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Inventory optimization for deteriorating goods with dynamic demand and deferred payment options

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Abstract

This research study creates an inventory model for degrading products that considers the trade credit and also realistic market characteristics like advertising-motivated initial demand and unstable holding costs. The demand is characterized by a two-phase nonlinear ramp, where the sales initiate at a positive level because of advertising, grow over time until achieving a maximum, and subsequently stabilize throughout the product's lifespan. To account for varying market conditions, three scenarios are examined: when the credit period is less than the demand peak, when it exceeds that point, and when it goes beyond the product's total lifespan. Numerical cases are employed to evaluate the results of these situations and to define the issue as a nonlinear constrained optimization. The results that can assist companies in handling deteriorating goods stock more efficiently within trade credit agreements.

Subject Classification: *90B05.*

Keywords: *Economic order quantity, Inventory, Nonlinear ramp curve demand, Optimum cycle length, Payment postponement.*

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1. Introduction

The demand in this research study shadows a two-stage ramp pattern. Sales start at a positive level since the early promotion, rise to a peak, and then settle into a steady phase for the rest of the product's life. To cover different business situations, it is considered at three cases: when the credit period ends before highest demand, when it crosses the peak, and when it even goes past the product's full life span. In real practice, suppliers often support retailers by offering trade credit, letting them delay payment for some fixed time. This usually encourages retailers to order more, which is important for short-life products. The delayed payment also helps their cash flow since there is no interest during the credit period. But once the allowed time passes, interest charges usually start, and late payments can turn into a burden. Another important point is the cost of carrying inventory storage, handling, insurance, and the slow drop in product value. Many older models assume these holding costs stay constant, but that is not always true now. When firms consider these economic changes, they can design inventory policies that stay financially workable and more stable in today's uncertain markets.

1.1 *Objective of the Study*

The major purpose of this research is to concept an inventory model that accurately describes actual company activities by incorporating several dynamic features. The model considers: (i) a non-linear ramp-shaped pattern of demand behaviour; (ii) holding costs that increase steadily over time; (iii) a constant rate of product deterioration; (iv) the impact of tolerable payment delays; (v) costs of advertising that impact both demand and profit; and (vi) the existence of shortages with full backlogs. By combining these components, the study aims to provide a dense and useful framework for improving inventory management in realistic and dynamic market backgrounds.

2. Literature Review

Aggarwal and Jaggi [1] extended the idea of adding credit terms to inventory systems to perishable items and credit period decisions for demand-sensitive products. Jammal, Sarker, and Wang [2] and Sarker, Jammal and Wang [3] looked at deteriorating items with shortages and explored how delayed payments influence ordering behaviour. Later work by Jaggi, Goyal, and Goel [4], added different credit policies and demand structures to make these models more realistic. Singh and Pattnayak [5] examined quadratic demand and varying deterioration rates, showing how these features interact with payment delays. Credit-based inventory models also appeared in specialized fields: Shi, Zhang, and Zhou [6] analysed two phase demand under delayed payments. Singh et al. [7] used a Weibull deterioration setting to capture more complex product life cycles. Research then expanded into supply chain coordination. Mondal, Garai, and Roy [8] offered a more flexible model combining time-varying demand, Weibull decay, and full credit options. Saranya and Chandrasekaran [9], Palanivelu and Chandrasekaran [10] focused advertising-driven demand, and two-phase demand with payment delays. Dubey and Kumar [11] proposed an inventory model with sensitivity analysis to assess parameter impacts under uncertain environments. Neutrosophic frameworks have been applied to network optimization problems, where Pratyusha and Kumar [12] solved the critical path problem using computational tools, and Tripathi and Kumar [13] developed a multi-objective linear programming approach for the neutrosophic minimal cost flow problem. Additionally, Chen and Lee [14] optimized a modified inventory model by jointly determining process variance and quality investment decisions. Across these works, permissible payment delays consistently emerge as a major factor influencing order quantities, total cost, and coordination between supply-chain partners.

3. Notations and Presumptions

3.1 Presumptions

- The demand rate follows a time-dependent function represented by a non-linear ramp curve.
- The deterioration rate remains constant with no replacements during the cycle.
- The planning horizon is assumed to extend indefinitely.
- The replenishment rate is considered sufficiently large and not subject to any upper bound.
- The model centres on a single-item inventory system.
- Suppliers offer tolerable payment delays to retailers.
- Constant rate of advertisement cost is considered.
- In the first model, the permissible adjournment in payment is shorter than or falls within the inventory cycle length.

