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Solid transportation problem : A Lagrange's interpolation polynomial strategy for handling improbability and multi-objective decision-making

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Abstract

Mathematical optimization problems that simultaneously take into account several, typically conflicting, transportation planning objectives are known as multi-objective transportation problems (MOTPs). Unlike classic transportation problems, where only one objective is taken into consideration, such as lower costs, MOTP aims to find the most effective solutions to multiple objectives simultaneously. Decision-making processes in real situations are cumbersome within transportation networks, transmission vehicles, or delivery setups. Transportation in real life cannot be portrayed in distinctly defined terms because logistical challenges are not just limited to cost alone. Based on supply, demand, and conveyance capacity, we examined a multi-choice multi-objective solid transportation problem (MC-MOSTP) in which the objective function was analyzed. The suggested model used Lagrange's interpolation polynomial approach to deal with uncertainty, and a numerical example was used to validate the model's applicability.

Subject Classification: *90B06, 90C29, 97N50.*

Keywords: *Solid transportation problem, Lagrange's interpolation approach, Multi-choice random parameter, Multi-objective programming.*

1. Introduction

Among various applications, the transportation field is one of the best-known uses of linear programming [1]. It can be applied using multiple strategic tools such as derived transportation structures. These techniques are utilized across various areas such as supply management, inventory regulation, logistics operations, and manufacturing arrangement. In the standard transportation model, the main factors considered are cost, supply, and demand. On the other hand, due to intense market competition, these criteria may not always be precisely defined. Product pricing can fluctuate frequently or be influenced by variations in the manufacturing process. Furthermore, an unreliable and indistinct relationship between supply and demand may arise from incomplete information regarding product transportation.

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Decision-making is highlighted throughout various fields, including statistics, mathematics, psychology, philosophy, and economics. It is essential to acknowledge the critical function of transportation in distribution networks. To support manufacturers in fulfilling customer requirements, the transportation problem (TP) aims to minimize the expenses involved in moving products from production points to end users. The transportation problem (TP) involves supply, demand, and price considerations. These factors involve the transportation problem (TP). A number of vehicles facilitate the movement of items from their points of origin to their destinations, creating cost effectiveness and meeting deadlines. The fundamental theoretical underpinnings of the transportation problem (TP) were first presented by Hitchcock [2] and subsequently extensively examined by numerous scholars in the literature. A widely recognized Transportation Problem (TP), sometimes referred to as the three-dimensional TP, first came up by Haley [3]. The above content presents a more detailed and improved interpretation of the solid transportation problem (STP). The main aim of the STP is to achieve minimum total transportation cost optimally allocates products from their source to their destination utilizing cargo technology for transportation.

The STP (Figure 1) is a critical issue that the industry needs to address. There may be delays and congestion, or connectivity may also be limited by poor transport infrastructure. A city could have a major transport problem if its public transport infrastructure is old and unable to cope with the demand. As a result, travelers may face overcrowded buses, longer wait times, and frustration. Solving large transportation problems is exceptionally difficult due to the number of people, goods, and businesses that need to be transported from one location to another. It also involves detailed strategic planning, substantial infrastructure investments, and high levels of administration and execution to ensure the effective transportation of people and goods.

In order to better represent variations in market prices for aggregate commodities, incorporating multi-choice supply and demand elements is essential. Healy [4] was the first to introduce multi-choice programming to solve zero–one linear programming challenges. In this work, multi-choice programming models with fuzzy constraints are developed and solved to address situations that involve several selection options and uncertain (fuzzy) parameters. It uses an interpolation approach that combines fuzzy goal programming with the ϵ-constraint technique, enabling the transition from probabilistic constraints to logical and fuzzy programming frame works.

It applies an interpolation-based method that merges fuzzy goal programming with the ϵ-constraint approach, allowing probabilistic constraints to be transformed into logical and fuzzy programming formulations.

This results in a broader class of mixed-integer nonlinear programs, known as MCFS-MOTPs. The approach was initially introduced by Sayed and Baky [5], who proposed that MOTPs could be addressed using chance-constrained programming, incorporating fuzzy goal programming and the global criteria technique. This strategy supports finding solutions that reconcile multiple objectives according to the decision-maker's priorities.

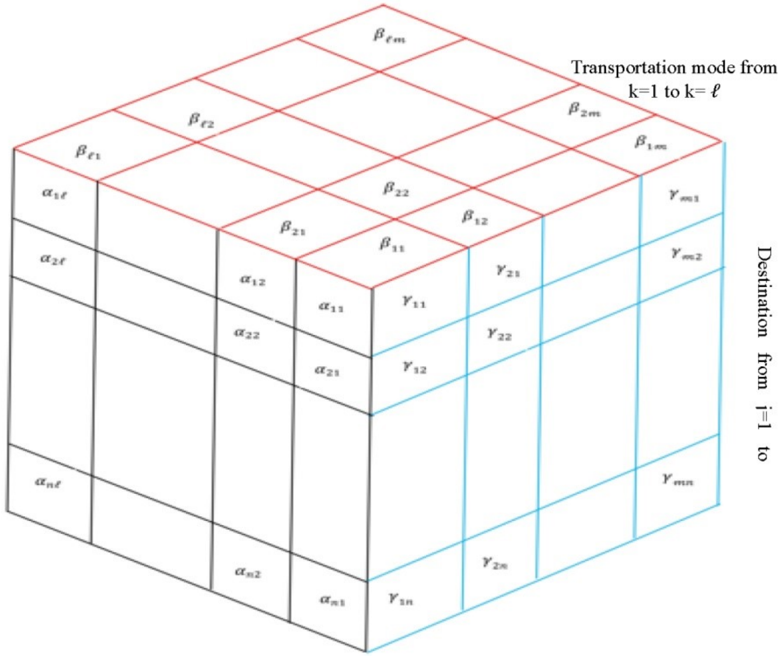

Figure 1
Tabular illustration of requirement(n), deliver (m), and transportation (ℓ)

They proposed a distinct solution to the MOTPs by utilizing a weighted form of goal programming, as reported by Joshi et al. [6]. This method focuses on finding the preferred compromise solutions. The approach is explained further through an accompanying numerical example.

