\alpha e^{-a\left(\frac{\omega_0\eta}{C_s}\right)^b}\cdot\left[1+aT_2^b-\theta\left(T_2-\frac{\omega_0\eta}{C_s}\right)\right]$$
$$+(h_n+d_c)\cdot D\cdot\left(-\frac{1}{2}(T_3-T_2)+\frac{1}{2}T_2-\frac{\theta}{2}(T_3-T_2)^2\right) \tag{18}$$

The Total cost function's second derivatives are listed below:

$$\frac{d^2TC}{d\eta^2}=h_n\cdot\left(\frac{2\omega_0}{C_s}\right)$$
$$+(f_n+h_n+d_c)\cdot\{(\omega_0\alpha e^{-a\left(\frac{\omega_0\eta}{C_s}\right)^b}+\omega_0\alpha\eta)\cdot\left[-\frac{\omega_0}{C_s}-a\left(\frac{\omega_0\eta}{C_s}\right)^b+\theta\left(T_2\cdot\frac{\omega_0}{C_s}-\frac{\omega_0\eta}{C_s}\right)\right]$$
$$+\omega_0\alpha\cdot[\left(T_2-\frac{\omega_0\eta}{C_s}\right)+\frac{aT_2^{b+1}-a\left(\frac{\omega_0\eta}{C_s}\right)^{b+1}}{b+1}-\frac{\theta}{2}\left(T_2\left(T_2-2\cdot\frac{\omega_0\eta}{C_s}\right)+\left(\frac{\omega_0\eta}{C_s}\right)^2\right)]$$
$$+\omega_0\alpha e^{-a\left(\frac{\omega_0\eta}{C_s}\right)^b}\cdot\left[-ab\left(\frac{\omega_0\eta}{C_s}\right)^{b-1}-\theta\cdot T_2\cdot\left(\frac{\omega_0}{C_s}\right)\right]\} \tag{19}$$

$$\frac{\partial^2 TC}{\partial T_2^2} = (f_n + h_n + d_c) \cdot \omega_0 \eta \alpha e^{-a\left(\frac{\omega_0 \eta}{C_s}\right)^b} \cdot (abT_2^{b-1} - \theta) + (h_n + d_c)D \cdot (1 + \theta(T_3 - T_2)) \quad (20)$$

$$\frac{\partial^2 TC}{\partial \eta \, \partial T_2} = (f_n + h_n + d_c) \cdot \omega_0 \alpha e^{-a\left(\frac{\omega_0 \eta}{C_s}\right)^b} \left[1 + aT_2^b - \theta\left(T_2 - \frac{\omega_0 \eta}{C_s}\right)\right] \quad (21)$$

Total Cost Optimization subject to to η and T_2

Start

Step 1: Assign appropriate numerals to all parameters, such as credit period, rate and cost components.

Step 2: Calculate the TC of two variables, namely η and T_2. That is $TC(\eta, T_2)$.

Step 3: Compute the first-order partial derivatives of $TC(\eta, T_2)$ w. r. to η and T_2:

$$\frac{\partial TC}{\partial \eta}, \frac{\partial TC}{\partial T_2}$$

Step 4: Evaluate the first-order partial derivatives by Solve the following equations

$$\frac{\partial TC}{\partial \eta} = 0, \frac{\partial TC}{\partial T_2} = 0$$

simultaneously to extract all stationary points (η_c, T_{2c}) within the feasible domain.

Step 5: Find the second-order partial derivatives at each critical point:

$$\frac{\partial^2 TC}{\partial \eta^2}, \frac{\partial^2 TC}{\partial T_2^2}, \frac{\partial^2 TC}{\partial \eta \partial T_2}$$

Apply the second derivative (Hessian matrix) test to verify whether each point is a local minimum.

Step 6: If the second derivative conditions confirm convexity over the domain, declare the local minimum as the global minimum.

Step 7: Initialize $\eta^* = \eta_c$ and $T_2^* = T_{2c}$ for which the total cost is minimised.

Compute the optimal total cost: $TC^* = TC(\eta^*, T_2^*)$.

Step 8: Register η^*, T_2^*, and TC^* as the optimal order quantity, replenishment time, and optimal (minimum) cost of the system in the under-considered environment.