- In the second model, the permissible adjournment in payment extends beyond the inventory cycle length.

3.2 Notations

- R (t) – Deterministic Demand. $R(t)=\begin{cases} d_0+d_1t-d_2t^2\ , & 0<t\le\tau \\ d(\tau) & ,\ \tau\le t<T_i \end{cases}$

  Where $d(\tau)=d_0+d_1\tau-d_2\tau^2$ and $\tau\le\dfrac{d_1}{2d_2}$.

- h_0+h_1t - Charge of Holding Inventory per unit per time.
- CO – Charge of an order per cycle.
- AC-Advertisement Cost
- IDC – Charge of Deterioration inventory per cycle.
- ICC- Charge of carrying the inventory per cycle.
- η - Constant rate of deterioration.
- $q_{11,}q_{12,}q_2$ - EOQ of the models
- $In_{11}(t), In_{12}(t), In_2(t)$ - the level of inventory at any time t.
- IE_{11}, IE_{12}, IE_2 - Earned interest during the period of tolerable delay in payments.
- IC_{11}, IC_{12}, IC_2 - Interest charged later the time of tolerable delay in payments.
- P_d - Tolerable payment delay period
- $TIC_{11}, TIC_{12}, TIC_2$ - Aggregate inventory cost incurred per unit of time
- T_{11}, T_{12}, T_2 - The time span allocated for replenishing inventory within each cycle

4. Inventory Modelling

4.1 *Inventory Model*

$$\frac{d\mathrm{In(t)}}{dt}=-\eta\mathrm{In(t)}-D(\mathrm{t}) \tag{1}$$

$$\frac{d\mathrm{In(t)}}{dt}=-\eta\mathrm{In(t)}-(d_0+d_1t-d_2t^2), \qquad 0<t\le\tau \tag{2}$$

$$\frac{d\text{In(t)}}{dt} = -\eta \text{In(t)} - (d_0 + d_1\tau - d_2\tau^2), \qquad \tau < \text{t} \le T_1 \tag{3}$$

$$\frac{dIn(t)}{dt} = -(d_0 + d_1 t - d_2 t^2), \qquad T_1 < t \le t_1 \tag{4}$$

With the environments,

$$In(0) = q_i \quad \text{and} \quad In(T_1) = 0 \tag{5}$$

From the equations (2), (3) and (4)

$$\text{In}(t) = \left(\frac{2d_0 + d_1\eta - d_2\eta^2}{\eta^3}\right)(1 - e^{-\eta t_1}) + qe^{-\eta t_1} + \frac{d_0 t_1^2}{\eta} - \frac{t_1(2d_0 + a_1\eta)}{\eta^2} \qquad 0 < t \le \tau \tag{6}$$

$$I(t) = \frac{1}{\eta}\left(\left(d_2\tau^2 - d_1\tau - d_0\right)\left(1 - e^{-\eta t_1}\right) + \eta q e^{-\eta t_1}\right) \qquad \tau \le t < T_1 \tag{7}$$

$$I(t) = \frac{1}{\eta}\left(\left(d_2\tau^2 - d_0 - d_1\tau\right)\left(1 - e^{-\eta t_1}\right) + \eta q e^{-\eta t_1}\right) \qquad T_1 \le t < t_1 \tag{8}$$

On resolving with (5)

$$q = \frac{1}{\eta}\left(d_0 + d_1\tau - d_2\tau^2\right)\left(e^{\eta t_1} - 1\right) \tag{9}$$

(6) , (7) and (8) will become,

$$I(t) = \left(1 - e^{-\eta t_1}\right)\left(\frac{2d_0 + d_1\eta - d_2\eta^2}{\eta^3}\right) + \frac{1}{\eta}\left(d_0 + d_1\tau - d_2\tau^2\right)\left(1 - e^{-\eta t_1}\right) + \frac{d_0 t_1^2}{\eta} - \frac{t_1(2d_0 + d_1\eta)}{\eta^2} \qquad 0 < t \le \tau \tag{10}$$

$$\text{In}(t) = \left(d_0 + d_1\tau - d_2\tau^2\right)\left(e^{-\eta t_1} - 1\right) + \left(d_2\tau^2 - d_0 - d_1\tau\right)\left(e^{-\eta t_1} - 1\right) \qquad \tau \le t \le T_1 \tag{11}$$

$$I(t) = \frac{1}{\eta}\left(d_0 + d_1\tau - d_2\tau^2\right)\left(e^{\eta t_1} - t_1 - 1\right) \qquad T_1 \le t < t_1 \tag{12}$$