A procedure for addressing complicated transportation scenarios, particularly when the objective involves the quotient of two linear functions, is described in [7]. The researchers focus on the cost ratios of transportation problems under uncertainty and multiple objectives to optimize such problems. Identification of the most promising trade-offs is conducted using fuzzy programming and weighted sum methods [8]. Joshi et al. [9] applied neutrosophic sets to model uncertain multi-objective transportation problems and solved them using neutrosophic compromise programming. In [10], a fractional objective (cost per unit) is proposed to address uncertainty in delivery and demand for transportation, applying a range-based approach to generate robust decisions. Interpolating polynomials and nonlinear mixed-integer programming are used to explain multi-choice linear programming problems [11].

In direction to explain the STP, a basic feasible solution must have nonzero values for at least $mn\ell - (m-1)(n-1)(\ell-1)$ variables, represented as χ_{ijk} (Figure 2). The transportation problem always has a feasible solution, whereas physical constraints don't always guarantee one. Nevertheless, it is ensured that there is a solution that satisfies all constraints—aside from the non-negativity constraints—and has a manageable number of nonzero variables. This type of solution is referred to as a "basic" solution. In this case, only $mn + n\ell + \ell m - m - n - \ell - 1$ equations (corresponding to nonzero χ_{ijk}'s values) are needed, leaving $m + n + \ell - 1$ variables set to zero [3].

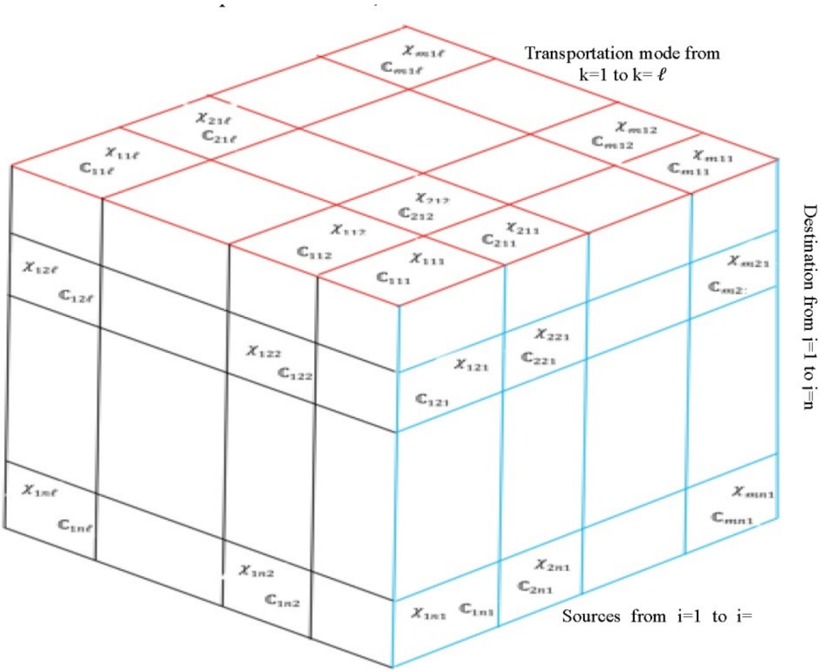

Figure 2
A tabular depiction of SFTP that shows the decision factors χ_{ijk} and the cost $\mathbb{C}_{ijk}$.

Lagrange's interpolating polynomial is utilized to identify an accurate decision from the multiple choices, hence mitigating computational complexity and other difficulties. It is additionally employed to substitute the multiple-choice parameters in the transportation problem (TP). By applying essential numerical techniques [12], we have formulated the nonlinear programming model and are working on solving it, which serves as the main focus of this study. This article explores the impact of rising fuel prices, road taxes, market volatility, and other key factors on the cost coefficients of objective function parameters while evaluating multiple options for real-world transportation problems.

The application of fuzzy artificial intelligence (AI) in manufacturing helps address decision-making challenges caused by uncertainties in transportation. It facilitates the selection of the best options that meet several goals by producing personalized solutions with corresponding confidence levels [13]. The current study introduces a novel approach for addressing uncertainty in transportation issues with multiple objectives [14]. A nonlinear model employing Lagrange's interpolating polynomial is presented to compute optimal parameters for cost minimization in multi-choice transportation problems, with LINGO software [15] used for obtaining the solution. In [16], the authors address a multi-objective transportation model involving unknown parameters using Newton's divided difference interpolation along with a ranking approach The study in [17] tackles demand uncertainty by using interpolating polynomials and incorporates fuzzy programming to explain a MOTP with multiple choice parameters. Reference [18] presents an approach that incorporates Lagrange's interpolating polynomial to address multi-level, multi-choice transportation problems characterized by uncertain parameters. This technique allows the underlying variability in cost and supply factors to be captured more accurately, thereby enhancing the robustness of the model. The study demonstrates that polynomial interpolation can effectively represent the range of possible parameter values within complex transportation systems. We analyzed the multi-choice multi-objective solid transportation issue (MC-MOSTP), taking into account supply, demand, and transportation capacity.

Due to inherent uncertainty, the objective functions include multi-choice coefficients. To manage uncertainties in the problem, the model applies Lagrange's interpolation polynomial, reducing multi-choice parameters to a single decision variable. This method ensures that the resulting solution is either optimal or very close to it. The study efforts on the MC-MOSTP, employing Lagrange interpolation for each multi-choice parameter while considering inherent uncertainties in supply, demand, and transportation capacities. Compression has been finalized and verified. Table 1 lists a few noteworthy MOTP research studies.

Table 1
A comparative analysis of the suggested methodology and existing models.

