Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 47 (2026), No. 5-B, pp. 2075–2082
DOI : 10.47974/JIOS-2298

Fuzzy set theory in multi-criteria engineering decision making

Zaware Sarika Nitin §
Department of Computer Engineering
AISSMS Institute of Information Technology
Pune 411001
Maharashtra
India

Pooja Ashok Shelar †
Department of Artificial Intelligence & Data Science
Vishwakarma Institute of Technology
Pune 411037
Maharashtra
India

Deepika Sharma *
Department of Computer Science
Noida International University
Greater Noida 203201
Uttar Pradesh
India

V. Dhanalakshmi ‡
Department of Computer Science
Meenakshi College of Arts and Science
Meenakshi Academy of Higher Education and Research
Chennai 600078
Tamil Nadu
India

§ *E-mail:* zaware@aissmsioit.org
† *E-mail:* pooja.shelar@vit.edu
* *E-mail:* deepika.sharma@niu.edu.in (Corresponding Author)
‡ *E-mail:* dhanalakshmiv@maher.ac.in

Sharyu Ikhar [@]
Researcher Connect Innovations and Impact Private Limited
Nagpur 440027
Maharashtra
India

Yogesh Nagargoje [#]
Researcher Connect Innovation and Impact Private Limited
Pune 411045
Maharashtra
India

Abstract

With the incorporation of fuzzy set theory into Multi-criteria decision-making (MCDM), the engineers have modified how they make decisions in consideration of uncertainty and imprecision. Where emotional considerations come into play together with numerical considerations in a given real life problem, a conventional decision making process does not apply well. At the time when Zadeh invented the fuzzy set theory, fuzzy set could handle vague and incomplete data and hence it is highly applicable in engineering. When we do not know what will happen, Fuzzy Analytic Hierarchy Process (FAHP) and the Fuzzy Technique of Order Preference by Similarity to Ideal Solution (Fuzzy TOPSIS) simplify the process of comparison and ranking of options. The paper examines such approaches and provides an explanation and examples mathematically and in the field of engineering.

Subject Classification: *03E72, 94D05.*

Keywords: *Fuzzy set theory, Multi-criteria decision-making, FAHP, Fuzzy TOPSIS, Uncertainty, Engineering decision-making.*

I. Introduction

The integration of the fuzzy set theory with MCDM has transformed the decision making process of engineers by providing them with a method of handling uncertainty and imprecision within a planned manner [1]. The real-life issues tend to be too complex to apply the traditional methods of decision-making since both qualitative and quantitative aspects have to be considered simultaneously [2, 3]. Gao et al. [4] had invented the fuzzy set theory, which is the method of describing uncertain, vague or missing data. This is why it is a perfect instrument in the process of making decisions engineering-wise, where the necessary data is not always that precise [4, 5]. MCDM approaches and in particular, those used to solve situations that are not clear, allow individuals to make decisions by comparing and ranking alternatives using a number of factors,

[@] *E-mail:* **sharayuikhar@gmail.com**
[#] *E-mail:* **symbiosisyogesh@gmail.com**

which may not always be consistent [6, 7]. These fuzzy MCDM techniques provide a more desirable, precise and comprehensive method of resolving technical problems. The Figure 1 presented represents multi criteria decision system.

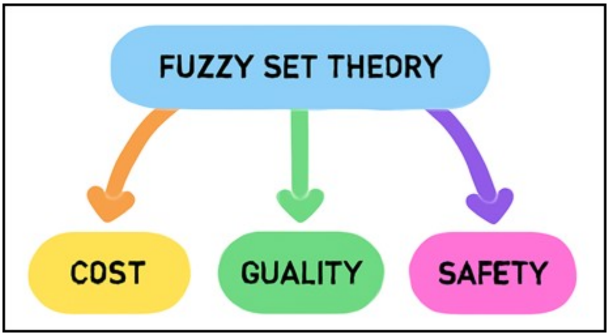

Figure 1
Fuzzy-Based Engineering Decision System

The paper will consider these techniques, provide mathematical equations, and demonstrate how they may be applied in various engineering disciplines [8, 9, 10]. It provides the decision-makers with valuable information on how to address complex, multi-criteria issues.

II. Fuzzy MCDM Techniques in Engineering Applications

A. *Fuzzy Analytic Hierarchy Process (FAHP)*

FAHP addresses the unclear, qualitative and quantifiable aspects of the hierarchical structures in such a way that any type of decision can be made in case of uncertainty [11, 12].