Inventory Carrying Cost

$$ICC = \int_0^{\tau}\left(h_1 t + h_2\right)\left(\left(1-e^{-\eta t_1}\right)\left(\frac{2d_0 + d_1\eta - d_2\eta^2}{\eta^3}\right) + \frac{1}{\eta}\left(d_2\tau^2 - d_0 - d_1\tau\right)\left(e^{-\eta t_1} - 1\right) + \frac{d_0 t_1^2}{\eta} - \frac{t_1(2d_0 + d_1\eta)}{\eta^2}\right)dt + \int_{\tau}^{T}\left(h_1 t + h_2\right)\left(\left(1-e^{-\eta t_1}\right)\left(\frac{2d_0 + d_1\eta - d_2\eta^2}{\eta^3}\right) + \frac{1}{\eta}\left(d_2\tau^2 - d_0 - d_1\tau\right)\left(e^{-\eta t_1} - 1\right) + \frac{d_0 t_1^2}{\eta} - \frac{t_1(2d_0 + d_1\eta)}{\eta^2}\right)dt \quad (13)$$

Deterioration Cost of Inventory

$$DCI = dq_i + d\int_0^{\tau}\left(d_2 t^2 - d_0 - d_1 t\right)dt + d\int_{\tau}^{T}\left(d_2\tau^2 - d_0 - d_1\tau\right)dt \quad (14)$$

Inventory Shortage Cost

$$SCI = s\int_{t_1}^{T}\left[\left(d_2\tau^2 - d_0 - d_1\tau\right)\left((T_1 - t_1\right)\right]dt \quad (15)$$

MODEL -I $P_D \le T_1$

The supplier is required to pay interest on the stored inventory at the rate *IC* whenever the credit period does not exceed the replenishment cycle. The allowable delay period can arise under two conditions: it may occur before the demand reaches its turning point, or after the turning point.

Case (i) $P_D(\text{delay period}) > \tau(\text{demand turning point})$

If the tolerable stay period befalls after the demand turning point,

Interest received during $\left[0, P_D\right]$,

$$EI_1 = PI_e \int_0^{P_D} R(T)\,dt$$

$$= PI_e\left\{d_0\tau P_d + \frac{d_1\tau}{2}\left(2P_d - d_1\right) + \frac{2d_1\tau^2}{3}\left(3P_d - \tau\right)\right\} \quad (16)$$

Interest charges due by the vendor during the period $\left[P_D, T\right]$

$$CI_1 = CI_c \int_{P_D}^{T} In(t)\,dt$$

$$= CI_c\left\{\int_{P_D}^{\tau}\left(\left(d_0 + d_1 t - d_2 t^2\right)\left(e^{-\eta t_1} - 1\right) + \left(d_0 + d_1\tau - d_2\tau^2\right)\left(1 - e^{-\eta t_1}\right)\right)dt\right\} \quad (17)$$

Case (ii) Demand turning point exceeds the tolerable delay of payment $\tau > P_D$

Interest earned during $\left[0, P_D\right]$,

$$EI_2 = PI_e \int_0^{P_D} D(t)\,dt$$

$$= PI_e \int_0^{P_D} \left(d_0 + d_1 t - d_2 t^2\right) dt \tag{18}$$

Interest cares due by the vendor during the period $\left[P_D, T\right]$

$$CI_2 = CI_c \int_{P_D}^{T} In(t)\,dt = CI_c \int_{\tau}^{T_1} \mathrm{In}(t)\,d\mathrm{t} + CI_c \int_{P_D}^{\tau} \mathrm{In}(t)\,d\mathrm{t}$$

$$= CI_c \left\{ \begin{aligned} &\int_{P_D}^{\tau} \left(\left(1 - e^{-\eta t_1}\right)\left(\frac{2d_0 + d_1\eta - d_2\eta^2}{\eta^3}\right) + \frac{1}{\eta}\left(d_0 + d_1\tau - d_2\tau^2\right)\left(1 - e^{-\eta t_1}\right) + \frac{d_0 t_1^2}{\eta} - \frac{t_1(2d_0 + d_1\eta)}{\eta^2} \right) dt \\ &+ \int_{\tau}^{T} \left(\left(d_0 + d_1\tau - d_2\tau^2\right)\left(e^{-\eta t_1} - 1\right) + \left(d_0 + d_1\tau - d_2\tau^2\right)\left(1 - e^{-\eta t_1}\right) \right) dt \end{aligned} \right\} \tag{19}$$