Reference (year)	**Problem type**	**Parameter**		**Multi-choice**	**Multi-objective**	**Methodology**
	Solid	**Supply and Demand**	**Conveyance**			
Bhatia (1978)	✓	✓	✓			Modified distribution method
Joshi (2023)		✓			✓	Goal programming employing a weighted sum approach
S. Acharya (2016)		✓		✓	✓	LIP

Roy (2015)		✓		✓		LIP
Joshi (2022)		✓			✓	The theory of neutrosophy
Joshi (2024)	✓	✓	✓	✓	✓	NDD
Joshi (2011)		✓			✓	Duality theory
This paper	✓	✓	✓	✓	✓	LIP

Where NDD denotes Newton's Divided Difference, LIP denotes Lagrange's Interpolating Polynomial, and FP denotes Fuzzy Programming.

The organization of this research paper is as follows. Section 1 introduces the topic and surveys the relevant literature. Section 2 presents the essential terminology, and Section 3 specifies the notational framework adopted in the analysis. Section 4 details the problem formulation, while Section 5 describes the methodology used to solve it. Section 6 illustrates the proposed approach through a numerical example. Section 7 analyzes the results and major findings, and Section 8 provides the concluding remarks of the study.

2. Basic Definitions

The following definitions outline key concepts relevant to optimization problems and different types of solutions (Figure 3):

2.1 *Feasible Solution*

A combination of decision variable values that satisfies every constraint of the problem, resulting in a legitimate solution to the optimization problem.

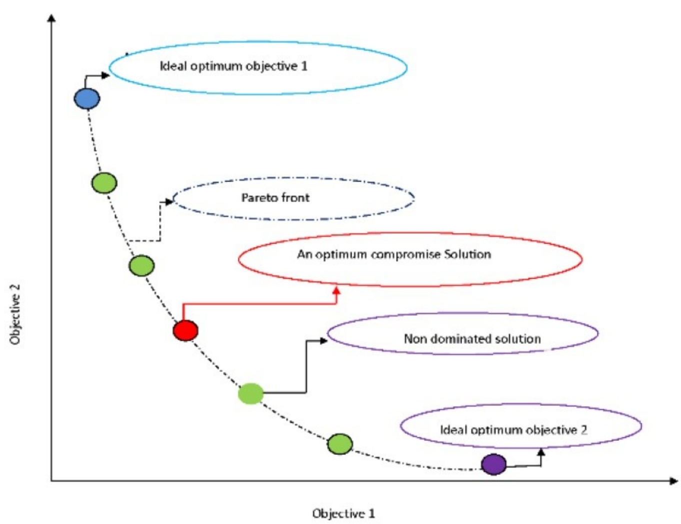

Figure 3
The Compromise Solution.

2.2 *Compromise Solution*

The compromise optimal solution is the closest possible solution to the ideal solution and is achieved through mutual compromise.

2.3 *Ideal Solution*

In a nor maximization problem, the optimal solution is achieved when the objective function reaches its minimum value.

2.4 *Anti-ideal Solution:*

It is the solution in a minimization problem where the objective function attains its peak value, representing the least favorable outcome.

2.5 *Pareto Optimal Solution*

In the context of multi-objective optimization, a solution is regarded as Pareto optimal when it is impossible to enhance the value of any one objective without simultaneously deteriorating the performance of at least one of the remaining objectives. In other words, such a solution represents a state of mutual compromise among all objectives, where no further improvement can be achieved without introducing a trade-off.

3. List of Notations Used in This Study

To ensure clarity and uniformity in our presentation and analysis, we provide a list of notations utilized in this research.

➢ $\mathcal{R}$: Represents the entire number of objective functions. ➢ m: Represents the total number of supply sources. ➢ n: Indicates the total number of demand destinations. ➢ ℓ: Represents the total number of conveyances. ➢ $\mathcal{Z}_r$: r^{th}objective function. ➢ β_{ki}: The required number of units to be transported from source i^{th} using the k^{th} mode of transportation. ➢ γ_{ij}: Quantity of units accessible at the i^{th} source, requested by the j^{th} destination.	➢ χ_{ijk}: Symbolizes the number of units transported from the i^{th} supply point to the j^{th} demand destination using the j^{th} mode of transportation. ➢ $\mathbb{C}_{ijk}(r)$: The cost of using the k^{th} transportation mode of the r^{th} objective function to move one shipment unit from the i^{th} supply source to the j^{th}demand destination. ➢ α_{jk}: The required number of units to be transported using the k^{th} mode of transportation to the j^{th}destination.

4. Mathematical Modeling of the Transportation Problem

Products from many manufacturing facilities must be delivered by a logistics business to diverse commercial destinations. This scenario has m manufacturing facilities, n retail stores, and l vehicles, with the k^{th} vehicle designated for transporting a uniform product from the i^{th}manufacturing facility to the j^{th} retail store. Let the amount of the product in units be denoted by χ_{ijk}. The mathematical formulation of the problem is based on multi-choice random parameters, with solid fractional transportation cost functions, which indicate that their values are not always constant.

A logistics company is responsible for delivering products from multiple manufacturing facilities to various retail destinations. In this scenario, there are $m$manufacturing facilities, $n$retail stores, and $l$vehicles, where the k-th vehicle is assigned to transport a specific product from the i-th manufacturing facility to the j-th retail store. The quantity of the product transported is represented by χ_{ijk}. The problem is mathematically formulated using multi-choice random parameters and incorporates solid fractional transportation cost functions, reflecting the fact that the costs are not always constant.

