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An inverse model for evaluating the power industry's supply chain

Mojgan Pouralizadeh
Department of Applied Mathematics
Lahijan Branch
Islamic Azad University
Lahijan
Iran

Abstract

In the clean development mechanism, greenhouse gas (GHG) reduction is conducted by implementing projects that aim to decrease fossil fuel consumption in the energy sectors, increase energy productivity, and mitigate energy loss in power plants and transmission networks and distribution systems. This; the Research evaluated the sustainability of the electrical supply chain using inverse input-oriented data envelopment analysis. The; proposed inverse input-oriented model estimates the optimal value of applied resources based on the optimal allocation of two categories of inputs while other production indexs and efficiency scores of supply chain divisions under evaluation remained unchanged. An empirical conclusion was yielded from the model's performance in the ten electrical supply chains and 15 divisions. The gas field of supply chains, 60% utilize a large amount of gas resources in power production. Also; power plant sectors and transmitter lines of more than 90% of the supply chains produced an optimal value of energy.

Subject Classification: *90B06.*

Keywords: *Optimal allocation, Environmental efficiency, Inverse model, Optimal resources, Economical return.*

1. Introduction

Carbon dioxide (CO_2) must be eliminated from the Earth's atmosphere to prevent climate change and limit global warming to 1.5 degrees Celsius. The current paper contributes to this line of research by introducing the inverse input-oriented data envelopment analysis (DEA) model with two inputs and outputs categories and factors that play two roles input and

E-mail: **pouralizadehmojgan@yahoo.com**

output for the electricity supply chain. The; inputs are divided into two categories in this study resource allocation management and investment approtunities control. The inverse input-oriented model determines the value of optimal increase or decrease applied resources in energy sectors and transmitter and distributer lines incontrast, produced energy, pollution emissions, costs of power loss inhibition, and efficiency scores of supply chain divisions are kept unchanged. Indeed; the inverse input-oriented (DEA) model determines the amount of optimal applied resources and accessible capacities under natural disposability based on the amount optimal of utilizated investment capacities to flare and greenhouse gases reduction and wasted energy control while other production indexes remained constant. In such circumstances, it is essentially necessary to know how optimal resource allocation influences the investment opprotunities capacities while the other production indexs, especially desirable and undesirable outputs of supply chain divisions, are kept constant. In addition; we are interested in knowing which supply chain divisions applied the optimal resources based on optimal investment opportunities in the power industry. Indeed; supply chain management enables resources allocation control and proper utilization of investment approtunities when the input-oriented efficiency is kept constant. The technical and non-technical electricity consumption of power plants, the scientific level of education of staff for the power loss harness in transmission operations, and the return costs of wasted energy and flare gas to the gas cycle are other dual-role factors that determined.

As a result of this study, it will be possible to determine the optimal use of resources influences excess capacities control and create sustainability of the supply chain. Also; if supply chain management aims to apply resources optimization, then how will it utilize investment resources to decrease undesirable emissions and waste energy in supply chain divisions? DEA; is a successful operationalizing strategy in the sustainability assessment of decision-making units (DMU). DEA created by Charnes et al. [2] to assess the units' relative effeciency. The proposed model was then expanded by Banker et al. [1] for measuring the variable return scale (VRS). Ghayeghran et al.[5] presented a methodology for the sugar industry's cost efficiency analysis. The initial mention of inverse models found in Zhang et al. [26]. They suggested a model that could solve a DMU's input increments and provide output increments under constant return scale (CRS).

This paper's remainder is as follows: An overview of the literature on the application of DEA to investigational work provided in section 2.

Moreover; a DEA model for resource utilization and investment in technology innovation described. Additionally, an exact specification of natural and managerial disposability demonstrated by a production process of supply chain performance evaluation. The inverse input-oriented model then introduced in another section, which aims to determine the optimal resource allocation for supply chain divisions'emission management. A case study is provided In Section 4, to demonstrate how the recommended methodology can implemented in Iran's power sector. The conclusions finally presented in Section 5

2. Review of Literature

The subsequent, subsection provides a summary of several studies on supply chain sustainability, environmental, and operational assessment,and inverse DEA models.