Inventory Cost

$$ITC_{11} = \frac{1}{T_{11}}\left(OC + AC + ICC + DCI + SCI - EI_{11} + CI_{11}\right)$$

$$= \frac{1}{T_{11}} \left\{ \begin{aligned} &k_1 + k_2 + \int_0^{\tau} \left(h_1 t + h_2\right) \mathrm{In}(t)\,d\mathrm{t} + \int_{\tau}^{T} \left(h_1 t + h_2\right) \mathrm{In}(t)\,d\mathrm{t} \\ &+ d\left(q_1 - \int_0^{\tau} \left(d_0 + d_1 t - d_2 t^2\right) dt - \int_{\tau}^{T} \left(d_0 + d_1\tau - d_2\tau^2\right) dt \right) \\ &+ s\int_{t_1}^{T} \left[\left(d_0 + d_1\tau - d_2\tau^2\right)\left((t_1 - T_1\right) \right] dt \\ &- \left(PI_e \int_0^{\tau} \left(d_0 + d_1 t - d_2 t^2\right) dt + \int_{\tau}^{P_D} \left(d_0 + d_1\tau - d_2\tau^2\right) dt \right) \\ &+ cI_c \left\{ \int_{P_D}^{\tau} \left(\left(d_0 + d_1\tau - d_2\tau^2\right)\left(e^{-\eta t_1} - 1\right) + \left(d_0 + d_1\tau - d_2\tau^2\right)\left(1 - e^{-\eta t_1}\right) \right) dt \right\} \end{aligned} \right\} \tag{20}$$

$$ITC_{12} = \frac{1}{T_{12}}\left(OC + AC + ICC + DCI + SCI - EI_{12} + CI_{12}\right)$$

$$= \frac{1}{T_{12}} \left\{ \begin{array}{l} k_1 + k_2 + \int_0^{\tau} \left(h_1 t + h_2\right) \mathrm{In}(t)\, dt + \int_{\tau}^{T} \left(h_1 t + h_2\right) \mathrm{In}(t)\, dt \\ +d\left(q_2 + \int_0^{\tau} \left(d_2 t^2 - d_0 - d_1 t\right) dt + \int_{\tau}^{T} \left(d_2 \tau^2 - d_0 - d_1 \tau\right) dt \right) \\ +s\int_{t_1}^{T} \left[\left(d_2 \tau^2 - d_0 - d_1 \tau\right)\left((t_1 - T_1\right) \right] dt + PI_e \int_0^{P_D} \left(d_2 t^2 - d_0 - d_1 t\right) dt \\ +cI_c \left\{ \begin{array}{l} \int_{P_D}^{\tau} \left(\left(1 - e^{-\eta t_1}\right)\left(\frac{2d_0 + d_1 \eta - d_2 \eta^2}{\eta^3} \right) + \frac{1}{\eta}\left(d_2 \tau^2 - d_0 - d_1 \tau\right)\left(e^{-\eta t_1} - 1\right) + \frac{d_0 t_1^2}{\eta} - \frac{t_1 (2d_0 + d_1 \eta)}{\eta^2} \right) dt \\ + \int_{\tau}^{T} \left(\left(d_2 \tau^2 - d_0 - d_1 \tau\right)\left(1 - e^{-\eta t_1}\right) - \left(d_2 \tau^2 - d_0 - d_1 \tau\right)\left(1 - e^{-\eta t_1}\right) \right) dt \end{array} \right. \end{array} \right\} \tag{21}$$

Model –II $P_D > T_2$

The model-I appropriate delay time, where the acceptable delay period is longer than the inventory cycle duration.

The renewal cycle length is less than the tolerable stay period, then the interest that will be charged becomes zero.

