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Optimal quality investment and specification limits settings with 100% inspection and sampling inspection for quality protection

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Abstract

Shin et al. [2] proposed the process parameters and tolerance designs model. Their model firstly obtained the process mean and standard deviation. Then the tolerance model is formulated for obtaining the optimal tolerance.

Product inspection is a short-term method for assuring the shipment quality. One should consider a long-term method for improving quality, e.g., quality investment. In this paper, the authors address the extension of Shin et al.'s [2] tolerance model with quality investment.

Subject Classification: *60E05 Probability distributions: general theory.*

Keywords: *Product inspection, Quality investment, Specification limits.*

1. Introduction

The quality design, statistical process control, and acceptance sampling inspection are the traditional statistical quality control methodology. Taguchi [1] proposed the quality loss function for evaluating the product.

Previous researchers have addressed the optimal process mean and process standard deviation settings. Shin et al. [2] proposed the integrated model with process parameters and tolerance settings. Goethals and Cho [3] presented the design of experiment methodology for obtaining the estimates of process parameters under the multiple quality characteristics.

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Their model can determine the optimal control factor settings and process target values. Goethals and Cho [4] considered the multiple mixed quality characteristics with experimental design for obtaining the predictable process means settings. Their trade-off model addressed the process variability, tolerance, and process cost and adopted the non-zero skewness normal distribution application. Boylan and Cho [5] further presented the robust parameters design for the mass production process. They also adopted Monte Carlo simulation for comparing the different performances of estimators. Boylan et al. [6] considered the robust parameter design model with Monte Carlo simulation for obtaining the precise estimators of process parameters under the constrained resource. Their model is available for addressing the true optimal process parameters settings. Raza et al. [7] considered the expected gross income, profit and product uniformity as the three objective functions for determining the economic selection of process mean.

The control charts and process capability index are two important tools for SPC. We use them for controlling process and improvement. One adopts the control charts for process control and uses the PCI for process improvement. One expects to obtain six sigma quality of product. Hence, SPC is available for quality improvement. There are some works about PCI, e.g., Kane [8], Chan et al. [9], Boyles [10], and Pearn et al. [11].

Wu and Govindaraju [12] proposed the computer-aided method for designing the parameters of sampling inspection plan. Their model needs to consider the identification of the quality characteristic function and measurement error adjustment. Recently, the acceptance sampling plans integrating the application of process capability index has been proposed, e.g., Hsu et al. [13]. Their model considered the economic sample size satisfying the specified producer's and consumer's risks. Chen and Chou [14] proposed the manufacturing target selection with sampling inspection and PCI. Chen and Chou [15] determined the parameters of inspection plan with consumer's risk in the quantitative quality control model. Banihashemi et al. [16] presented the resubmitted and multiple dependent state inspection plan with process yield and Taguchi's quality loss. To minimize the average number of sample size is the objective function. The constraints include the producer's risk and consumer's risk based on yield index. Bose and Guha [17] addressed the decision criteria for the economic production lot problem with on-line inspection and full inspection. They obtained the conclusion that the on-line inspection can reduce the holding cost and improve the expected profit.

In this paper, the authors extend Shin et al.'s [2] work for tolerance model with quality investment and tolerance by considering 100% and sampling inspections.

2. Review of Mathematical Model

Shin et al.'s [2] model firstly considers the process parameters optimization. The closed-form solutions of optimal process mean μ and standard deviation σ can be obtained by satisfying Hessian positive definite matrix. Then the optimal process parameters are the input variables of subsequent model.

In the phase II, 100% inspection tolerance optimization with minimum expected total cost for determining the optimal tolerance coefficient δ.