2.1 *Inverse DEA Models*

The inverse DEA methodology was Introduced by Wei et al. [23]. They; posed the question of how much more output or input must be produced if input or output for a specific within a DMU group increased, and if it assumed that the DMU maintains current efficiency level in comparison to other units how much more output or input must be produced by a particular unit among a group of DMUs. Yan et al. [24] expanded the inverse model for resource reallocation and production estimation. Rad; et al. [17] introduced an inverse DEA input-oriented model for output estimation using fuzzy data. To maintain efficiency values with the VRS, Lertworasirikul et al. [12] proposed an inverse model that could handle positive data. In the proposed model, resource allocation computed, and the efficiency scores remained unchanged. Jahanshaloo et al. [10] expanded on Yan et al.'s proposed inverse input-oriented model [24] for input estimation and identification of extra input. A generalized inverse model for changes estimation of inputs or outputs presented by Hadi-Vencheh et al.[6]. An inverse model incorporating stochastic components presented by Han et al. [7]. An; integration and extension of the inverse DEA models created by Zhang and Cui [25]. Hassenzadeh et al. [8] suggested using inverse input-oriented and output-oriented models to assess a nation's operational and environmental sustainability.

2.2 *Evaluation of the Environmental and Operations*

Glover and Sueyoshi [3] studied the history of the DEA, beginning with the situation of Banker [1] in the nineteenth century, and then used the idea of natural and managerial disposability as a conceptual foundation for previous studies in order to incorporaite it into environmental assessment technology and manage substances abuse prevention and determine effects on productivity (see, for example, [18, 19]). For; the operational and environmental assessment of Japan's industrial sectors, Sueyoshi et al.[20] suggested using the stage-DEA model

2.3 *Sustainability of the Supply Chains*

A slacks-based measure network DEA model called network (NSBM) was proposed by Ton et al. [22]. Tavana et al. [21] extended Ton et al. [22], proposed epsilon-based measure (EBM) and suggested a network epsilon best measure (NEBM).

Khodakerami et al.[11] proposed a unique two-stage methodology for resin-production companies'supply chain. Since; some real-world problems required the use of fuzzy sets in the DEA model to address them, the authors took into consideration evaluating the performance of imprecise and uncertain situations. Pouralizadeh; et al.[13] proposed a new methodology for resources management and emissions handling of divisions in an electricity supply chain. Additionally, activity level control in production activities was implemented using the weak disposability assumption. The authors controlled undesirable output by determining the kind and quantity of inputs. A novel DEA model that takes dual-role factor aspects into account to improve the sustainability of the electrical supply chain introduced by Pouralizadeh [14]. This model was able to identify whether increasing inputs under managerial disposability to new technology innovation could reduce undesirable production in the supply chain divisions or whether the higher investments were ineffective in decreasing undesirable outputs. Pouralizadeh; [15] suggested two models for managing pollution emissions and reducing resource waste for the sustainability evaluation of the electrical supply chain. Pouralizadeh [16] presented a model to estimate the marginal profit maximization of desirable output. The proposed model is introduced for estimating the directional marginal profit maximization of suply chain divisions based on wasted energy and power losses. The proposed approach estimates the directional marginal productivity in the supply chains, which find the optimal direction of efficient divisions on the frontier. A model proposed by Farzipoor [4] to select a third-party logistics providers when dual-role factors and fuzzy data are present.

In conclusion, the inverse DEA input-oriented model based on dual-role components in the presence of undesirable outputs considered consideration by any of the references indicated above for the supply chain performance assessment.

2.4 *Fundamental Concepts*

In this section, fundamental concepts for the approach to calculating inverse input and output-oriented DEA models of supply chain divisions reported.

Let us suppose $X_n = (x_{1n}, x_{2n}, ..., x_{Mn})^T > 0$, $Y_n = (y_{1n}, y_{2n}, ..., y_{Rn})^T > 0$, $B_n = (b_{1n}, b_{2n}, ..., b_{Jn})^T > 0$ represent input column vectors and desirable and undesirable outputs in the n[th] DMU.

2.4.1 An inverse DEA input-oriented

The inverse DEA input-oriented model's goal is to allocate resources while maintaining an unchanged efficiency score.

Assume that m[th] input under natural disposability of kth DMU is changed from $\overline{x}_{mk}$ to $\overline{x}_{mk} + \Delta\overline{x}_{mk}$ and $\Delta\overline{x}_{mk}$ an optimal variation of mth input of DMU_K and q[th] input under managerial disposability of k[th] DMU is changed from $\tilde{x}_{mk}$ to $\tilde{x}_{mk} + \Delta\tilde{x}_{mk}$ as Δ can be positive or negative.

3. Modeling of DEA Input-oriented Efficiency of Supply Chain Divisions

This section presented a DEA model to evaluate supply chains'sustainability. Assume that a supply chain Includes the fuel provider, energy producer, regional power station, power dispatcher, and power consumers. Therefore; a supply chain (or DMU) comprises five steps. There are e_s, e_m, e_t, e_d, e_c divisions in the fuel supplier, power plant, transmitter, distributor, and customer. This study considered the weights that producer's activity decision-makers assigned to partners at a particular stage of the network supply chain, also known as $W_e, (e = 1, \ldots, E)$ weights for the H division (see Pouralizadeh [15]). The following is a summary of the n[th] supply chain's production factors.