Model 1:

Minimize $$\mathcal{Z}_r = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{\ell} \mathbb{C}_{ijk}(r)\chi_{ijk}, \quad (1)$$

Subject to $$\sum_{i=1}^{m} \chi_{ijk} = \alpha_{jk}. \quad (2)$$

$$\sum_{j=1}^{n} \chi_{ijk} = \beta_{ki}. \quad (3)$$

$$\sum_{k=1}^{\ell} \chi_{ijk} = \gamma_{ij}. \quad (4)$$

$$\sum_{j=1}^{n} \alpha_{jk} = \sum_{i=1}^{m} \beta_{ki}, \sum_{k=1}^{\ell} \beta_{ki} = \sum_{j=1}^{n} \gamma_{ij}, \sum_{i=1}^{m} \gamma_{ij} = \sum_{k=1}^{\ell} \alpha_{jk}. \quad (5)$$

$$\sum_{j=1}^{n} \sum_{k=1}^{\ell} \alpha_{jk} = \sum_{k=1}^{\ell} \sum_{i=1}^{m} \beta_{ki} = \sum_{i=1}^{m} \sum_{j=1}^{n} \gamma_{ij}. \quad (6)$$

$$\chi_{ijk} \geq 0, \forall\, i, j, k. \quad (7)$$

Where $\mathbb{C}_{ijk}(r) = (\mathbb{C}_{ijk}^{1}(r), \ldots, \mathbb{C}_{ijk}^{s}(r))$.

5. Solution Methodology

This section provides an overview of approaches to solving transportation problems, including multiple solutions and different objectives. Initially, multivariate parameterization is performed using the Lagrangian interpolating polynomial method. Subsequently, a deterministic form of the probabilistic constraints is obtained using probabilistic restriction techniques. One approach is to construct a Lagrangian sum with multi-choice parameters, given a predefined number of nodes. Each node reflects the value of one function within the multi-choice parameter set. Lagrangian decision polynomials transform multi-choice parameters and select a single parameter from multiple choice parameters using a random numerical method.

Table 2

The Cost of Transportation at The TP Node Points.

Node point	Z_{ijk}	0	1	2	... ...	$S-1$
Transportation cost	$f(Z_{ijk}) = \mathbb{C}_{ijk}$	$\mathbb{C}_{ijk}^{1}$	$\mathbb{C}_{ijk}^{2}$	$\mathbb{C}_{ijk}^{3}$	... ...	$\mathbb{C}_{ijk}^{S}$

The objective function for Model 1 can be expressed as follows:

Minimize $\mathcal{Z}_r = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{\ell} \mathbb{C}_{ijk}(r)\chi_{ijk}$

At each node S, the interpolating polynomial attains functional values denoted by

$$\mathbb{C}_{ijk}(r) = (\mathbb{C}_{ijk}^{1}(r), \dots, \mathbb{C}_{ijk}^{s}(r)),$$

where the positions of the nodes are represented by $0,1,2, \dots, S-1$. The dataset provided in Table 2 is approximated using a Lagrange interpolating polynomial $P_{S-1}(Z_{ijk})$of degree $(S-1)$. This polynomial is defined as follows:

$$P_{S-1}(Z_{ijk}) = \frac{(Z_{ijk}-1)(Z_{ijk}-2)\dots(Z_{ijk}-S+1)}{(-1)^{(S-1)}(S-1)!}\mathbb{C}_{ijk}^{1}(r)$$
$$+\frac{(Z_{ijk})(Z_{ijk}-2)\dots(Z_{ijk}-S+1)}{(-1)^{(S-2)}(S-2)!}\mathbb{C}_{ijk}^{2}(r) + \frac{(Z_{ijk})(Z_{ijk}-1)(Z_{ijk}-3)\dots(Z_{ijk}-S+1)}{(-1)^{(S-3)}(S-3)!2!}\mathbb{C}_{ijk}^{3}(r)$$
$$+\dots+\frac{(Z_{ijk})(Z_{ijk}-1)(Z_{ijk}-2)\dots(Z_{ijk}-S+2)}{(S-1)!}\mathbb{C}_{ijk}^{S}(r),$$
$$\forall\, j = 1,2, \dots, n, i = 1,2, \dots, m, \quad k = 1,2, \dots, \ell. \tag{8}$$

Consequently, Model 1's objective function is rewritten as follows by utilizing Eq. (8):

Minimize

$$\mathcal{Z}_r = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{\ell} \Bigg[\frac{(Z_{ijk}-1)(Z_{ijk}-2)\dots(Z_{ijk}-S+1)}{(-1)^{(S-1)}(S-1)!}\mathbb{C}_{ijk}^{1}(r)$$
$$+\frac{(Z_{ijk})(Z_{ijk}-2)\dots(Z_{ijk}-S+1)}{(-1)^{(S-2)}(S-2)!}\mathbb{C}_{ijk}^{2}(r) + \frac{(Z_{ijk})(Z_{ijk}-1)(Z_{ijk}-3)\dots(Z_{ijk}-S+1)}{(-1)^{(S-3)}(S-3)!2!}\mathbb{C}_{ijk}^{3}(r)$$
$$+\dots+\frac{(Z_{ijk})(Z_{ijk}-1)(Z_{ijk}-2)\dots(Z_{ijk}-S+2)}{(S-1)!}\mathbb{C}_{ijk}^{S}(r)\Bigg]\chi_{ijk}$$

and hence, Model 1 is equivalent to following model:

Model 2:

Minimize

$$\mathcal{Z}_r = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{\ell} \Bigg[\frac{(Z_{ijk}-1)(Z_{ijk}-2)\dots(Z_{ijk}-S+1)}{(-1)^{(S-1)}(S-1)!}\mathbb{C}_{ijk}^{1}(r)$$
$$+\frac{(Z_{ijk})(Z_{ijk}-2)\dots(Z_{ijk}-S+1)}{(-1)^{(S-2)}(S-2)!}\mathbb{C}_{ijk}^{2}(r) + \frac{(Z_{ijk})(Z_{ijk}-1)(Z_{ijk}-3)\dots(Z_{ijk}-S+1)}{(-1)^{(S-3)}(S-3)!2!}\mathbb{C}_{ijk}^{3}(r)$$
$$+\dots+\frac{(Z_{ijk})(Z_{ijk}-1)(Z_{ijk}-2)\dots(Z_{ijk}-S+2)}{(S-1)!}\mathbb{C}_{ijk}^{S}(r)\Bigg]\chi_{ijk}$$