End

5. Numerical Verification

5.1 *Numerical Illustration*

A numerical example is provided to assess the model's reliability. The following parameter values are taken into account in a particular environment: The initial weight of the livestock is set as $\omega_0 = 50$ grams, and the target weight after the intensive feeding process is $\omega_1 = 1500$ grams. The ratio of good-quality items after screening is assumed to be $\alpha = 0.96$. Livestock growth is estimated using a Weibull distribution with shape and scale parameters a=0.8 and b=0.8, respectively. The purchasing cost of the livestock per gram per period is $p_n = 0.45$, while the screening cost is $s_n = 0.01$ per gram per period. In addition, the feeding cost is $f_n = 0.9$, the holding cost is $h_n = 0.05$. An annual interest rate of $f_p = 0.12$. The rate of discount $E = 0.08$ is applied to prepayments. The buyer makes prepayments over $n = 3$ equally spaced intervals within a period of $k = 2$. The rate of deterioration of the livestock is assumed to be $\theta = 0.001$ per gram per period. The screening rate is $C_s = 600000$, and per cycle inventory setup cost is $S_e = 3000$. Demand rate for the inventory is considered to be $D = 550000$ grams per period. We have arrived the optimum total cost $TC^* = 10385.90$ money units for the optimum ordering quantity $\eta^* = 24.6679$ items and ameliorating period $T_2^* = 0.1207$ days.

5.2 *Sensitivity Analysis*

Sensitivity of the parameters like demand rate, proportion of good items and number of prior instillments or advanced payments on total system cost is discussed. Moreover, it is discussed and listed out in Table 2 that how the purchasing cost, Feeding cost and screening cost are influencing the total system cost.

Effect of Increased Demand Rate: The rate of increase of demand rate influences the system total cost in increasing trend, as the purchasing optimum ordering quantity is in increasing trend to satisfy more customers arrival. In order to achieve this optimality, one must go with lesser duration of amelioration processing time. Increasing trend of optimum total cost on the increase of demand rate is depicted in Figure 7.

Effect of Increased proportion of Good Items: The rate of increase of proportion of good items influences the system total cost in decreasing trend, as the optimum initial ordering quantity is decreased and the optimum duration of amelioration too decreased. Decreasing trend of optimum total cost on the increase of proportion of good items is depicted in Figure 6.

Effect of Increase of Number of Prior Installments: The increase of number of installments influences the system optimum total cost in increasing trend, as the optimum initial ordering quantity is increased and the optimum duration of amelioration too increased. Increasing trend of optimum total cost on the increase of number of installments is depicted in Figure 8.

Effect of Increased Purchasing Cost: The increasing trend of purchasing cost results in increase of optimum system total cost as the optimum total initial ordering quantity is increased and the optimum ameliorating period too

increased. Such trend between purchasing cost and the system optimum total cost is depicted pictorially in Figure 3.

Effect of Increased Screening cost: The increasing trend of screening cost results in increase of optimum system total cost. Since this effect force the system to opt for more number of optimum initial ordering quantity and it influences the increased optimum amelioration process interval. Such trend between screening cost and the system optimum total cost is depicted pictorially in Figure 5.

Effect of Increased Feeding cost: The increasing trend of feeding cost results in decrease of optimum system total cost. Since this effect force the system to opt for lesser number of optimum initial ordering quantity and it influences the lesser optimum amelioration process interval. Such trend between feeding cost and the system optimum total cost is depicted pictorially in Figure 4.

Table 2

Sensitivity Analysis of Parameters on T_2, η, and Total Cost (TC)

Parameter	Values	T_2	η	TC
P_n	0.40	0.1017	21.5498	10270.98
	0.45	0.1113	23.1416	10326.51
	0.50	0.1207	24.6679	10385.90
	0.55	0.1299	26.1375	10449.01
	0.60	0.1390	27.5571	10515.71
F_n	0.80	0.1290	27.9069	10435.34
	0.85	0.1247	26.1973	10409.26
	0.90	0.1207	24.6679	10385.90
	0.95	0.1171	23.2924	10364.86
	1.00	0.1137	22.0491	10345.82
S_n	0.01	0.1207	24.6679	10385.90
	0.02	0.1226	24.9681	10398.31
	0.03	0.1245	25.2660	10410.87
	0.04	0.1263	25.5617	10423.58
	0.05	0.1282	25.8553	10436.44
α	0.94	0.1225	25.4444	10398.13
	0.95	0.1216	25.0512	10391.98
	0.96	0.1207	24.6679	10385.90
	0.97	0.1199	24.2943	10380.02
	0.98	0.1190	23.9300	10374.28
D	900000	0.1217	22.5454	10352.37