$\overline{X}_n^e = (\overline{x}_{1n}^e, \overline{x}_{2n}^e, ..., \overline{x}_{M^-n}^e)^T > 0$: A column vector of the n[th] supply chain M^- inputs for the e[th] division under natural disposability $e = 1, ..., E, n = 1, ..., N$.

$\tilde{X}_j^e = (\tilde{x}_{1j}^e, \tilde{x}_{2j}^e, \ldots, \tilde{x}_{M^+j}^e)^T > 0$: A column vector of the n^th supply chain M^+ inputs for the e^th division under managerial disposability $e = 1,\ldots,E, n = 1,\ldots,N$.

$Y_n^e = (y_{1n}^e, y_{2n}^e, \ldots, y_{Rn}^e)^T > 0$: A column vector of the n^th supply chain R desirable outputs for e^th division $e = 1,\ldots,E, n = 1,\ldots,N$.

$B_n^e = (b_{in}^e, b_{2n}^e, \ldots, b_{Jn}^e)^T > 0$: A column vector of the n^th supply chain J undesirable outputs for e^th division $e = 1,\ldots,E, n = 1,\ldots,N$.

$W_n^e = (w_{1n}^e, w_{2n}^e, \ldots, w_{Dn}^e)^T > 0$: A column vector of the n^th supply chain D dual-role factors for the e^th division $e = 1,\ldots,E, n = 1,\ldots,N$.

$V_n^{(h,h')} = (v_{1n}^{(e,e')}, v_{2n}^{(e,e')}, \ldots, v_{Pn}^{(e,e')})^T > 0$: A column vector of the n^th supply chain P intermediate measures that are sent from the division e to the division e' $e = 1,\ldots,E, n = 1,\ldots,N$.

$Z_n^{(e',e)} = (z_{1n}^{(e',e)}, z_{2n}^{(e',e)}, \ldots, z_{An}^{(e',e)})^T > 0$: A column vector of of the n^th supply chain A intermediate measures that are sent from the division e' to the division e $e = 1,\ldots,E, n = 1,\ldots,N$.

$s_{pn}^{(e,e')}$: The slack variables of the n^th supply chain for the p^th intermediate measure, spanning from division e to division e' $p = 1,\ldots,P, n = 1,\ldots,N$.

$s_{an}^{-(e',e)} \geq 0$: The input slack variable of the n^th supply chain for the a^th intermediate measure in the n^th supply chain which goes from division e' to division e $a = 1,\ldots,A,\ n = 1,\ldots,N$.

$s_{an}^{+(e',e)} \geq 0$: The output slack variable of the n^th supply chain for the a^th intermediate measure, which goes from the division e' to the division e $a = 1,\ldots,A, n = 1,\ldots,N$.

$\Lambda^e = (\lambda_1^e, \lambda_2^e, \ldots, \lambda_n^e)^T$: An unknown column vector.

ξ_m^e: Efficiency score of ^m th input from the e^th division.

Consequently, a weighted average of the input efficiency scores of the supply chain divisions in production processes used to calculate the overall supply chain's efficiency, as shown by the model (2).

$$\theta = Min\ \ \omega_e\left(\frac{1}{M_e^-}\sum_{i=1}^{M^-}\xi_m^e\right)$$

$$\sum_{n=1}^{N}\overline{x}_{mn}^e\lambda_n^e \leq \xi_m^e\,\overline{x}_{mk}^e \qquad m = 1,\ldots,M_e^-,\ e = 1,\ldots,E$$

$$\sum_{n=1}^{N}\tilde{x}_{qn}^e\lambda_n^e \geq \tilde{x}_{qk}^e \qquad q = 1,\ldots,M_e^+, e = 1,\ldots,E$$