Subject to $\sum_{i=1}^{m} \chi_{ijk} = \alpha_{jk}, \sum_{j=1}^{n} \chi_{ijk} = \beta_{ki}, \sum_{k=1}^{\ell} \chi_{ijk} = \gamma_{ij}.$

$\sum_{j=1}^{n} \alpha_{jk} = \sum_{i=1}^{m} \beta_{ki},\ \sum_{k=1}^{\ell} \beta_{ki} = \sum_{j=1}^{n} \gamma_{ij},\ \sum_{i=1}^{m} \gamma_{ij} = \sum_{k=1}^{\ell} \alpha_{jk}.$

$\sum_{j=1}^{n} \sum_{k=1}^{\ell} \alpha_{jk} = \sum_{k=1}^{\ell} \sum_{i=1}^{m} \beta_{ki} = \sum_{i=1}^{m} \sum_{j=1}^{n} \gamma_{ij}.$

$\chi_{ijk} \geq 0, \forall\, i, j, k.$

Properties: ➢ The polynomial passes exactly through all given points. ➢ It is unique for a given set of points. ➢ The degree of the polynomial is at most $S-1$. **Advantages:** ➢ Simple to understand and apply. ➢ Guarantees exact interpolation at the given points. ➢ Useful for small datasets.	**Limitations:** ➢ Can be computationally expensive for large datasets. ➢ May exhibit oscillatory behavior (Runge's phenomenon) for high-degree polynomials. ➢ Sensitive to errors in input data. **Applications:** ➢ Function approximation. ➢ Numerical integration (e.g., Newton-Cotes formulas). ➢ Curve fitting in computer graphics. ➢ Signal processing.

6. Numerical Example

An example is illustrated in Tables 3 and 4, which present the values of $v$for the primary and additional objective functions for conveyances $k = 1,2,$and 3. The MC-MOSTP is explained using LINGO version 18.0.

Minimize $\mathcal{Z}_1 = \{15,18,20\}\chi_{111} + \{15,16,18,19\}\chi_{121} + \{25,26,27\}\chi_{131} + \{19,21,25\}\chi_{211} + \{14,17,19,22\}\chi_{221} + \{12,15,19\}\chi_{231} + \{28,29\}\chi_{311} + \{10,12,13\}\chi_{321} + \{10,12,13\}\chi_{331} + \{16,19,22\}\chi_{112} + \{25,27,30\}\chi_{122} + \{19,20\}\chi_{132} + \{27,30\}\chi_{212} + \{20,21,25\}\chi_{222} + \{13,15,19\}\chi_{232} + \{15,18,20,24\}\chi_{312} + \{12,15\}\chi_{322} + \{16,17,19\}\chi_{332} + \{9,10,11\}\chi_{113} + \{16,17\}\chi_{123} + \{11,18,20\}\chi_{133} + \{18,19,21\}\chi_{213} + \{13,16,18\}\chi_{223} + \{11,18,20\}\chi_{233} + \{14,25,16,17\}\chi_{313} + \{26,29\}\chi_{323} + \{12,15,18\}\chi_{333}$

Minimize $\mathcal{Z}_2 = \{16,19,20\}\chi_{111} + \{18,19,21\}\chi_{121} + \{12,16,18,21\}\chi_{131} + \{14,16\}\chi_{211} + \{20,21,23,26,29\}\chi_{221} + \{13,16,18\}\chi_{231} + \{13,17\}\chi_{311} + \{14,17,19,22\}\chi_{321} + \{15,16\}\chi_{331} + \{20,21,22,25\}\chi_{112} + \{14,16,17\}\chi_{122} + \{10,12,13,15,17\}\chi_{132} + \{9,12,15\}\chi_{212} + \{17,18\}\chi_{222} + \{12,13,15,\}\chi_{232} + \{15,18,20,24\}\chi_{312} + \{12,15\}\chi_{322} + \{16,17,19\}\chi_{332} + \{9,10,11\}\chi_{113} + \{16,17\}\chi_{123} + \{11,18,20\}\chi_{133} + \{12,15,17\}\chi_{213} + \{10,13,18\}\chi_{223} + \{12,13,15\}\chi_{233} + \{11,12,15,17,19\}\chi_{313} + \{20,22,25\}\chi_{323} + \{23,25,27,30\}\chi_{333}$

Subject to, $\chi_{111} + \chi_{211} + \chi_{311} = 15,\ \chi_{112} + \chi_{212} + \chi_{312} = 18$
$\chi_{113} + \chi_{213} + \chi_{313} = 20,\ \chi_{121} + \chi_{221} + \chi_{321} = 8$
$\chi_{122} + \chi_{222} + \chi_{322} = 12,\quad \chi_{123} + \chi_{223} + \chi_{323} = 9$
$\chi_{131} + \chi_{231} + \chi_{331} = 11,\quad \chi_{132} + \chi_{232} + \chi_{332} = 8$
$\chi_{133} + \chi_{233} + \chi_{333} = 16,\quad \chi_{111} + \chi_{121} + \chi_{131} = 16$
$\chi_{211} + \chi_{221} + \chi_{231} = 13,\quad \chi_{311} + \chi_{321} + \chi_{331} = 15$
$\chi_{112} + \chi_{122} + \chi_{132} = 9,\quad \chi_{212} + \chi_{222} + \chi_{232} = 15$

$$\chi_{312} + \chi_{322} + \chi_{332} = 14, \qquad \chi_{113} + \chi_{123} + \chi_{133} = 10$$
$$\chi_{213} + \chi_{223} + \chi_{233} = 17, \qquad \chi_{313} + \chi_{323} + \chi_{333} = 18$$
$$\chi_{111} + \chi_{112} + \chi_{113} = 10, \qquad \chi_{121} + \chi_{122} + \chi_{123} = 6$$
$$\chi_{131} + \chi_{132} + \chi_{133} = 9, \qquad \chi_{211} + \chi_{212} + \chi_{213} = 21$$
$$\chi_{221} + \chi_{222} + \chi_{223} = 10, \qquad \chi_{231} + \chi_{232} + \chi_{233} = 14$$
$$\chi_{311} + \chi_{312} + \chi_{313} = 22, \qquad \chi_{321} + \chi_{322} + \chi_{323} = 13$$
$$\chi_{331} + \chi_{332} + \chi_{333} = 12$$
$$\chi_{ijk} \geq 0, \qquad i, j, k = 1,2,3.$$

The subsequent equivalent mathematical model is constructed based on the numerical example above by using the method "Lagrange Interpolating Polynomial" presented in the subsection.

