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On directed signed cordial labeling

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Abstract

A measurable collection of dots and a collection of ordered pairings of different dots make up a directed graph, commonly known as a digraph $\mathcal{D}$. Each (u,v) pair is denoted by $\overrightarrow{uv}$ where u is the arc's beginning dot and v its terminal dot, is an arc. A digraph having V dots and A arcs is called $D(V, A)$. For each vertex $v \in V$, the indegree $d^-(v)$ of v indicates how many arcs have v as their terminal dot and the out degree $d^+(v)$ of v indicates how many arcs have v indicates how many arcs have v as their starting dot. In this article, we explored the presence of di-signed cordiality for certain families of digraphs and defined a new style of labeling known as directed signed cordial labeling (DSCL), also known as di-signed cordial labeling.

Subject Classification: *05C78.*

Keywords: *Di-graph, Di triangular snake, Di helm, Di closed helm.*

1. Introduction

Concepts are easier to understand when research use diagrammatic depiction. When applied to these data sets, the geometric depiction of graph topologies provides a potent tool for data interpretation and understanding.

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Diagrams with points and lines connecting them can be used to describe problems that occur in numerous areas. Mathematical modelling will now be based on **Graph Theory**. A graph label that, depending on a specific situation, assigns integers to the lines, dots, or both.

Because of its practical applications and theoretical significance, graph labeling has been researched in great detail. Rosa [1] presented early fundamental work on vertex valuations, while structural characteristics of particular graph classes were examined in [13], [2]. Cahit [4] introduced cordial labeling, which turned out to be a less effective substitute for gracious labeling. Babujee and Loganathan, [5] provides a thorough overview of graph labelingmet hods. In [7], [10], [3], cycle-related and general graph families were examined for additional advancements in signed and product cordial labeling.

Several extensions of cordial and signed cordial labeling have been reported in the literature, particularly in the context of discrete structures and cryptographic applications in the papers [11], [8], [12] and [9].

The Signed Cordial labeling(SCL) and its extension, the signed product cordial labeling, are one of important binary labeling scheme in graph theory that contribute to current research advancements. Many graphs have been demonstrated to accept signed cordial labeling, according to the literature. Devaraj and Delphy [6] presented the idea of SPCL.

In this finding, we extend the notion of SCL to directed graphs, which we call directed SCL (or di-signed CL). A di-signed graph is a directed graph that takes this type of labeling. To facilitate a thorough and understandable comprehension of this work, the following definitions are offered.

2. Preliminaries

Definition 2.1: A graph $\mathrm{G}^* = (\mathcal{V}^*, \mathcal{E}^*)$ is Signed Cordial (SC) $\exists$ an line labeling $\zeta_1^* : \mathcal{E}^*(\mathrm{G}^*) \to \{+1, -1\}$ such that the corresponding derived dot labeling $\zeta_1^* : \mathcal{V}^*(\mathrm{G}^*) \to \{+1, -1\}$ satisfies the following conditions:

1. The label of each dot v^* in $\mathcal{V}^*(\mathrm{G}^*)$, defined as the multiplication of those values of all line's incident with v^*.
2. The obtained values are balanced with respect to both dots and edges, that is,

$$|v_{\varsigma_1}(+1) - v_{\varsigma_1}(-1)| \leq 1$$

$|e_{\varsigma_{1^*}}(+1) - e_{\varsigma_{1^*}}(-1)| \leq 1$, in which $v_{\varsigma_1}(\iota)$ and $e_{\varsigma_{1^*}}(\iota)$ is the count of dots and lines labeled ι respectively, $\iota \in \{+1, -1\}$.

If such a labeling exists, the graph G^* is called a SCG [5].

Note: As opposed to SCL, which allot labels to the lines and induces labels for the dots, Signed Product cordial labeling (SPCL) allots labels to the dots and induces labels for the lines. This makes SPCL the opposite of SCL [7].

Definition 2.2: A di-triangular snake $\overrightarrow{\mathcal{T}_\rho}$ is defined as a triangular snake $\mathcal{T}_\rho$ in which the edges of every complete graph of order 3 subgraphs are consistently oriented either clockwise or anticlockwise.