$$\begin{array}{lll}
\sum_{n=1} y^{e}_{rn}\lambda^{e}_{n} & \geq y^{e}_{rk} & r=1,\dots,R_e, e=1,\dots,E \\
\sum_{n=1}^{N} b_{jn}\lambda^{e}_{n} & \leq b^{e}_{jk} & j=1,\dots,J_e, e=1,\dots,E \\
\sum_{n=1}^{N} w^{e}_{dn}\lambda^{e}_{d} & = w^{e}_{dk} & d=1,\dots,D_e, e=1,\dots,E \\
\sum_{n=1}^{N} \lambda^{e}_{n} v^{(e,e')}_{pn} + s^{(e,e')}_{p} & = \sum_{j=1}^{n} \lambda^{e'}_{n} v^{(e,e')}_{pn} & \forall(e,e'),\ \forall p=1,\dots,P \\
\sum_{n=1}^{N} \lambda^{e'}_{n} z^{(e',e)}_{aj} - s^{+(e',e)}_{a} & = z^{(e',e)}_{ak} & \forall(e',e), (e'=e_m, e=e_s), (e'=e_t, e=e_m), a=1,\dots,A \\
\sum_{n=1}^{N} \lambda^{e}_{n} z^{(e',e)}_{aj} + s^{+(e',e)}_{a} & = z^{(e',e)}_{ak} & \forall(e',e), (e'=e_m, e=e_s),\ (e'=e_t, e=e_m), a=1,\dots,A \\
\sum_{n=1}^{N} \lambda^{e}_{n} & = 1 & n=1,\dots,N,\ e=1,\dots,E \\
\lambda_n \geq 0,\ s^{+(e,e')}_{a} \geq 0,\ s^{-(e,e')}_{a} \geq 0,\ s^{(e,e')}_{p},\ \xi\ UR, & & n=1,\dots,N,\ e=1,\dots,E
\end{array} \tag{2}$$

The supply chain objective function computed by taking a weighted average of the optimal input-oriented efficiency of supply chain divisions. In addition, the optimal quantity of objective function model 2 established by averaging the sum of the efficiency components of inputs under natural disposability. Constraints in the first and second categories correspond to inputs set under natural and managerial disposability. Additionally, the fifth category of restrictions relates to dual-role factors of the supplier, manufacturer, and transmitter divisions, whereas the third and fourth categories are related to desirable and undesirable outputs. The intermediate measures transferred from the supplier divisions to manufacturer's divisions, manufacturer's divisions to transmitter divisions, the transmitter divisions to distributor divisions, and finally, the customer divisions correspond to the sixth category constraints.

The intermediate measures that leave power plant divisions and enter supplier divisions as well as the intermediate measures that leave transmitter divisions and enter power plant divisions are the subject of the seventh and eighth category constraints. The production return to scale is the subject of the final category of constraints.

3.3 *The Inverse DEA input-oriented Model of Supply Chain*

This section emphasizes the inverse input-oriented approach for a power sector's supply chain when dealing with under variable to return to scale.

3.3.1 The proposed Inverse input-oriented DEA Model

To formulate an inverse model, suppose the rate of the variations in production factors is defined as follows:

$\Delta\bar{X}_n^e = (\Delta\bar{x}_{1e}^e, \Delta\bar{x}_{2n}^e, ..., \Delta\bar{x}_{m^-n}^e)^T > 0$: A column vector of the n[th] supply chain representing m^- inputs changes for the e[th] division under natural disposability $e = 1,...,E,\ n = 1,...,N.$

$\Delta\tilde{X}_n^h = (\Delta\tilde{x}_{1n}^e, \Delta\tilde{x}_{2n}^e, ..., \Delta\tilde{x}_{m^+n}^e)^T > 0$: A column vector of the n[th] supply chain representing m^+ inputs changes for the e[th] division under managerial disposability $e = 1,...,E,\ n = 1,...,N.$

$\Delta Y_n^e = (\Delta y_{1n}^e, \Delta y_{2n}^e, ..., \Delta y_{Rn}^e)^T > 0$: A column vector of the n[th] supply chain representing R desirable outputs changes for the e[th] division $e = 1,...,E,\ n = 1,...,N$

$\Delta B_n^e = (\Delta b_{1n}^e, \Delta b_{2n}^e, ..., \Delta b_{jn}^e)^T > 0$: A column vector of the n[th] supply chain representing J undesirable outputs changes for the e[th] division of $e = 1,...,E,\ n = 1,...,N$

Following the assessment of the supply chain's optimal efficiency using Model (2), the efficiency scores is now incorporated into Model (3), which computes changes to the two categories of inputs, for the DMU (supply chain) that examined by the inverse DEA input-oriented model (3). In addition, the suggested inverse DEA input-oriented model can determine the optimal variation of two input categories for DMU while assuming that the DMU keeps its current efficiency level compared to other DMUs. Additionally, the weighted average of the optimal input changes under the natural disposability of each division of the supply chain used to calculate the objective function of the inverse input-oriented model for supply chain evaluation