	950000	0.1212	23.6110	10369.20
	1000000	0.1207	24.6679	10385.90
	1050000	0.1203	25.7168	10402.48
	1100000	0.1198	26.7579	10418.95
n	1	0.1138	23.5432	10341.54
	2	0.1190	24.3898	10374.62
	3	0.1207	24.6679	10385.90
	4	0.1216	24.8063	10391.59
	5	0.1221	24.8890	10395.02

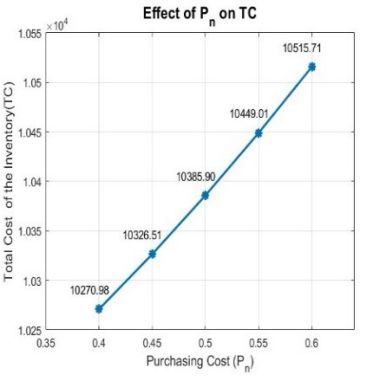


Figure 3
***P_n* vs TC**

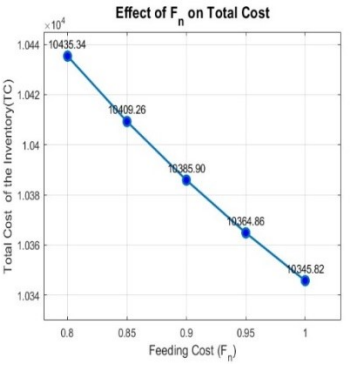


Figure 4
***F_n* vs TC**

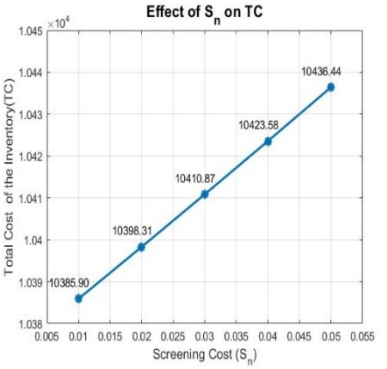


Figure 5
***S_n* vs TC**

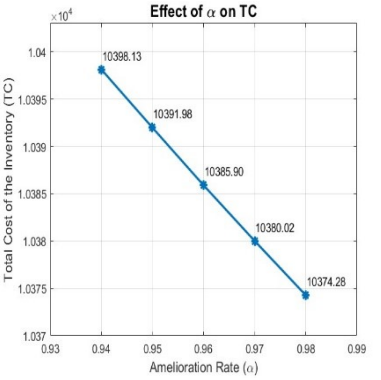


Figure 6
***α* vs TC**

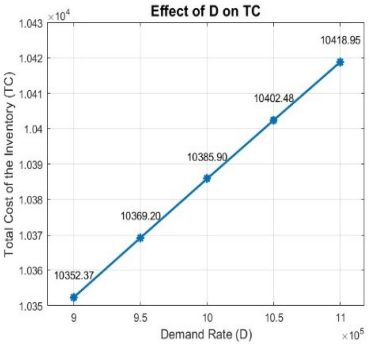


Figure 7
D vs TC

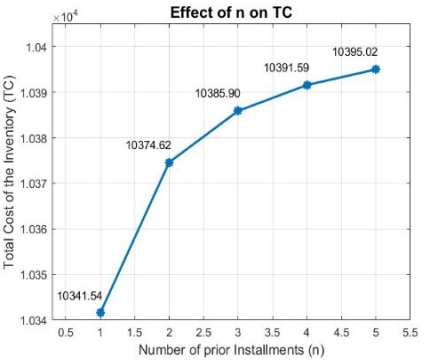


Figure 8
n vs TC

6. Conclusion and Scope for Future Work

This study projects the effect of the advance payment option in finite instalments in order to optimise the total cost of the ameliorating inventory system. Essential screening processes are initiated at the beginning of the inventory cycle, as it eases the unnecessary costs of caring for and growing unworthy items. Once all livestock reached the desired maturity age, sold out to incurs revenue. Item deterioration is unavoidable. Supplier grants the retailer to pay the purchase cost in finite instalments prior to the replenishment epoch. With this constrained environment, the total system cost is arrived. Numerical representation enhances the optimisation processes of the system's total cost by fixing appropriate parameter values. Analytic optimisation helps to investigation of the sensitivity of the important cost functions and essential parameters. Constraints of this work lead to many new avenues for the researchers to extend the current work by including the following: Consideration of pre, post and on-time payment options gives more managerial insights to investors in their strategic decision-making process in optimising total cost. An integrated single or multi-server queue facility and server facility with prioritised customer entry will make the current article more adaptable to real-time applications. Consideration of a Multi-warehouse facility with constrained.

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