3. Modified Shin et al.'s [2] Model

3.1 *Notations*

k_1	quality loss coefficient
μ	unknown parameter of process mean
σ	unknown parameter of standard deviation
T	target value of process mean
C_0	constant in the process cost
C_1	coefficient of μ in the process cost
C_2	coefficient of σ in the process cost
C_3	coefficient of $\mu\sigma$ in the process cost
$TC_1(\mu,\ \sigma)$	process total cost incurred without inspection
$TC_2(Inv,\ \delta)$	total cost incurred with 100% inspection
$TC_2(Inv,\ \delta, n, d_0)$	total cost incurred with sampling inspection
$\phi(\cdot)$	standard normal p.d.f.
$\Phi(\cdot)$	standard normal c.d.f.
C_R	rejection unit cost
a_0	constant in the manufacturing cost
a_1	coefficient of the tolerance range in the manufacturing cost
LSL	lower tolerance limit

USL	upper tolerance limit
μ_0	known process mean in the phase I
σ_0	known standard deviation in the phase I
Inv	quality investment
μ_1	improved process mean
σ_1	improved standard deviation
C_{pm}	process capability index
K_m	specified C_{pm} value
β_1	exponential declined coefficient of process mean through quality investment
α_1	exponential declined coefficient of standard deviation through quality investment
β	customer's risk
Y	normal quality characteristic
$f(y)$	the p.d.f. of Y without quality investment,

$$f(y)=\frac{1}{2\sqrt{\pi}\sigma}\exp(-\frac{1}{2}(\frac{y-\mu}{\sigma})^2),\ \ -\infty<y<\infty$$

$f(y,Inv)$	the p.d.f.of Y with quality investment,

$$f(y,Inv)=\frac{1}{2\sqrt{\pi}\sigma_1}\exp(-\frac{1}{2}(\frac{y-\mu_1}{\sigma_1})^2),\ \ -\infty<y<\infty$$

σ_T	target value of standard deviation
δ	coefficient of tolerance
b	sample size
X	number of non-conformance
d_0	clearance number
LF_1	expected quality loss in the rejected lot
LF_0	expected quality loss for un-inspected unit in the accepted lot
N	lot size
$P(X\le d_0)$	accepted probability of inspected lot
$LTPD$	lot tolerance percent defective

$P_a(p = LTPD)$ accepted probability of inspected lot
I_c inspection unit cost

3.2 *Modified Model*

According to Shin et al. [2], the modified model for process parameters settings model and quality investment and specification limits settings model is as follows:

3.2.1 Phase I. Process Parameters Settings Model

One needs to determine the initial process mean and standard deviation as follows:

Minimize

$$E[TC_1(\mu,\ \sigma)] = \int_{-\infty}^{\infty} k_1(y-T)^2 f(y)dy + C_0 + C_1\mu + C_2\sigma + C_3\mu\sigma$$
$$= k_1[(\mu-T)^2 + \sigma^2] + C_0 + C_1\mu + C_2\sigma + C_3\mu\sigma \qquad (1)$$

Set $\frac{\partial E[TC_1(\mu,\ \sigma)]}{\partial \mu} = 0$ and $\frac{\partial E[TC_1(\mu,\ \sigma)]}{\partial \sigma} = 0$. We have the optimal $\mu_0 = \frac{4k_1^2 T - 2k_1C_1 + C_2C_3}{4k_1^2 - C_3^2}$ and $\sigma_0 = \frac{2k_1C_2 + 2k_1C_3T - C_1C_3}{-4k_1^2 + C_3^2}$. The specified values of $d\mu_0$ and σ_0 are the input variables of phase II.

3.2.2 Phase II. Quality Investment and Specification Limits Settings Model

One needs to obtain the optimal quality investment and specification limits through modified model with 100% or sampling inspection.

(1) 100% Inspection

The expected total cost includes the quality loss, rejection cost, manufacturing cost, and quality investment. Mathematical model is to have

Minimize

$$E[TC_2(Inv,\ \delta)]$$
$$= \int_{LSL}^{USL} k_1(y-T)^2 f(y, Inv)dy + C_R[\int_{-\infty}^{LSL} f(y, Inv)dy + \int_{USL}^{\infty} f(y, Inv)dy]$$
$$+ a_0 + a_1(USL - LSL) + Inv$$
$$= k_1[(\mu_1 - T)^2][\Phi(\tfrac{USL-\mu_1}{\sigma_1}) - \Phi(\tfrac{LSL-\mu_1}{\sigma_1})] - 2k_1\sigma_1(\mu_1 - T)[\phi(\tfrac{USL-\mu_1}{\sigma_1})$$