Note: Throughout this article, the orientation we considered anticlockwise.

Definition 2.3. An *alternating helm*, denoted by $\overrightarrow{\mathcal{AH}_\rho}$, is obtained from a helm graph $\mathcal{H}_\rho$ by assigning the following orientations:

1. All cycle lines are directed uniformly (clockwise or anticlockwise),
2. Each spoke lines is directed toward the hub dot, and
3. All pendant lines are directed toward the pendant dot.

Definition 2.4: A di closed helm, is obtained from the closed helm graph $\mathcal{CH}_\rho$ by orienting all the lines in both the outer and inner cycles clockwise, and directing every spoke line towards the hub dot and it is denoted by $\overrightarrow{\mathcal{CH}_\rho}$.

Note: In the present discussion, we assume that the cycle in di helm and di closed helm are orientated clockwise. Also, we represent odd as *od* and even as *evn*.

Definition 2.5: A digraph is defined as a graph whose lines possess directionality, generally indicated by an arrow from one dot to another. An illustration of a directed graph is shown in Figure 1

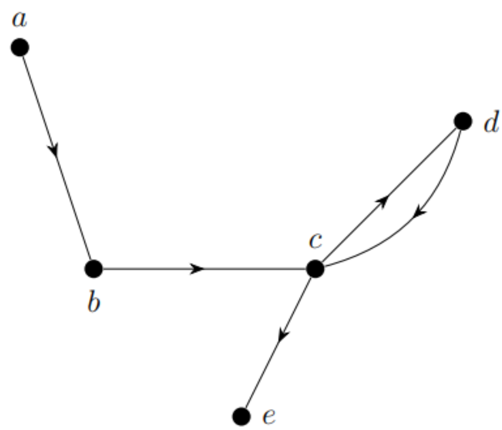

Figure 1
A directed graph

We next define DSCL as follows:

Definition 2.6: A di-graph $D^*\ (V', A')$ is said to be directed signed cordial (DSC) if $\exists$ an arc function $f_1^{\#} : \mathcal{A}\,' \rightarrow \{+1, -1\}$ in which the derived dot function

$f_1 : \mathcal{V}' \to \{+1, -1\}$ is obtained by assigning to each dot v^* in $\mathcal{V}'$ is the multiplication of those values of all in-degree arcs incident with v^*. Furthermore, the following requirements need to be met:

1. $v^*_{f_1}(+1) - v^*_{f_1}(-1) \leq 1$,
2. $a^*_{f_1^\#}(+1) - a^*_{f_1^\#}(-1) \leq 1$,
3. The minimum in-degree of each vertex is one,

where $v^*_{f_1}(\iota)$ and $a^*_{f_1^*}(\iota)$ represent the count of dots and lines with ι respectively, $\iota \in \{+1, -1\}$.

Note: The most required condition to prove a graph is DCG is 3.

3. Main Results

This article generally defines the dot and line collection of a di-triangular snake, alternating helm, and di-closed helm as follows:

$$\mathcal{V}'\left(\overrightarrow{\mathcal{T}_\rho}\right) = \begin{cases} v_\iota^* \text{ dots of the path: } 1 \leq \iota \leq \rho + 1 \\ u_\tau^* \text{ top dots } \left(\mathcal{V}_{top}\right)\text{: } 1 \leq \tau \leq \rho \end{cases}$$

$$\mathcal{A}'\left(\overrightarrow{\mathcal{T}_\rho}\right) = \{v_\iota^* v_{\iota+1}^*, u_\iota^* v_\iota^*, v_{\iota+1}^* u_\iota^* \colon 1 \leq \iota \leq \rho\}$$

$$\mathcal{V}'\left(\overrightarrow{\mathcal{AH}_\rho}\right) = \begin{cases} v_0^* \text{ dot of the hub} \\ v_\iota^* \text{ cycle dots: } 1 \leq \iota \leq \rho \\ u_\tau^* \text{ hanging dots:} 1 \leq \tau \leq \rho \end{cases}$$