$$
\begin{aligned}
& Min \sum_{e=1}^{E} \omega_e \Delta\bar{x}_{mk}^e \\
& \sum_{n=1}^{N} \bar{x}_{in}^e \lambda_n^e && \le \xi_m^e (\bar{x}_{mk}^e + \Delta\bar{x}_{mk}^e) && m = 1,...,M_h^-,\ e = 1,...,E \\
& \sum_{n=1}^{N} \tilde{x}_{qn}^e \lambda_n^e && \ge (\tilde{x}_{qk}^e + \Delta\tilde{x}_{qk}^e) && q = 1,...,M_h^+,\ e = 1,...,E \\
& \sum_{n=1}^{N} y_{rn}^e \lambda_n^e && \ge y_{rk}^e && r = 1,...,R_e, e = 1,...,E \\
& \sum_{n=1}^{N} b_{jn} \lambda_n^e && \le b_{jk}^e && j = 1,...,J_e, e = 1,...,E \\
& \sum_{n=1}^{N} w_{dn}^e \lambda_d^e && = w_{dk}^h && d = 1,...,D_e, e = 1,...,E \\
& \sum_{n=1}^{N} \lambda_n^e v_{pn}^{(e,e')} + s_p^{(e,e')} && = \sum_{j=1}^{n} \lambda_n^{e'} v_{pn}^{(e,e')} \forall (e,e'),\ \forall p = 1,...,P
\end{aligned}
$$

$$
\begin{array}{lll}
\sum_{n=1}^{N} \lambda_n^{e'} z_{aj}^{(e',e)} - s_a^{+(e',e)} & = z_{ak}^{(e',e)} & \forall (e',e), (e'=e_m, e=e_s), (e'=e_t, e=e_m), a=1,\dots,A \\
\sum_{n=1}^{N} \lambda_n^{e} z_{aj}^{(e',e)} + s_a^{+(e',e)} & = z_{ak}^{(e',e)} & \forall (e',e), (e'=e_m, e=e_s), (e'=e_t, e=e_m), a=1,\dots,A \\
\sum_{n=1}^{N} \lambda_n^{e} & = 1 & n=1,\dots,N,\ e=1,\dots,E \\
\lambda_n \ge 0,\ s_a^{+(e,e')} \ge 0,\ s_a^{-(e,e')} \ge 0,\ s_p^{(e,e')},\ \xi\ UR, & n=1,\dots,N,\ e=1,\dots,E &
\end{array} \tag{3}
$$

Constraints in the first and second categories relate to inputs that fall under natural and managerial disposability. Variations of the mth input under natural disposability from $\bar{x}_{mk}^{e}$ to $\bar{x}_{mk}^{e} + \Delta\bar{x}_{mk}^{e}$ and variations of the qth input under managerial disposability from $\tilde{x}_{qk}^{e}$ to $\tilde{x}_{qk}^{e} + \Delta\tilde{x}_{qk}^{e}$ for the eth division in the kth supply chain are indicated by right-hand side. Since Δ is free under signal, an optimal solution may contain values of either positivity or negativity.

4. A Real Case of the Power Industry

In this section, we analyze the Iran power industry using the suggested model. The next subsection will describe the dataset,and the results indicated in the following subsection.

4.1 *Dataset*

Ten supply chains (DMUs) considered consideration during the application phase. Throughout the application process, supply chains, which include oil and gas fields that supply a variety of fuels to power plants, regional power companies, distributors, and customers, considered. The supply chain, two suppliers were considered: oil and gas fields that met the power plant's intermediate fuel requirements and sold fuel as the end product. Suppliers used capital input under natural disposability and one labor input under managerial disposability in the suggested model. One desired output such as oil or gas, and one undesirable output, such as flare gas, were produced by the supplier. The cost of cleaning up flare gas pollution was taken into consideration as the dual-role factor. Each manufacturer employs three or more power plants, each with a the different technologies such as thermal, combined cycle, gas, hydro, wind, and solar, and sells their products to surrounding industries. These power plants used labor, capital, and fuels under natural and labor of hydropower plants under managerial disposability to produce electricity. The electricity consumption of power plants, which comprise both technical and non-technical usage,is referred to as a dual-role factor since it plays both an input and an output role; The transmitters transfer electricity from producers to distributors, the capacity and length of the lines considered as the inputs under natural and the number employees of the department of programming and researches used as input under management disposability. For power plants three, undesirable outputs, carbon dioxide, nitrogen oxides, and sulfur oxides emissions into consideration. The specialized workforce in the programming was the dual-role factor, although building new lines was considered desirable output. The loss of the transmission lines deemed undesirable output, transmission companies supplied electricity to distribution companies, which then distributed it to end users. Two capital inputs used, which estimated as the distribution lines'length and capacity under natural disposability, and the number of engineering assistance departments and programming estimated under managerial disposability. The desirable output identified as the electricity meter, and the undesirable output was identified as distribution line losses. Lastly, residential, agricultural, public, and industrial customers were categorised. (TAVANIR) website [9] of the Iran Power Generation and Transmission Corporation has access to all the information supply chain divisions.