Assume A = (aij) is a fuzzy pairwise comparison matrix with a ij indicating the fuzzy comparison between criterion i and criterion j.

$$\boldsymbol{a_{ij} = [\, l_{ij}, m_{ij}, u_{ij}]}$$

Fuzzy Consistency Index (CI):

Consistency index CI is then computed to test the consistency of fuzzy pairwise comparison matrix A:

$$\boldsymbol{CI} = \left(\frac{\boldsymbol{1}}{\boldsymbol{n(n-1)}}\right) * \frac{\boldsymbol{\Sigma(i=1\ to\ n)\Sigma(j=1, j \neq i\ to\ n) a_{ij}}}{\boldsymbol{a_{ji}}} \tag{1}$$

Fuzzy Priority Vector:

$$\boldsymbol{w_i} = \frac{\boldsymbol{\Sigma(j=1\ to\ n) a_{ij}}}{\boldsymbol{n}} \tag{2}$$

Fuzzy Consistency Ratio (CR):

$$\boldsymbol{CR} = \frac{\boldsymbol{CI}}{\boldsymbol{RI}} \tag{3}$$

Fuzzy Aggregation of Criteria Weights:

$$W_{agg} = \Sigma(i = 1 \ to \ n)w_i * a_{ij} \tag{4}$$

and Wagg is aggregate fuzzy priority of criteria.

B. *Fuzzy TOPSIS*

Fuzzy TOPSIS is based on the original TOPSIS approach with the inclusion of a fuzzy logic that compares options relative to the best and the worst possible answers in addition to considering uncertainty.

Fuzzy Decision Matrix (FDM):

The entries of it are fuzzy numbers indicating the extent to which the i-th option fits the j-th standard.

$$x_{ij} = \left[l_{ij}, m_{ij}, u_{ij} \right] \tag{5}$$

The fuzzy based decision matrix is as below:

$$X_{tilde} = \left(x_{tilde_{ij}} \right) \text{ is calculated by:}$$

$$x_{tilde_{ij}} = \frac{x_{ij}}{\sqrt{\Sigma(i=1 \ to \ m)x_{ij}^2}} \tag{6}$$

$$A^+ = \left[min\left(x_{tilde_{ij}} \right) \right], A^- = \left[max\left(x_{tilde_{ij}} \right) \right] \tag{7}$$

III. Mathematical Formulation

A. *Representation of criteria and alternatives in fuzzy domain*

The criteria and options are presented in the form of a fuzzy set and a fuzzy number is assigned to each criterion to indicate the fuzziness [13, 14].

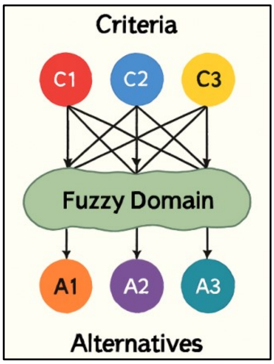

Figure 2
Fuzzy Domain Representation of Criteria and Alternatives

In Figure 2, shows the fuzzy criteria and alternative relationships in decision making situations. This allows you to have options in instances where you are not sure of the result.

Normalized Fuzzy Decision Matrix (NFDM):

Fuzzy decision matrix to be normalized as: $\boldsymbol{X_{tilde}} = \left(\boldsymbol{x_{tilde_{ij}}}\right)$ is calculated as:

$$\boldsymbol{x_{tilde_{ij}}} = \frac{x_{ij}}{\sqrt{\Sigma(i=1\ to\ m)x_{ij}^2}} \tag{8}$$

For the Fuzzy Decision Matrix as Weighted Normalized (WNFDM):

The ideal solution for Fuzzy and it represented by:

$$\boldsymbol{A^+} = \left[\boldsymbol{min}\left(\boldsymbol{x_{tilde_{ij}}}\right)\right] \tag{9}$$

Fuzzy Negative-Ideal Solution:

The fuzzy negative-ideal solution is represented by:

$$\boldsymbol{A^-} = \left[\boldsymbol{max}\left(\boldsymbol{x_{tilde_{ij}}}\right)\right] \tag{10}$$

B. *Membership function modeling*

Usually in fuzzy logic, a membership function (MF) transforms an item x in the context space X into a real number in [0,1]. This is how happy or like you belong to a fuzzy set you are.