$$\mathcal{A}'\left(\overrightarrow{\mathcal{AH}_\tau}\right) = \{v_0^* v_i^*, v_i^* v_{i+1}^*, v_i^* u_i^* \colon 1 \leq i \leq \rho$$

$$\mathcal{V}'\left(\overrightarrow{\mathcal{CH}_\rho}\right) = \begin{cases} v_0^* \text{ dot of the hub} \\ v_\iota^* \text{ inner cycle dots: } 1 \leq \iota \leq \rho \\ u_\tau^* \text{ outercycle dots: } 1 \leq \tau \leq \rho \end{cases}$$

$$\mathcal{A}'\left(\overrightarrow{\mathcal{CH}_\rho}\right) = \{v_0^* v_\iota^*,\ v_\iota^* v_{\mathrm{i}+1}^*,\ v_\mathrm{i}^* u_\mathrm{i}^*, v_\mathrm{i}^* u_{\mathrm{i}+1}^* \colon 1 \leq \mathrm{i} \leq \rho$$

Theorem 3.1: The Di-triangular snake $\overrightarrow{\mathcal{T}_\rho}, \rho \geq 3$ is DSCG.

Proof: The di-triangular graph of order ρ is denoted by $\mathcal{G}^*$. The arc function $f_a^\# : \mathcal{A}'(\mathcal{G}^*) \to \{+1, -1\}$ of $\mathcal{G}^*$ is defined as:

Case 1: $\rho \equiv 0 \bmod 2$

$$f_a^\#(v_\iota^* u_\iota^*) = \begin{cases} +1 \ \iota \text{ od} \\ -1 \ \iota \text{ evn} \end{cases}$$

$$f_a^\#(v_{\iota+1}^* u_\iota^*) = \begin{cases} +1 \ \iota \text{ evn} \\ -1 \ \iota \text{ od} \end{cases}$$

Case 2: $\rho \equiv 1 \bmod 2$

$$f_a^\#(v_\iota^* u_\iota^*) = \begin{cases} +1 \ \iota \text{ evn} \\ -1 \ \iota \text{ od} \end{cases}$$

$$f_{\mathrm{a}}^{\#}(v_{\iota+1}^{*}u_{\iota}^{*}) = \begin{cases} +1 & \iota \text{ od} \\ -1 & \iota \text{ evn} \end{cases}$$

where $1 \leq \iota \leq \rho$.

The following is the definition of the arc of the path of G^{*}:

Case 1: $\rho \equiv 0,1,2 \bmod 4$

$$f_{\mathrm{a}}^{\#}(v_{\iota}^{*}v_{\iota+1}^{*}) = \begin{cases} +1 & \iota = 4c+1 \text{ or } 4c+2 \\ -1 & \iota = 4c \text{ or } 4c+3 \end{cases} \quad c \in \mathbb{Z}$$

where $1 \leq \iota \leq \rho$.

Case 2: $\rho \equiv 3 \bmod 4$

$$f_{\mathrm{a}}^{\#}(v_{\iota}^{*}v_{\iota+1}^{*}) = \begin{cases} +1 & \iota = 4c+1 \text{ or } 4c+2 \\ -1 & \iota = 4c \text{ or } 4c+3 \end{cases} \quad c \in \mathbb{Z}$$

where $1 \leq \iota \leq \rho - 2$.

$$f_{\mathrm{a}}^{\#}(v_{\iota}^{*}v_{\iota+1}^{*}) = \begin{cases} +1 & \iota = \rho - 1 \\ -1 & \iota = \rho \end{cases}$$

The following is the derived dot label of G^{*}:

Case 1: $\rho \equiv 0 \bmod 2$

$$f_{1}^{\#}(u_{\iota}^{*}) = \begin{cases} +1 & \iota \text{ evn} \\ -1 & \iota \text{ od} \end{cases}$$

Case 2: $\rho \equiv 1 \bmod 2$

$$f_{1}^{\#}(u_{\iota}^{*}) = \begin{cases} +1 & \iota \text{ od} \\ -1 & \iota \text{ evn} \end{cases}$$

where $1 \leq \iota \leq \rho$.