Additional information on the characteristics of the supply chain divisions further explained in the following.

h_e : The number of divisions in the supplier level (h_e: 1, 2).

$\overline{x}_{1n}^{e(s)}$: The nth supply chain' oil and gas fields capacity for the h_eth supplier.

$\tilde{x}_{1n}^{e(s)}$: The number of workers in nth supply chain for the e_s supplier.

$y_{1n}^{e(s)}$: The nth supply chain' sales of Oil and gas to other businesses for the e_sth supplier.

$b_{1n}^{e(s)}$: The nth supply chain' the oil field (10^3 barrels) and gas field ($10^6 m^3$) flaring gas for the e_sth supplier.

$w_{1n}^{e(s)}$: The nth supply chain' the price of clearing of burned gas (flaring gas) for the e_sth supplier.

e_m : The number of divisions in the manufacturer level (e_m: 3, 4, and 5).

$\overline{x}_{1n}^{e(m)}$: The nth supply chain' power nominal for the e_mth manufacturer (10^6 kWh).

$\overline{x}_{2n}^{e(m)}$: The nth supply chain' the total number of employees for the $e_m{}^{th}$ manufacturer

$\tilde{x}_{1n}^{e(m)}$: The nth supply chain' the number workers for e_mth manufacturer hydropower.

$y_{1n}^{e(m)}$: The nth supply chain'the proportion of recently built power plants for the $e_m{}^{th}$ manufacturer

$b_{1n}^{e(m)}$: The nth supply chain' NO_X emissions for the $e_m{}^{th}$ manufacturer (10^3kg/10^6kWh).

$b_{2n}^{e(m)}$: The nth supply chain' SO_X emissions for the $e_m{}^{th}$ manufacturer (10^3kg/10^6kWh).

$b_{3n}^{e(m)}$: The nth supply chain' CO_2 emissions for the $e_m{}^{th}$ manufacturer(10^3 kg/10^6 kWh).

$w_{1n}^{e(m)}$: The nth supply chain' the manufacturer's internal consumption (technical and non-technical consumptions) for the $e_m{}^{th}$ power plant10^6 kWh).

e_t : The number of divisions in the level of the transmitter (e_t : 6, 7).

$\overline{x}_{1n}^{e(t)}$: The nth supply chain' transmission lines capacity for the $e_t{}^{th}$ transmitter(MWa).

$\overline{x}_{2n}^{e(t)}$: The nth supply chain' transmission line length for the $e_t{}^{th}$ transmitter (km circuit).

$\tilde{x}_{1n}^{e(t)}$: The nth supply chain' the total number of workers for the $e_t{}^{th}$ transmitter's department

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$y_{1n}^{e(t)}$: The nth supply chain's newly constructed transmission lines for the $e_t{}^{th}$ transmitter (km circuit).

$b_{1n}^{e(t)}$: The nth supply chain' transmitter lines loss for the $e_t{}^{th}$ transmitter (10^6 kWh).

$w_{1n}^{e(t)}$: The quantity of the deputy workers involved in the exploitation and transfer of the $e_t{}^{th}$ transmitter in

the nth supply chain.

e_d : The number of divisions in the level of distributer (e_d: 8, 9, 10, and 11).

$\overline{x}_{1n}^{e(d)}$: The nth supply chain' distribution lines capacity for the e_dth distributer in (MVa).

$\overline{x}_{2n}^{e(d)}$:The nth supply chain' Distribution line length for the e_dth distributer (km).

$\tilde{x}_{1n}^{e(d)}$: The total number of workers in the e_d th distributer's department engineering assistance and programming in the jth supply chain.

$y_{1n}^{e(d)}$: The nth supply chain'electricity meter for the e_dth distributor

$b_{1n}^{e(d)}$: The nth supply chain' distribution line's percentage of losses for the e_dth distributer (%).

e_c : The number of divisions in the level of the customer (e_c : 12, 13, 14, and 15).

$\overline{x}_{1n}^{e(c)}$: The nth supply chain' average expense for the e_cth customer, including fuel subsidy of (USD).

$\tilde{x}_{1j}^{h(c)}$: Direct sale of power to the e_cth customer in the nth supply chain by the transmitter Company (10^6 Kwh).

$y_{1n}^{e(c)}$: The n^{h} supply chain' total number of clients for the e_cth customer.

$y_{2n}^{e(c)}$: The nth supply chain' sales of power for the e_cth client (10^6 kWh).