Minimize $\mathcal{Z}_1 = [-0.5u_{111}^2 + 3.5u_{111} + 15]\chi_{111} + [-0.3333u_{121}^3 + 1.5u_{121}^2 - 0.1666u_{121} + 15]\chi_{121} + [\, u_{131} + 25]\chi_{131} + [u_{211}^2 + u_{211} + 26]\chi_{211} + [0.3333u_{221}^3 - 1.5u_{121}^2 + 4.1666u_{221} + 14]\chi_{221} + [0.5u_{231}^2 + 2.5u_{231} + 12]\chi_{231} + [u_{311} + 28]\chi_{311} + [-0.5u_{321}^2 + 2.5u_{321} + 10]\chi_{321} + [-0.5u_{321}^2 + 2.5u_{321} + 10]\chi_{321} + [3u_{112} + 16]\chi_{112} + [0.5u_{122}^2 + 1.5u_{122} + 25]\chi_{122} + [u_{132} + 19]\chi_{132} + [3u_{212} + 27]\chi_{212} + [1.5u_{222}^2 - 0.5u_{222} + 20]\chi_{222} + [u_{232}^2 + u_{232} + 13]\chi_{232} + [0.5u_{312}^3 - 2u_{312}^2 + 4.5u_{312} + 15]\chi_{312} + [3u_{322} + 12]\chi_{322} + [0.5u_{332}^2 + \ 0.5u_{332} + 16]\chi_{332} + [u_{113} + 9]\chi_{113} + [u_{123} + 16]\chi_{123} + [-2.5u_{133}^2 + 9.5u_{133} + 11]\chi_{133} + [0.5u_{213}^2 + 0.5u_{213} + 18]\chi_{213} + [-0.5u_{223}^2 + 3.5u_{223} + 13]\chi_{223} + [2.5u_{233}^2 + 9.5u_{233} + 11]\chi_{233} + [u_{313} + 14]\chi_{313} + [3u_{323} + 26]\chi_{323} + [3u_{333} + 12]\chi_{333}$

Minimize $\mathcal{Z}_2 = [-v_{111}^2 + 4v_{111} + 16]\chi_{111} + [0.5v_{121}^2 + 0.5v_{121} + 18]\chi_{121} + [0.16667v_{131}^3 + 1.5v_{131}^2 + \ 5.333v_{131} + 12]\chi_{131} + [2v_{211} + 14]\chi_{211} + [-0.0416\, v_{221}^4 + 0.25\, v_{221}^3 + 0.4166\, v_{221}^3 + 0.75v_{221} + 20]\chi_{221} + [0.5v_{231}^2 + 3.5v_{231} + 13]\chi_{231} + [4v_{311} + 13]\chi_{311} + [-0.3333v_{321}^3 - 1.5v_{321}^2 + 4.1667v_{321} + 14]\chi_{321} + [v_{331} + 15]\chi_{331} + [0.333v_{112}^3 - v_{112}^3 + 1.6667v_{112} + 20]\chi_{112} + [-0.5v_{122}^2 + 2.5v_{122} + 14]\chi_{122} + [-0.125v_{132}^4 + 1.08333v_{132}^3 - 2.875v_{132}^2 + 3.9167v_{132} + 10]\chi_{132} + [3v_{212} + 9]\chi_{212} + [v_{222} + 17]\chi_{222} + [0.5v_{232}^2 + 0.5v_{232} + 12]\chi_{232} + [4v_{312} + 11]\chi_{312} + [0.273v_{322} + 10]\chi_{322} + [0.1667v_{332}^3 - 0.5v_{332}^2 + \ 2.833v_{332} + 9]\chi_{332} + [0.5v_{113}^2 + 0.5v_{113} + 20]\chi_{113} + [4v_{123} + 15]\chi_{123} + [2v_{133} + 11]\chi_{133} + [-0.5v_{213}^2 + 3.5v_{213} + 12]\chi_{213} + [v_{223}^2 + 2v_{223} + 10]\chi_{223} + [0.5v_{233}^2 + 0.5v_{233} + 12]\chi_{233} + [0.1667v_{313}^4 - 0.5v_{313}^2 + 4.333v_{313}^2 + 2v_{313} + 11]\chi_{313} + [0.5v_{333}^2 + 1.5v_{323} + 20]\chi_{323} + [0.1667v_{333}^3 - 0.5v_{333}^2 + 2.33v_{333} + 23]\chi_{333}$

Subject to, $\chi_{111} + \chi_{211} + \chi_{311} = 15, \quad \chi_{112} + \chi_{212} + \chi_{312} = 18$
$$\chi_{113} + \chi_{213} + \chi_{313} = 20, \qquad \chi_{121} + \chi_{221} + \chi_{321} = 8$$
$$\chi_{122} + \chi_{222} + \chi_{322} = 12, \qquad \chi_{123} + \chi_{223} + \chi_{323} = 9$$
$$\chi_{131} + \chi_{231} + \chi_{331} = 11, \qquad \chi_{132} + \chi_{232} + \chi_{332} = 8$$