The following is the derived dot of the path of G^{*}:

Case 1: $\rho \equiv 0 \bmod 2$

$$f_{1}^{\#}(v_{1}^{*}) = +1,\ f_{1}^{\#}(v_{2}^{*}) = -1 \text{ and } f_{1}^{\#}\left(v_{\rho+1}^{*}\right) = -1$$

$$f_{1}^{\#}(v_{\iota}^{*}) = \begin{cases} +1 & \iota = 4c \text{ or } 4c+3 \\ -1 & \iota = 4c+1 \text{ or } 4c+2 \end{cases} \quad c \in \mathbb{Z}$$

where $3 \leq \iota \leq \rho$.

Case 2: $\rho \equiv 1 \bmod 2$

$$f_{1}^{\#}(v_{1}^{*}) = -1,\ f_{1}^{\#}(v_{2}^{*}) = +1$$

$$f_{1}^{\#}(v_{\iota}^{*}) = \begin{cases} +1 & \iota = 4c+1 \text{ or } 4c+2 \\ -1 & \iota = 4c \text{ or } 4c+3 \end{cases} \quad c \in \mathbb{Z}$$

where $3 \leq \iota \leq \rho + 1$.

Both the dot and arc condition is met. Hence the graph G^{*} is (DSCG). A di-triangular snake with $\rho = 3$ is illustrated in Figure 2.

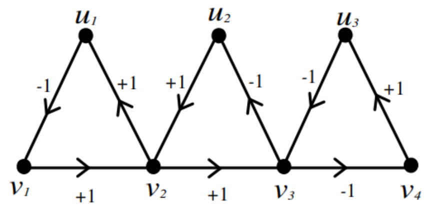

Figure 2
Directed triangular snake $\vec{T}_3$

Theorem 3.2: *The alternative helm* $\overrightarrow{\mathcal{AH}_\rho}, \rho \geq 3$ *is di-signed cordial graph (DSCG).*

Proof: The alternative helm graph $\overrightarrow{\mathcal{AH}_\rho}$, be $\mathcal{G}^*$. The definition of the arc set of G^* is

$$f_{a}^{\#}\left(\overrightarrow{v_0^* v_\iota^*}\right) = \begin{cases} +1\ \iota\ \text{od} \\ -1\ \iota\ \text{evn} \end{cases}$$

$$f_{a}^{\#}\left(\overrightarrow{v_\iota^* v_{\iota+1}^*}\right) = +1$$

$$f_{a}^{\#}\left(\overrightarrow{v_\iota^* u_\iota^*}\right) = -1$$

where $1 \leq \iota \leq \rho$.

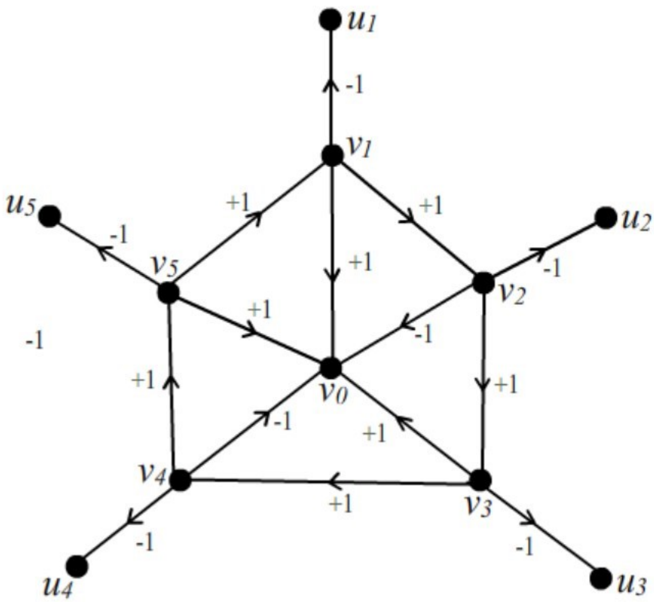

Figure 3
Alternative helm $\overrightarrow{\mathcal{AH}}_5$

The following is derived dot label:

$$f_1^{\#}(v_0^*) = \begin{cases} +1 \ \rho = 4c \text{ or } 4c+1 \\ -1 \ \rho \ = 4c+2 \text{ or } 4c+3 \end{cases} \ c \in \mathbb{Z}$$

$$f_1^{\#}(v_\iota^*) = +1 \text{ and } f_1^{\#}(u_\iota^*) = -1$$

where $1 \ \leq \ \iota \ \leq \ \rho$.