$$-\phi(\frac{LSL-\mu_1}{\sigma_1})]-k_1\sigma_1^2[(\frac{USL-\mu_1}{\sigma_1})\phi(\frac{USL-\mu_1}{\sigma_1})-(\frac{LSL-\mu_1}{\sigma_1})\phi(\frac{LSL-\mu_1}{\sigma_1})-\Phi(\frac{USL-\mu_1}{\sigma_1})$$
$$+\Phi(\frac{LSL-\mu_1}{\sigma_1})]+C_R\{1-\Phi(\frac{USL-\mu_1}{\sigma_1})+\Phi(\frac{LSL-\mu_1}{\sigma_1})]\}+a_0+2a_1\delta\sigma_0+Inv \quad (2)$$

subject to

$$LSL=\mu_0-\delta\sigma_0 \quad (3)$$
$$USL=\mu_0+\delta\sigma_0 \quad (4)$$
$$f(y,Inv)=\frac{1}{\sqrt{2\pi}\sigma_1}\exp(-\frac{1}{2}(\frac{y-\mu_1}{\sigma_1})^2),\ \ LSL<y<USL \quad (5)$$
$$\mu_I^2=T^2+(\mu_0^2-T^2)\exp(-\beta_1 Inv) \quad (6)$$
$$\sigma_1^2=\sigma_T^2+(\sigma_0^2-\sigma_T^2)\exp(-\alpha_1 Inv) \quad (7)$$
$$C_{pm}=\frac{USL-LSL}{6\sqrt{(\mu_1-T)^2+\sigma_1^2}}=\frac{2\delta\sigma_0}{6\sqrt{(\mu_1-T)^2+\sigma_1^2}}=K_m \quad (8)$$

The parameters (Inv,δ) need to be determined with minimum $E[TC_2(Inv,\delta)]$ under the specified C_{pm}. Its solution can be found as follows:

Step 1. Set the maximum *Inv* be $Inv_{\max}$.

Step 2. Set the initial $Inv = 0$. To find the corresponding δ satisfying Eq. (8) and $E[TC_2(Inv,\delta)]$ from Eq. (2).

Step 3. Let $Inv = Inv + 1$, go to Step 2 until $Inv = Inv_{\max}$. We obtain (Inv,δ) with minimum $E[TC_2(Inv,\delta)]$.

(2) Sampling Inspection

All units are inspected when the lot is rejected. If the inspected unit is the defective, then it will be rejected and replaced by conforming one. The expected total cost of phase II with the quality loss for the accepted lot, quality loss for the rejected lot, rejection cost, manufacturing cost, inspection cost, and quality investment. The model is as follows:

Minimize

$$E[TC_2(Inv,\ \delta,n,d_0)]$$
$$=\ \{[nI_c+npC_R+(N-n)LF_0+nLF_1]P(X\le d_0)+[NI_c+NpC_R$$
$$+NLF_1]P(X>d_0)+(a_0+2a_1\delta\sigma_0)N+Inv\}/N$$
$$=\ \{[(N-n)LF_0+nLF_1]P(X\le d_0)+NLF_1P(X>d_0)$$

$$+NC_R[1-\Phi(\tfrac{USL-\mu_1}{\sigma_1})+\Phi(\tfrac{LSL-\mu_1}{\sigma_1})]\cdot[N-(N-n)P(X\le d_0)]$$
$$+(a_0+2a_1\delta\sigma_0)N+Inv+[NI_c-I_c(N-n)P(X\le d_0)]\}/N$$
$$=\ [(1-\tfrac{n}{N})LF_0+\tfrac{n}{N}LF_1]P(X\le d_0)+LF_1\cdot P(X>d_0)$$
$$+C_R[1-\Phi(\tfrac{USL-\mu_1}{\sigma_1})+\Phi(\tfrac{LSL-\mu_1}{\sigma_1})][1-(1-\tfrac{n}{N})]P(X\le d_0)$$
$$+[a_0+2a_1\delta\sigma_0]+\frac{Inv}{N}+[I_c-I_c(1-\tfrac{n}{N})P(X\le d_0)] \tag{9}$$