$$\chi_{133}+\chi_{233}+\chi_{333}=16, \qquad \chi_{111}+\chi_{121}+\chi_{131}=16$$
$$\chi_{211}+\chi_{221}+\chi_{231}=13, \qquad \chi_{311}+\chi_{321}+\chi_{331}=15$$
$$\chi_{112}+\chi_{122}+\chi_{132}=9, \qquad \chi_{212}+\chi_{222}+\chi_{232}=15$$
$$\chi_{312}+\chi_{322}+\chi_{332}=14, \qquad \chi_{113}+\chi_{123}+\chi_{133}=10$$
$$\chi_{213}+\chi_{223}+\chi_{233}=17, \qquad \chi_{313}+\chi_{323}+\chi_{333}=18$$
$$\chi_{111}+\chi_{112}+\chi_{113}=10, \qquad \chi_{121}+\chi_{122}+\chi_{123}=6$$
$$\chi_{131}+\chi_{132}+\chi_{133}=9, \qquad \chi_{211}+\chi_{212}+\chi_{213}=21$$
$$\chi_{221}+\chi_{222}+\chi_{223}=10, \qquad \chi_{231}+\chi_{232}+\chi_{233}=14$$
$$\chi_{311}+\chi_{312}+\chi_{313}=22, \qquad \chi_{321}+\chi_{322}+\chi_{323}=13$$
$$\chi_{331}+\chi_{332}+\chi_{333}=12$$
$$0\le u_{111}\le 2;\ 0\le u_{121}\le 3;\ 0\le u_{131}\le 2;\ 0\le u_{211}\le 2;$$
$$0\le u_{221}\le 3;\ 0\le u_{231}\le 2;\ 0\le u_{311}\le 1;\ 0\le u_{321}\le 2;$$
$$0\le u_{331}\le 2;\ 0\le u_{112}\le 2;\ 0\le u_{122}\le 2;\ 0\le u_{132}\le 1;$$
$$0\le u_{212}\le 1;\ 0\le u_{222}\le 2;\ 0\le u_{232}\le 2;\ 0\le u_{312}\le 3;$$
$$0\le u_{322}\le 1;\ 0\le u_{332}\le 2;\ 0\le u_{113}\le 2;\ 0\le u_{123}\le 1;$$
$$0\le u_{133}\le 2;\ 0\le u_{213}\le 2;\ 0\le u_{223}\le 2;\ 0\le u_{233}\le 2;$$
$$0\le u_{313}\le 3; 0\le u_{323}\le 1; 0\le u_{333}\le 2;$$
$$0\le v_{111}\le 2;\ 0\le v_{121}\le 2;\ 0\le v_{131}\le 3;\ 0\le v_{211}\le 1;$$
$$0\le v_{221}\le 4;\ 0\le v_{231}\le 2;\ 0\le v_{311}\le 1;\ 0\le v_{321}\le 3;$$
$$0\le v_{331}\le 1;\ 0\le v_{112}\le 3;\ 0\le v_{122}\le 2;\ 0\le v_{132}\le 4;$$
$$0\le v_{212}\le 2;\ 0\le v_{222}\le 1;\ 0\le v_{232}\le 2;\ 0\le v_{312}\le 1;$$
$$0\le v_{322}\le 1;\ 0\le v_{332}\le 4;\ 0\le v_{113}\le 2;\ 0\le v_{123}\le 1;$$
$$0\le v_{133}\le 2;\ 0\le v_{213}\le 2;\ 0\le v_{223}\le 2;\ 0\le v_{233}\le 2;$$
$$0\le v_{313}\le 4;\ 0\le v_{323}\le 2;\ 0\le v_{333}\le 3;$$
$$\chi_{ijk}\ge 0, \qquad i,j,k=1,2,3 \quad u_{ijk},v_{ijk}\in Z^{+}.$$

Table 3

The transportation cost $\mathbb{C}_{ijk}(r)$ for both the first and second objective functions.

		j = 1		j = 2		j = 3	
$i=1$	κ = 1	$\mathbb{C}_{111}(1)=\{15,18,20\}$	$\mathbb{C}_{111}(2)=\{16,19,20\}$	$\mathbb{C}_{121}(1)=\{15,16,18,20\}$	$\mathbb{C}_{121}(2)=\{18,19,21\}$	$\mathbb{C}_{131}(1)=\{25,26,27\}$	$\mathbb{C}_{131}(2)=\{12,16,18,19\}$
	κ = 2	$\mathbb{C}_{112}(1)=\{16,19,22\}$	$\mathbb{C}_{112}(2)=\{20,21,22,25\}$	$\mathbb{C}_{122}(1)=\{25,27,30\}$	$\mathbb{C}_{122}(2)=\{14,16,17\}$	$\mathbb{C}_{132}(1)=\{19,20\}$	$\mathbb{C}_{132}(2)=\{10,12,13,15,17\}$
	κ = 3	$\mathbb{C}_{113}(1)=\{9,10,11\}$	$\mathbb{C}_{113}(2)=\{20,21,23\}$	$\mathbb{C}_{123}(1)=\{16,17\}$	$\mathbb{C}_{123}(2)=\{15,19\}$	$\mathbb{C}_{133}(1)=\{11,18,20\}$	$\mathbb{C}_{133}(2)=\{11,13,15\}$
$i=2$	κ = 1	$\mathbb{C}_{211}(1)=\{19,21,25\}$	$\mathbb{C}_{211}(2)=\{14,16\}$	$\mathbb{C}_{221}(1)=\{14,17,19,22\}$	$\mathbb{C}_{221}(2)=\{20,21,23,26,29\}$	$\mathbb{C}_{231}(1)=\{12,15,19\}$	$\mathbb{C}_{231}(2)=\{13,16,18\}$
	κ = 2	$\mathbb{C}_{212}(1)=\{27,30\}$	$\mathbb{C}_{212}(2)=\{9,12,15\}$	$\mathbb{C}_{222}(1)=\{20,21,25\}$	$\mathbb{C}_{222}(2)=\{17,18\}$	$\mathbb{C}_{232}(1)=\{13,15,19\}$	$\mathbb{C}_{232}(2)=\{12,13,15\}$
	κ = 3	$\mathbb{C}_{213}(1)=\{18,19,21\}$	$\mathbb{C}_{213}(2)=\{12,15,17\}$	$\mathbb{C}_{223}(1)=\{13,16,18\}$	$\mathbb{C}_{223}(2)=\{10,13,18\}$	$\mathbb{C}_{233}(1)=\{11,18,20\}$	$\mathbb{C}_{233}(2)=\{12,13,15\}$
$i=3$	κ= 1	$\mathbb{C}_{311}(1)=\{28,29\}$	$\mathbb{C}_{311}(2)=\{13,17\}$	$\mathbb{C}_{321}(1)=\{10,12,13\}$	$\mathbb{C}_{321}(2)=\{14,17,19,22\}$	$\mathbb{C}_{331}(1)=\{10,12,13\}$	$\mathbb{C}_{331}(2)=\{15,16\}$