Mathematical definition:

$$\boldsymbol{\mu A} : \boldsymbol{X} \rightarrow [\boldsymbol{0}, \boldsymbol{1}]$$

In the majority of cases, membership functions are linear according to such numerical traits as cost, speed, etc.

(a) Linear Increasing MF (higher values better):

$$\boldsymbol{\mu Ai(x)} = \begin{cases} \boldsymbol{0}\ \boldsymbol{x \le a} \\ \frac{\boldsymbol{xi - ai}}{\boldsymbol{bi - ai}}\ \boldsymbol{a < x < b} \\ \boldsymbol{1}\ \boldsymbol{x \ge b} \end{cases}$$

(b) Linear Decreasing MF (lower values better):

$$\boldsymbol{\mu Ai(x)} = \begin{cases} \boldsymbol{0}\ \boldsymbol{x \le a} \\ \frac{\boldsymbol{bi - xi}}{\boldsymbol{bi - ai}}\ \boldsymbol{a < x < b} \\ \boldsymbol{1}\ \boldsymbol{x \ge b} \end{cases}$$

Triangular or trapezoidal forms are used for language terms like "low," "medium," and "high."

(a) Triangular MF:

$$\boldsymbol{\mu Ai(x)} = \begin{cases} \boldsymbol{0}\ \boldsymbol{x \le a} \\ \frac{\boldsymbol{xi - ai}}{\boldsymbol{bi - ai}}\ \boldsymbol{a < x \le b} \\ \frac{\boldsymbol{xi - 1}}{\boldsymbol{bi - ai}}\ \mathbf{b < x < c} \\ \boldsymbol{0} \quad \boldsymbol{x \ge c} \end{cases}$$

(b) Trapezoidal MF:

$$\mu Ai(x) = \begin{cases} 0 & x \leq a \\ \frac{xi-1}{bi-ai} & a < x \leq b \\ 1 & b < x < c \\ \frac{di-xi}{di-ci} & c \leq x < d \\ 0 & x \geq c \end{cases}$$

IV. Result Discussion

Table 1 displays the results of FAHP. It indicates the performance of four alternatives (A, B, C, and D) on three different categories with varying weight.

Table 1

FAHP Results for Alternative Ranking

Alternative	Criterion 1 Score (Weight = 40%)	Criterion 2 Score (Weight = 30%)	Criterion 3 Score (Weight = 30%)	Overall Weighted Score
A	70 %	80 %	60 %	0.73
B	60 %	90 %	70 %	0.75
C	80 %	60 %	80 %	0.76
D	70 %	70 %	90 %	0.77

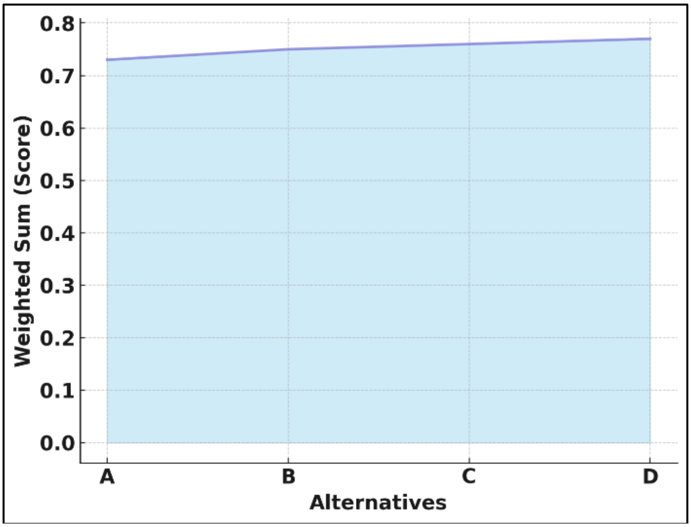

Figure 3
Comparison of Weighted Scores for Alternatives

The weighted value of each option demonstrates its suitability to the specified requirements, considering the degree of importance of each of them.

The highest score (77%) is received by alternative D, which indicates that it performs the best in all of the weighted categories. It is followed by C (76%), B (75%), and A (73%). Figure 3 also shows the weighted score of different alternatives in different criteria, which option is the best based on the considered factors.

This list reveals the effectiveness of FAHP method in comparison of options when decision-making factors receive non-uniformly distributed weights.