subject to

$$LSL=\mu_0-\delta\sigma_0 \tag{10}$$
$$USL=\mu_0+\delta\sigma_0 \tag{11}$$
$$f(y,Inv)=\frac{1}{\sqrt{2\pi}\sigma_1}\exp(-\tfrac{1}{2}(\tfrac{y-\mu_1}{\sigma_1})^2),\ \ LSL<y<USL \tag{12}$$
$$\mu_1^2=T^2+(\mu_0^2-T^2)\exp(-\beta_1 Inv) \tag{13}$$
$$\sigma_1^2=\sigma_T^2+(\sigma_0^2-\sigma_T^2)\exp(-\alpha_1 Inv) \tag{14}$$
$$LF_0=\int_{-\infty}^{\infty}k_1(y-T)^2 f(y,Inv)dy=k_1[(\mu_1-T)^2+\sigma_1^2] \tag{15}$$
$$P_a(p=LTPD)=\beta \tag{16}$$

$$C_{pm}=\frac{USL-LSL}{6\sqrt{(\mu_1-T)^2+\sigma_1^2}}=\frac{2\delta\sigma_0}{6\sqrt{(\mu_1-T)^2+\sigma_1^2}}=K_m \tag{17}$$

$$LF_1=\frac{\int_{LSL}^{USL}k_1(y-T)^2 f(y,\,Inv)dy}{\int_{LSL}^{USL}f(y,\,Inv)dy}$$
$$=\{k_1[(\mu_1-T)^2][\Phi(\tfrac{USL-\mu_1}{\sigma_1})-\Phi(\tfrac{LSL-\mu_1}{\sigma_1})]-2k_1\sigma_1(\mu_1-T)[\phi(\tfrac{USL-\mu_1}{\sigma_1})$$
$$-\phi(\tfrac{LSL-\mu_1}{\sigma_1})]-k_1\sigma_1^2[(\tfrac{USL-\mu_1}{\sigma_1})\phi(\tfrac{USL-\mu_1}{\sigma_1})-(\tfrac{LSL-\mu_1}{\sigma_1})\phi(\tfrac{LSL-\mu_1}{\sigma_1})$$
$$-\Phi(\tfrac{USL-\mu_1}{\sigma_1})+\Phi(\tfrac{LSL-\mu_1}{\sigma_1})]\}/[\Phi(\tfrac{USL-\mu_1}{\sigma_1})-\Phi(\tfrac{LSL-\mu_1}{\sigma_1})] \tag{18}$$

The combination $(Inv,\ \delta,\ n,\ d_0)$ need to be determined with minimum $E[TC_2(Inv,\delta,\ n,\ d_0)]$ under the specified C_{pm} and consumer's risk. Its solution can be found as follows:

Step 1. Set the maximum d_0 be $d_{\max}$ and maximum *Inv* be $Inv_{\max}$. Set the initial $Inv=0$ and $d_0=0$.

Step 2. For the given $LTPD,\ \beta,$ and $d_0,$ we have the n satisfying Eq. (16).

Step 3. One can obtain the δ satisfying Eq. (17) and $E[TC_2(Inv,\delta, n, d_0)]$ from Eq. (9).

Step 4. Let $Inv = Inv + 1$, go to Step 3 until $Inv = Inv_{\max}$.

Step 5. Let $d_0 = d_0 + 1$, go to Step 2 until $d_0 = d_{\max}$. We obtain (Inv, δ, n, d_0) with minimum $E[TC_2(Inv,\delta, n, d_0)]$.

4. Numerical Example with Sensitivity Analysis

Consider the following parameters: $k = 100$, $T = 50$, $C_0 = 100$, $C_1 = 6.1$, $C_2 = -3.1$, $C_3 = -3.85$, $a_0 = 100$, $a_1 = -0.2$, $C_R = 100$, $\alpha_1 = 0.5$, $\beta_1 = 0.1$, $\sigma_T = 0.5$, $K_m = 1$, $LTPD = 5\%$, $\beta = 10\%$, $I_c = 1$, and $N = 500$.

The result of original Shin et al.'s (2010) model is $\mu_0 = 49.9883$, $\sigma_0 = 0.9778$, $\delta = 1.0269$, $LSL = 48.9842$, $USL = 50.9924$, and $E[TC_2(\delta)] =$ 150.3168.

By solving Eqs. (1)-(8), the optimal solution for 100% inspection is $\mu_0 = 49.9883$, $\sigma_0 = 0.9778$, $\mu_I = 49.9943$, $\sigma_I = 0.5202$, $Inv = 7.0665$, $\delta = 1.5962$, $LSL = 48.4276$, $USL = 51.5491$, and $E[TC_2(Inv, \delta)]$ = 133.6104.