	$\kappa = 2$	$\mathbb{C}_{312}(1) = \{15,18,20,24\}$	$\mathbb{C}_{312}(2) = \{11,15\}$	$\mathbb{C}_{322}(1) = \{12,15\}$	$\mathbb{C}_{322}(2) = \{10,13\}$	$\mathbb{C}_{332}(1) = \{16,17,19\}$	$\mathbb{C}_{332}(2) = \{9,11,12,13,15\}$
	$\kappa = 3$	$\mathbb{C}_{313}(1) = \{14,15,16,17\}$	$\mathbb{C}_{313}(2) = \{11,12,15,17,19\}$	$\mathbb{C}_{323}(1) = \{26,29\}$	$\mathbb{C}_{323}(2) = \{20,22,25\}$	$\mathbb{C}_{333}(1) = \{12,15,18\}$	$\mathbb{C}_{333}(2) = \{23,25,27,30\}$

Table 4

The Number of Units γ_{ij}, α_{jk} and $\beta_{\kappa i}$.

		$\mathbf{j = 1}$	$\mathbf{j = 2}$	$\mathbf{j = 3}$	$\boldsymbol{\beta}_{\mathbf{k\kappa i}}$
$\boldsymbol{i} = \mathbf{1}$	$\kappa = 1$	$\gamma_{11} = 10$	$\gamma_{12} = 6$	$\gamma_{13} = 9$	$\beta_{11} = 16$
	$\kappa = 2$				$\beta_{21} = 9$
	$\kappa = 3$				$\beta_{31} = 10$
$\boldsymbol{i} = \mathbf{2}$	$\kappa = 1$	$\gamma_{21} = 21$	$\gamma_{22} = 10$	$\gamma_{23} = 14$	$\beta_{12} = 13$
	$\kappa = 2$				$\beta_{22} = 15$
	$\kappa = 3$				$\beta_{32} = 17$
$\boldsymbol{i} = \mathbf{3}$	$\kappa = 1$	$\gamma_{31} = 22$	$\gamma_{32} = 13$	$\gamma_{33} = 12$	$\beta_{13} = 15$
	$\kappa = 2$				$\beta_{23} = 14$
	$\kappa = 3$				$\beta_{33} = 18$
$\boldsymbol{\alpha}_{\mathbf{j}\kappa}$		$\alpha_{11} = 15$	$\alpha_{21} = 8$	$\alpha_{31} = 11$	
		$\alpha_{12} = 18$	$\alpha_{22} = 12$	$\alpha_{32} = 8$	
		$\alpha_{13} = 20$	$\alpha_{23} = 9$	$\alpha_{33} = 16$	

7. Results and discussion

The researchers and engineers formulated the multi-objective function, inclusive of restrictions, in explicit form and illustrated it through numerical examples. Costs are treated as random variables with multiple possible values, and the corresponding results were obtained using LINGO 18.0 software, as presented in Table 5.

8. Conclusions

This study aims to investigate the explanatory framework for the multivariate transportation problem, considering multivariate expenditure characteristics. The objective function's cost coefficients may be initially ambiguous in many real-world transportation problems because of a number of affecting factors. Considering all of these aspects, a multivariate transport pattern has been designed because the charging coefficients the objective functions and parameters are multivariate. In mathematical programming, multivariate parameters are not handled by any direct methodLagrange's interpolation polynomial is employed for the first time in this research to handle MOSTP's multivariate parameters. The approach's major advantages in analyzing multivariate transportation problems are presented. The MC-MOSTP under consideration is answered by using LINGO 18.0 software.

Table 5

Each of Z_1 and Z_2 was evaluated independently, without considering the effects of the other objective.

Ideal solution	Anti-ideal solution	Ideal solution	Anti-ideal solution	Variables						
Z_1	Z_1	Z_2	Z_2	i	j	k	χ_1		χ_2	
							u_{ijk}	χ_{ijk}	v_{ijk}	χ_{ijk}
1690.898	1873.647	1281.494	1366.875	1	1	1	0	2	0	6
						2	0	8	2	3
						3	1.2346	0	0	1
					2	1	0.0566	4	2	0
						2	0	1	0	6
						3	0	1	1	0
					3	1	1.2346	0	1.2346	0
						2	1	0	1.2346	0
						3	0	9	0	9
				2	1	1	0	13	0	6
						2	0	5	0	14
						3	0	3	0	1
					2	1	0	0	1.1807	0
						2	0.1667	2	0	1
						3	0	8	0	9
					3	1	0	4	0	7
						2	0	8	0	0
						3	0	6	0	7
				3	1	1	1	0	0	3
						2	0	5	0	1
						3	0	17	0.2664	18
					2	1	0	4	3	8
						2	0	9	0	5
						3	0.7349	0	1.0604	0
					3	1	0	111	0	4
						2	0.8651	0	0	8
						3	0	1	1.1112	0

The model's validity could be increased by conducting additional research that integrates multiple-choice or uncertain variables concerning supply, demand, and transportation attributes. Researchers might also investigate how well the model works in further domains, such as supply chain management and economics. We provide a new method for dealing with MC-MOSTPs, which makes it a useful tool for industrial decision makers. Modern manufacturing is dynamic; therefore, success in an increasingly globalized environment necessitates new methods of decision-making.

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