V. Conclusion

Fuzzy set theory is a powerful decision making approach in engineering in cases where one has to work with uncertainty and poor data. Such useful techniques as FAHP and Fuzzy TOPSIS can be used to check and rank various options considering numerous factors. Fuzzy logic can assist individuals in decision making as they provide them with more stable and precise answers. As engineering problems become increasingly complex, fuzzy MCDM will keep increasing the accuracy and efficiency of decision-making processes in a broad scope of areas.

References

[1] Y. Chen and D. Ling, “A framework for assessment of logistics enterprises’ safety standardization performance based on prospect theory,” *Journal of Operational Intelligence*, vol. 2, pp. 153–166 (2024).

[2] N. Jan, J. Gwak, M. Deveci, V. Simic, and J. Antucheviciene, “Mathematical analysis of big data analytics under bipolar complex fuzzy soft information,” *Applied Soft Computing*, vol. 157, p. 111481 (2024).

[3] D. Farooq, “Application of Pythagorean fuzzy analytic hierarchy process for assessing driver behavior criteria associated to road safety,” *Journal of Soft Computing and Decision Analysis*, vol. 2, pp. 144–158 (2024).

[4] D. Gao, W. Xie, R. Cao, J. Weng, and E. W. M. Lee, “The performance of cumulative prospect theory’s functional forms in decision-making behavior during building evacuation,” *International Journal of Disaster Risk Reduction*, vol. 99, p. 104132 (2023), doi: 10.1016/j.ijdrr.2023.104132.

[5] A. Hussain, Y. Liu, K. Ullah, M. Rashid, T. Senapati, and S. Moslem, “Decision algorithm for picture fuzzy sets and Aczel Alsina aggregation operators based on unknown degree of weights,” *Heliyon*, vol. 10, p. e27548 (2024).

[6] C. N. Wang, N. L. Nhieu, and C. M. Chen, "Charting sustainable logistics on the 21st-Century Maritime Silk Road: a DEA-based approach enhanced by risk considerations through prospect theory," *Humanities and Social Sciences Communications*, vol. 11, p. 398 (2024), doi: 10.1057/s41599-024-02928-3.

[7] A. R. Krishnan, M. M. Kasim, R. Hamid, and M. F. Ghazali, "A modified CRITIC method to estimate the objective weights of decision criteria," *Symmetry*, vol. 13, no. 973 (2021).

[8] T. C. Chu and T. H. P. Le, "Evaluating and selecting agricultural insurance packages through an AHP-based fuzzy TOPSIS method," *Soft Computing*, vol. 26, pp. 7339–7354 (2022).

[9] H. K. Hammoud and Z. Al-Khafaji, "Neighborhood search algorithm for optimal reliability allocation with cost-reliability data for components," *Journal of Information and Optimization Sciences*, vol. 46, no. 5, pp. 1527–1533 (2025), doi: 10.47974/JIOS-1788.

[10] N. Karanjekar, A. Thute, A. Ninawe, A. Kawalkar, and Y. Meshram, "A review design analysis and development of drive shaft for automobile application with optimization after design includes weight reduction," *International Journal of Technical Advances and Research in Mechanical Engineering* (IJTARME), vol. 14, no. 1, pp. 35–40 (May 2025).

[11] S. Das, G. Nayak, L. Saba, M. Kalra, J. S. Suri, and S. Saxena, "An artificial intelligence framework and its bias for brain tumor segmentation: A narrative review," *Computers in Biology and Medicine*, vol. 143, p. 105273 (2022).

[12] A. Godbole, R. Dhabliya, V. Deshpande, S. A. Sivakumar, B. M. Shankar, and V. Khetani, "Ethical hacking and penetration testing strengthening cybersecurity posture through offensive security measures," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 27, no. 4, pp. 1295–1305 (2024).

[13] V. Deshpande, D. Khubalkar, A. Dhablia, M. J. Pasha, D. Dhabliya, and Y. Gandhi, "Enhancing financial transaction security with lightweight cryptographic algorithms," *Journal of Discrete Mathematical Sciences and Cryptography*, vol. 27, no. 2-B, pp. 741–751 (2024).

[14] I. Mohan, M. Venkataraman, and K. Thilagavathi, "Investigating utilizations of harmonic univalent functions employing the Prabhakar fractional operator within specific categories," *J. Interdiscip. Math.*, vol. 27, no. 5, pp. 1099–1108 (2024).

Received April, 2025