Interest earned during $\left[0, P_D\right]$,

$$EI_2 = PI_e \left\{ \int_0^{T_2} D\left(t\right) t\, dt + \int_{T_2}^{P_D} D\left(t\right)\left(P_D - T\right) dt \right\} \tag{22}$$

$$= PI_e \left\{ \int_0^{\tau} \left(d_0 + d_1 t - d_2 t^2\right) t dt - \int_{\tau}^{T_2} \left(d_2 \tau^2 - d_0 - d_1 \tau\right) t dt + \int_{T_2}^{P_D} \left(T - P_D\right)\left(d_2 \tau^2 - d_0 - d_1 \tau\right) dt \right\} \tag{23}$$

Interest charged during the time period $\left[0, P_D\right]$,

$$CI_2 = 0 \tag{24}$$

$$ITC_2 = \frac{1}{T_2}\left(OC + AC + ICC + IDC + SCI - EI_2 + CI_2\right)$$

$$= \frac{1}{T_2} \left\{ \begin{array}{l} k_1 + k_2 + \int_0^{\tau} \left(h_1 t + h_2\right) \mathrm{In}(t)\, dt + \int_{\tau}^{T_2} \left(h_1 t + h_2\right) \mathrm{In}(t)\, dt \\ +d\left[q_2 + \int_0^{\tau} \left(d_2 t^2 - d_0 - d_1 t\right) dt + \int_{\tau}^{T_2} \left(d_2 \tau^2 - d_0 - d_1 \tau\right) dt \right] + s\int_{T_1}^{t_1} \left(\left(d_2 \tau^2 - d_0 - d_1 \tau\right)\left((T - t_1\right)\right) dt \\ +PI_e \left\{ \int_0^{\tau} t\left(d_2 t^2 - d_0 - d_1 t\right) dt + \int_{\tau}^{T_2} t\left(d_2 \tau^2 - d_0 - d_1 \tau\right) dt + \left(P_D - T\right) \int_{T_2}^{P_D} \left(d_2 \tau^2 - d_0 - d_1 \hat{o}\right) dt \right\} \end{array} \right\} \tag{25}$$

4.2 *Methodology of Solving*

To achieve the optimum overall cost, the model requires that the first-order derivatives of the total cost function with respect to the decision variables to be zero, ensuring that the gradient vanishes at the optimum. Additionally, to confirm a local minimum, the Hessian matrix of the total cost curve must be positive definite or at least positive semidefinite in the case of a weak minimum. The resulting first-order conditions are highly nonlinear in the time variables t and T, obtaining closed-form solutions is generally unrealistic. The model is solved using a numerical approach: realistic parameter values are assigned, and MATLAB R2020 is employed to calculate the ideal time cycle and the associated decision variables.

5. Numerical Examples

Several numerical examples are provided to demonstrate the sensibleness and functionality of the proposed inventory model. Every illustration utilizes separate parameter value sets, allowing an investigation of how changes in input elements influence the final outcomes. This method offers a distinct view of the model's performance over various possible scenarios, emphasizing its versatility and efficiency in diverse operational environments.

Example: 1

The parameters values as $d_0 = 50$, $d_1 = 200$, $d_2 = 100$, $\tau = 0.75$, $d = 20$, $p = 50$, $h_1 = 0.5$, $h_2 = 3$, $P_d = 1$, $\eta = 0.02$ $i_e = 0.08$, $k_1 = 500$, $k_2 = 1000$, $c = 30$, $i_c = 0.25$, $s = 30$, The model produces the best cycle duration, the lowest total expense, and the related ideal order quantity $T_{11}^* = 1.8577$, $TIC_{11}^* = 689.39$ and $q_{11}^* = 205.42$. These crucial outcomes assist in determining the optimal operating conditions for the inventory system

Example: 2

The parameter values as $d_0 = 50$, $d_1 = 200$, $d_2 = 90$, $\eta = 0.02$, $\tau = 0.75$, $h_1 = 0.5$, $d = 20$, $h_2 = 3$, $k_1 = 500$, $k_2 = 1000$, $P_d = 0.35$, $i_e = 0.08$, $s = 30$, $i_c = 0.25$, $p = 50$ $c = 30$, The analysis identifies the ideal cycle length, the lowest possible total cost, and the related order quantity that guarantees optimal system efficiency $T_{12}^* = 2.334$, $TIC_{12}^* = 746.88$ and $q_{12}^* = 102.55$.