Both the dot and arc condition is met. Thus, the graph $\overrightarrow{\mathcal{AH}_\rho}$ is (DSCG). An alternating helm graph for $\rho = 5$ is depicted in Figure 3.

Theorem 3.3; *The di closed helm* $\overrightarrow{\mathcal{CH}_\rho}, \rho \ \geq \ 3$ *admits di- cordial graph.*

Proof: The di closed helm $\overrightarrow{\mathcal{CH}_\rho}$ be $\mathcal{G}^*$. The arc set of $\mathcal{G}^*$ is defined as.

$$f_a^{\#}\left(\overrightarrow{v_0^* v_\iota^*}\right) = -\ 1$$

$$f_a^{\#}\left(\overrightarrow{v_\iota^* v_{\iota+1}^*}\right) = +\ 1$$

$$f_a^{\#}\left(\overrightarrow{v_\iota^* u_\iota^*}\right) = +\ 1$$

$$f_a^{\#}\left(\overrightarrow{u_\iota^* u_{\iota+1}^*}\right) = -1$$

The derived dot label is as follow:

$$f_1^{\#}(v_0^*) = \begin{cases} +1 \ \rho \text{ evn} \\ -1 \ \rho \text{ od} \end{cases}$$

$$f_1^{\#}(v_\iota^*) = +1 \text{ and } f_1^{\#}(u_\iota^*) = -1$$

where $1 \ \leq \ \iota \ \leq \ \rho$.

Both the dot and arc condition is met. Hence the graph $\mathcal{G}^*$ is (DSCG). A di-closed helm graph is shown in Figure 4

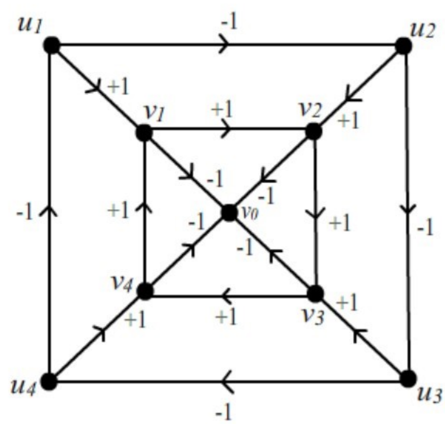

Figure 4
Di closed helm $\overrightarrow{\mathcal{CH}_4}$

4. Applications of di signed cordial labeling in energy distribution systems

The nodes in energy distribution networks are substations, and the edges are the power cables that connect them. To avoid underutilization of resources and overcrowding at each substation, efficient energy flow is essential. Depending on factors like voltage levels or power direction, edges (power lines) are labeled with $+1$ or -1 using disigned cordial labeling. The vertex label, which is determined by multiplying incoming edge labels at each node (indegree), shows whether the energy flow is balanced or unbalanced. In order to avoid overburdening, an optimal energy distribution is ensured by a $+1$ outcome for every node.

As an illustration:

1. Ensuring that power supply and demand are equal across substations is known as load balancing.
2. Fault tolerance: enables preventive actions by identifying crucial nodes that are vulnerable to overload.
3. Grid optimization: By streamlining flows, it reduces transmission energy loss.

This strategy improves efficiency and dependability in urban power management systems, smart grids, and renewable energy distribution.

5. Conclusion

By introducing disigned cordial labeling, a unique method that focuses on the product of indegree edge labels, we expanded the idea of graph labeling to directed graphs in this paper. We developed fundamental findings and showed how it may be used to energy distribution system. To find more profound theoretical characteristics and wider practical applications, we also suggest examining how this labeling behaves in organized interconnection networks such as grid graphs and torus topologies.

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