By solving Eqs. (9)-(18), the optimal solution for sampling inspection is $\mu_0 = 49.9883$, $\sigma_0 = 0.9778$, $\mu_I = 49.9984$, $\sigma_I = 0.5000$, $Inv = 19.2596$, $\delta = 1.5342$, $Inv = 19.2596$, $\delta = 1.5342$, $LSL = 48.4882$, $USL = 51.4885$, $d_0 = 1$, $n = 78$, and $E[TC_2(Inv, \delta, d_0, n)]$ = 142.368.

By comparing the above three results, one concludes that the modified model has the smaller expected total cost than that of original Shin et al.'s [2] one.

Tables 1-2 presents the results of sensitivity analysis for parameters. From Tables 1-2, we obtain the following results:

1. 100% inspection has the smaller expected total costt than that of sampling inspection.
2. Some parameters have a significant effect on quality investment, coefficient of tolerance, and expected total cost per unit for 100% inspection as follows:
   (1) the k, C_3, and α_1 have a significant effect on the quality investment;
   (2) the k, C_3, σ_T, and K_m have a significant effect on the coefficient of tolerance;

(3) the k, σ_T, and a_0 have a significant effect on the expected total cost.

3. Some parameters have a significant effect on quality investment, coefficient of tolerance, and expected total cost for sampling inspection as follows:

(1) the k, C_3, and α_1 have a significant effect on the quality investment;

(2) the k, C_3, σ_T, T, and K_m have a significant effect on the coefficient of tolerance;

(3) the k, σ_T, a_0, C_R, K_m, $LTPD$, β, and N, have a significant effect on the expected total cost;

(4) the K_m, $LTPD$, and β have a significant effect on the combination of parameters for sampling plan.

Table 1

Results for 100% inspection

k	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
120	49.9877	0.8148	49.9935	0.5169	6.3608	1.9034	137.7624
80	49.9913	1.2223	49.9960	0.5251	7.7531	1.2890	129.4413
T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
51	50.9887	0.9970	50.9945	0.5202	7.1720	1.5654	133.7151
49	48.9880	0.9585	48.9940	0.5203	6.9544	1.6284	133.5022
C_0	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
120	49.9883	0.9778	49.9943	0.5202	7.0665	1.5962	133.6104
80	49.9883	0.9778	49.9943	0.5202	7.0665	1.5962	133.6104
C_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
7.32	49.9822	0.9777	49.9912	0.5202	7.0656	1.5965	133.6145
4.88	49.9944	0.9779	49.9973	0.5203	7.0610	1.5961	133.6083
C_2	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
-2.48	49.9883	0.9747	49.9942	0.5202	7.0510	1.6012	133.5933
-3.72	49.9884	0.9809	49.9943	0.5202	7.0822	1.5912	133.6275
C_3	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
-3.08	49.9816	0.7852	49.9897	0.5202	5.7543	1.9880	132.3075
-4.62	49.9965	1.1704	49.9985	0.5202	7.9877	1.3334	134.5297
a_0	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$

Contd...

120	49.9883	0.9778	49.9943	0.5202	7.0665	1.5962	153.6104
80	49.9883	0.9778	49.9943	0.5202	7.0673	1.5962	113.6104
a_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
-0.16	49.9883	0.9778	49.9943	0.5202	7.0665	1.5962	133.6104
-0.24	49.9883	0.9778	49.9943	0.5202	7.0665	1.5962	133.6104
C_R	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
120	49.9883	0.9778	49.9943	0.5203	7.0624	1.5963	133.6646
80	49.9883	0.9778	49.9943	0.5202	7.0665	1.5962	133.5562
α_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
0.6	49.9883	0.9778	49.9937	0.5169	6.1893	1.5862	132.4036
0.4	49.9883	0.9778	49.9949	0.5251	8.2806	1.6112	135.3189
β_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
0.12	49.9883	0.9778	49.9950	0.5203	7.0625	1.5963	133.6095
0.08	49.9883	0.9778	49.9934	0.5202	7.0682	1.5962	133.6116
σ_T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
0.6	49.9883	0.9778	49.9941	0.6169	6.7278	1.8930	143.9497
0.4	49.9883	0.9778	49.9944	0.4250	7.3043	1.3042	125.1139
K_m	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta)]$
1.2	49.9883	0.9778	49.9943	0.5197	7.1211	1.9135	134.0356
0.8	49.9883	0.9778	49.9941	0.5223	6.8637	1.2822	132.4117