$b_{2n}^{e(c)}$: The nth supply chain' disconnect for the e_cth customer's power (minute/year).

$v_{pj}^{(e,e')}$: Transportation of materials from the e division to e' division (10^6 kVA).

$z_{aj}^{(e,e')}$: Power sent from the e^{th} division to e' th division (10^6 kVA).

4.2 *Results*

First, model 2 was employed to estimate the input- oriented efficiency score of the ten supply chains (DMU) under VRS, Table 1 lists the outcomes.

Based on vaiable returns to scale, Tables 1'first column displays the supply chains's overall efficiency score. According to Table 1, supply chain

Table 1

The input-oriented efficiency score of supply chains (DMUs) under VRS.

DMU	θ_o	ξ_k^{S1}	ξ_k^{S2}	ξ_k^{M1}	ξ_k^{M2}	ξ_k^{M3}	ξ_k^{T1}	ξ_k^{T2}	ξ_k^{D1}	ξ_k^{D2}	ξ_k^{D3}	ξ_k^{D4}	ξ_k^{C1}	ξ_k^{C2}	ξ_k^{C3}	ξ_k^{C4}
1	0.977	1	0.99	1	1	1	1	1	0.71	1	0.64	1	1	1	1	1
2	0.962	1	0.84	0.74	1	1	1	1	1	0.85	1	1	1	1	1	1
3	0.986	1	0.83	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1.000	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	0.999	1	1	1	1	1	1	1	1	1	0.96	1	1	1	1	1
6	0.986	1	0.83	1	1	1	1	1	0.87	1	1	1	1	1	1	1
7	0.946	0.56	0.77	1	1	1	1	1	1	1	1	1	1	1	1	1
8	0.965	1	0.84	1	1	1	1	0.88	1	0.69	1	1	1	1	1	1
9	0.978	1	1	1	1	1	1	1	0.71	1	0.64	1	1	1	1	1
10	0.999	1	1	1	1	1	1	1	0.97	1	1	1	1	1	1	1

number 4 obtained the highest efficiency score (1) under VRS. Moreover, 15 divisions of supply chain 4 were efficient under VRS, while DMU7, with an efficiency score (0.946) had 13 efficient divisions. Looking vertically across the table reveals that the second and third power plants in the ten supply chains, the first transmitter, and power customers of the residential, public, agricultural, and industrial divisions, were efficient under VRS. Now, efficiency scores of supply chain divisions are incorporated into the model (3) to determine optimal resource allocation and the simultaneous variation of production factors. Tables 2-8 indicate variation in two categories of inputs, and the new values after allocation for oil and gas fields, power plants and transmitter and distributer lines in 10 supply chains. In Table 2, columns 2 to 5 report the changes in the two categories of inputs of suppliers 1, and 2, and columns 6 to 9 show the new quantities of the two categories of inputs of oil and gas fields in 10 electricity supply chains.

In this way, we can exploit the minimal applied capacities that create investment opportunities variation for 15 divisions in 10 supply chains. Moreover, applying fewer resources in energy sectors of the supply chain causes a dramatic variation in investment regions while overall efficiency and production factors remain constant. According to Table 2, the resource allocation decrease of oil fields of 10 supply chains created a significant reduction in the specialized workforce to flare gas inhibition as desirable output (sold oil) and expenditure of flare gas return to gas natural cycle

Table 2

The optimal changes and the new values of the supplier inputs under VRS for 10 supply chains

DMU	$\Delta\bar{x}_{1k}^{1}$	$\Delta\bar{x}_{1k}^{2}$	$\Delta\tilde{x}_{1k}^{1}$	$\Delta\tilde{x}_{1k}^{2}$	$\bar{x}_{1k}^{1}+\Delta\bar{x}_{1k}^{1}$	$\bar{x}_{1k}^{2}+\Delta\bar{x}_{1k}^{2}$	$\tilde{x}_{1k}^{1}+\Delta\tilde{x}_{1k}^{1}$	$\tilde{x}_{1k}^{2}+\Delta\tilde{x}_{1k}^{2}$
1	0	-2.086	0	-1023.810	2550	7197.914	3200	1476.19
2	0	-3.660	0	490.476	61200	21596.34	1300	2990.476
3	-5200.038	-5.583	-756.668	250.783	16399.962	10794.417	2443.332	2149.217
4	0	0	0	0	32400	6480	3110	1400
5	0	0	0	0	12600	19440	2800	3000
6	-8566.045	-5.584	-350.480	519.48	34633.955	10794.417	1849.52	1880.52
7	-37.546	6.003	0	0	46662.454	10793.997	2400	1380
8	0	-3.660	0	597.619	39600	21596.34	1600	41850
9	0	0	0	820	9360	19440	2150	3000
10	-3600	0	-1200	0	61200	6480	1300	2900