Example: 3

Providing the parameter values as $d_0 = 50$, $d_1 = 200$, $d_2 = 100$, $\eta = 0.02$, $\tau = 0.75$, $h_1 = 0.5$, $d = 20$, $p = 50$, $s = 30$, $h_2 = 3$, $k = 500$, $c = 30$, $i_c = 0.25$, $i_e = 0.08$, $P_d = 1.8$, The assessment results in identifying the ideal restocking frequency, the cost threshold at which the system functions most effectively, and the order size that reaches this ideal $T_2^* = 3.3475$, $TIC_2^* = 887.28$ and $q_2^* = 285.22$.

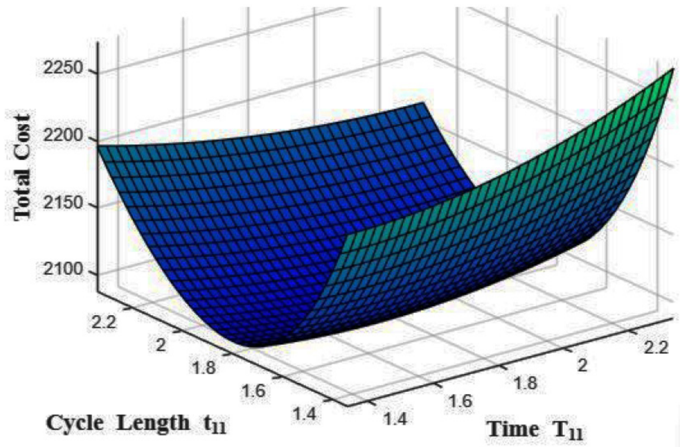

Figure 1

Model-I Inventory Total Cost Representation -case (i)

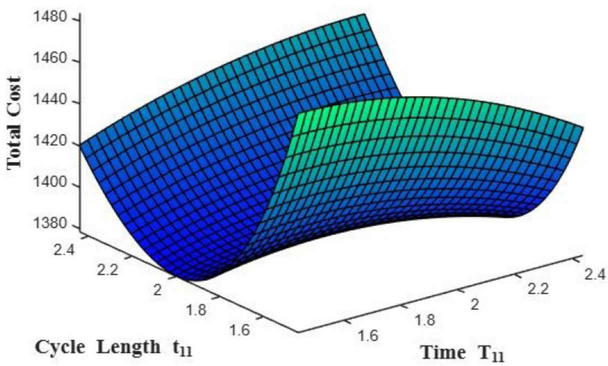

Figure 2

Model-I Inventory Total Cost Representation- case (ii)

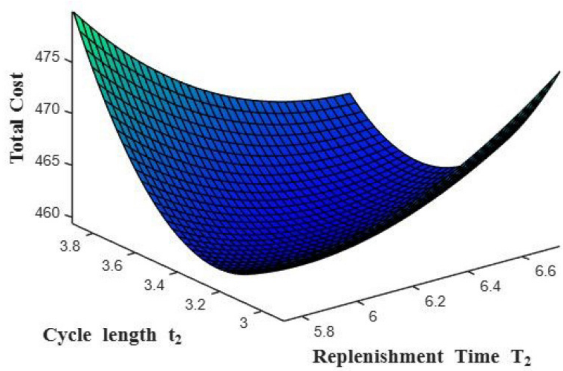

Figure 3

Model-II Inventory Total Cost Representation

When we looked at the three numerical examples side by side, it was pretty clear that Example 1 worked out as the best setup for the inventory system. It gives the lowest total cost with a decent cycle length and order quantity, so it feels like a well-balanced restocking plan. Example 2 doesn't do as well, the cycle time is a bit longer and the total cost also goes up, so it's not as efficient as the first one. Example 3 performs the weakest, with the biggest cycle duration and the highest overall cost among all three cases. So, from these results, Example 1 ends up giving the most cost-effective and practical approach under the given conditions. Also, from Figures 1, 2, and 3, we can see that the total-cost curves have a convex pattern. This shape basically shows that there is only one minimum point for each case, which helps confirm that the optimal solutions we calculated are valid for the different inventory settings.

5. Conclusion

This study gives useful guidance for both sellers and buyers in choosing suitable payment timings by making good use of allowable credit periods. Once the best replenishment time is identified, the total cost can be worked out through standard analytical steps, and the numerical examples show that the model can be applied in real inventory settings. The model can be adjusted for demand that increases slowly, depends on stock levels, or changes with price. Shortages can also be included to see how backorders affect system performance. More realistic deterioration patterns, like Weibull or Gamma forms, may also improve accuracy. Looking at such patterns together with trade-credit terms may offer deeper insights for industries facing uncertain markets and flexible payment conditions.

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