Table 2

Results for sampling inspection

k	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
120	49.9877	0.8148	49.9981	0.5000	18.6628	1.8410	147.3441	(1, 78)
80	49.9913	1.2223	49.9988	0.5000	20.1180	1.2273	137.3901	(1, 78)
T	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
51	50.9887	0.9970	50.9984	0.5000	19.3955	1.5046	142.3679	(1, 78)
49	48.9880	0.9585	48.9982	0.5000	19.2323	1.5651	142.3681	(1, 78)
C_0	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
120	49.9883	0.9778	49.9983	0.5000	19.2596	1.5342	142.3680	(1, 78)
80	49.9883	0.9778	49.9983	0.5000	19.2596	1.5342	142.3680	(1, 78)
C_1	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)

Contd...

7.32	49.9822	0.9777	49.9974	0.5000	19.1164	1.5345	142.3752	(1, 78)
4.88	49.9944	0.9779	49.9992	0.5000	19.7810	1.5340	142.3641	(1, 78)
C_2	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
-2.48	49.9883	0.9747	49.9983	0.5000	19.4706	1.5391	142.3680	(1, 78)
-3.72	49.9884	0.9809	49.9983	0.5000	19.2272	1.5294	142.3680	(1, 78)
C_3	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
-3.08	49.9816	0.7852	49.9969	0.5000	17.7334	1.9105	142.3729	(1, 78)
-4.62	49.9965	1.1704	49.9996	0.5000	20.4105	1.2817	142.3653	(1, 78)
a_0	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
120	49.9883	0.9778	49.9983	0.5000	19.2893	1.5342	162.3680	(1, 78)
80	49.9883	0.9778	49.9983	0.5000	19.2621	1.5342	122.3680	(1, 78)
a_1	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
-0.16	49.9883	0.9778	49.9983	0.5000	19.2596	1.5342	142.3680	(1, 78)
-0.24	49.9883	0.9778	49.9983	0.5000	19.2596	1.5342	142.3680	(1, 78)
C_R	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
120	49.9883	0.9778	49.9983	0.5000	19.2596	1.5342	145.8210	(1, 78)
80	49.9883	0.9778	49.9983	0.5000	19.2596	1.5342	138.9145	(1, 78)
α_1	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
0.6	49.9883	0.9778	49.9977	0.5000	16.3464	1.5342	142.3609	(1, 78)
0.4	49.9883	0.9778	49.9989	0.5000	23.7470	1.5343	142.3782	(1, 78)
β_1	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
0.12	49.9883	0.9778	49.9988	0.5000	19.3129	1.5342	142.3685	(1, 78)
0.08	49.9883	0.9778	49.9975	0.5000	19.3037	1.5343	142.3673	(1, 78)
σ_T	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
0.6	49.9883	0.9778	49.9983	0.6000	18.965	1.8410	153.3152	(1, 78)
0.4	49.9883	0.9778	49.9983	0.4001	19.441	1.2275	133.4128	(1, 78)
K_m	μ_0	σ_0	μ_I	σ_I	*Inv*	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
1.2	49.9883	0.9778	49.9984	0.5000	19.6528	1.8411	135.6711	(0, 46)
0.8	49.9883	0.9778	49.9982	0.5001	18.4996	1.2275	164.7200	(3, 134)

Contd...

$LTPD$	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
0.06	49.9883	0.9778	49.9983	0.5001	19.2621	1.5342	139.3140	(1, 65)
0.04	49.9883	0.9778	49.9983	0.5001	19.2064	1.5343	146.8928	(1, 97)
β	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
0.05	49.9883	0.9778	49.9983	0.5001	19.2089	1.5343	146.4139	(1, 95)
0.01	49.9883	0.9778	49.9983	0.5001	19.2596	1.5342	155.5581	(1, 33)
N	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
600	49.9883	0.9778	49.9984	0.5000	19.8156	1.5342	139.8004	(1, 78)
400	49.9883	0.9778	49.9983	0.5001	19.1017	1.5343	146.2191	(1, 78)
I_c	μ_0	σ_0	μ_I	σ_I	Inv	δ	$E[TC_2(Inv, \delta, d_0, n)]$	(d_0, n)
1.2	49.9883	0.9778	49.9983	0.5001	19.2596	1.5342	142.4025	(1, 78)
0.8	49.9883	0.9778	49.9983	0.5001	19.2596	1.5342	142.3335	(1, 78)

Conclusions

In this article, the authors have proposed the modified Shin et al.'s [2] model. Through the numerical results, the modified model has smaller expected total cost than that of original Shin et al.'s [2] one. Further work should address the modified model applied in the supply chain management with supplier.

References

[1] G. Taguchi, Introduction to Quality Engineering, *Asian Productivity Organization*, Tokyo (1986).

[2] S. Shin, P. Kongsuwon, and B. R. Cho, "Development of the parametric tolerance modeling and optimization schemes and cost-effective solutions," *European Journal of Operational Research*, vol. 207, pp. 1728-1741 (2010).

[3] P. L. Goethals and B. R. Cho, "The development of multi-response experimental designs for process parameter optimization," *International Journal of Quality & Reliability Management*, vol. 28, pp. 628-648 (2011).

[4] P. L. Goethals and B. R. Cho, "The optimal process mean problem: Integrating predictability and profitability into an experimental factor space," *Computers & Industrial Engineering*, vol. 62, pp. 851-869 (2012).

[5] G. L. Boylan and B. R. Cho, "Comparative studies on the high-variability embedded robust parameter design from the perspective of estimators," *Computers & Industrial Engineering,* vol. 164, pp. 442-452 (2013).

[6] G. L. Boylan, P. L. Goethals, and B. R. Cho, "Robust parameter design in resource-constrained environments: An investigation of trade-offs between costs and precision within variable processes," *Applied Mathematical Modelling*, vol. 37, pp. 2394-2416 (2013).

[7] S. A. Raza, F. C. Abdullakutty, S. Rathinam, and S. M. Govindaluri, "Multi-objective framework for process mean selection and price differentiation with leakage effects under price-dependent stochastic demand," *Computers & Industrial Engineering*, vol. 127, pp. 698-708 (2019).

[8] V. E. Kane, "Process capability indices," *Journal of Quality Technology*, vol. 18, pp. 41-52 (1986).

[9] L. K. Chan, S. W. Cheng, and F. A. Spring, "A new measure of process capability: Cpm," *Journal of Quality Technology,* vol. 30, pp. 162-175 (1988).

[10] R. A. Boyles, "The Taguchi capability index," *Journal of Quality Technology*, vol. 23, pp. 107-126 (1991).

[11] W. L. Pearn, S. Kotz, and N. L. Johnson, "Distributional and inferential properties of process capability indices," *Journal of Quality Technology,* vol. 24, pp. 216-231 (1992).

[12] H. Wu and K. Govindaraju, "Computer-aided variables sampling inspection plans for compositional proportions and measurement error adjustment," *Computers & Industrial Engineering*, vol. 72, pp. 239-246 (2014).

[13] B. M. Hsu, T.-C. Wang, and M.-H. Shu, "Lot-dependent sampling plans for qualifying long-term production capability with a one-sided specification," *Computers & Industrial Engineering,* vol. 146, p. 106583 (2020).

[14] C. H. Chen and C. Y. Chou, "Economic design of product and process parameters under the specified process capability index," *Quality Technology and Quantitative Management*, vol. 15, pp. 686-701 (2018).

[15] C. H. Chen and C. Y. Chou, "Optimal process mean setting by considering sampling inspection and process capability," *Journal of Information and Optimization Sciences*, vol. 41, pp. 1795-1801 (2020).

[16] A. Banihashemi, M. S. F. Nezhad, and A. Amiri, "A new approach in the economic design of acceptance sampling plans based on process yield index and Taguchi loss function," *Computers & Industrial Engineering*, vol. 159, p. 107155 (2021).

[17] D. Bose and A. Guha, "Economic production lot sizing under imperfect quality, on-line inspection and inspection error: Full vs. sampling inspection," *Computers & Industrial Engineering*, vol. 160, p. 107565 (2021).

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