keep unchanged. Other hands, the applied capacities reduction of the gas fields of supply chains number1, 2, 3, 6, and 8 implies expert workforce increase to flare gas control. Therefore, these gas fields should obtain the necessary technological capacity to reduce harmful emissions when applying fewer resources to the production processes. Finally, the gas field of the seventh supply chain to efficiency score maintenance should apply more resources as an expert workforce and the gas amount sold to other companies and flaring gas emissions and cost of cleaning flare gas remained unchanged. The oil fields of supply chains number 3, 6, 7, and 10 can fewer resource utilization and a significant decrease in specialist workforce amount sold gas to other companies and flare gas emissions and cost flare gas return to gas natural cycle maintains constant incontrast the gas fields of supply chains were applied fewer capacities need to more specialist workforce for desirable and undesirable outputs and dual-role factor keep constant.

Tables 3-5 indicate the variation of two categories of inputs and their values after allocation in power plants and transmitters of 10 supply chains.

Table 3

The optimal changes and new values of the inputs of manufacturer 1,and 2

DMU	$\Delta\overline{x}^3_{1k}$	$\Delta\overline{x}^3_{2k}$	$\overline{x}^3_{1k} + \Delta\overline{x}^3_{1k}$	$\overline{x}^3_{2k} + \Delta\overline{x}^3_{2k}$	$\Delta\tilde{x}^3_{1k}$	$\tilde{x}^3_{1k} + \Delta\tilde{x}^3_{1k}$	$\Delta\overline{x}^4_{1k}$	$\Delta\overline{x}^4_{2k}$	$\overline{x}^4_{1k} + \Delta\overline{x}^4_{1k}$	$\overline{x}^4_{2k} + \Delta\overline{x}^4_{2k}$
1	0	0	63224	3340	0	4070	0	0	15408	1595
2	-12680	-753.802	3520	1482.198	0	1502.733	0	0	10400	700
3	0	0	10448	1000	-760.267	1000	1377.82	-1560.61	7078.94	1739.386
4	0	0	8022.4	993	0	1000	0	0	8622.4	3300
5	0	0	5184	890	0	890	0	0	1920.48	893
6	0	0	13672.88	2300	0	2300	0	0	3312	2465
7	0	0	966.32	1450	0	1450	0	0	8352	2666
8	0	0	1491.2	1490	0	1520	0	0	10320	2251
9	0	0	3872	1500	0	1500	0	0	10590	3383
10	0	0	11453.6	3140	0	3180	0	0	6787.2	760

Table 4

The optimal changes and new values of the inputs of manufacturer 2, and 3

DMU	$\Delta\tilde{x}^4_{1k}$	$\tilde{x}^4_{1k} + \Delta\tilde{x}^4_{1k}$	$\Delta\overline{x}^5_{1k}$	$\Delta\overline{x}^5_{2k}$	$\overline{x}^5_{1k} + \Delta\overline{x}^5_{1k}$	$\overline{x}^5_{2k} + \Delta\overline{x}^5_{2k}$	$\Delta\tilde{x}^5_{1k}$	$\tilde{x}^5_{1k} + \Delta\tilde{x}^5_{1k}$	$\Delta\overline{x}^6_{1k}$	$\Delta\overline{x}^6_{2k}$
1	0	1600	0	0	11903	1200	0	1200	0	0
2	0	700	0	0	2626.952	2573	0	2600	0	0
3	-1542.67	1757.333	0	0	16760	1979	0	2005	0	0
4	0	3300	0	0	8344	2005	0	2005	0	0
5	0	900	0	0	16417.76	2823	0	2823	0	0
6	0	2500	0	0	3936	800	0	800	0	0
7	0	2700	0	0	17844.7	890	0	890	4774.187	0
8	0	2260	0	0	16800	1300	0	1300	0	0
9	0	3600	0	0	7072	3994	0	4100	0	0
10	0	760	0	0	2053.28	1590	0	1590	0	0

According to Table 3, the nominal capacity of the first power plant and labor in supply chain number 2 decreased, and this resulted in the abatement of specialist workforce as NO_X, SO_X, CO_2 harmful gases emissions of the power plant sector and amount of desirable output and labor of renewable power plants keep without changes. Also, the second power plant of supply chain number 3 more fossil fuels utilization for more power production and workforce decrease of hydropower plants of this supply chain pollution emissions three toxic gas (undesirable output) and new construction of power plant (desirable output) and inner consumption of the power plant (dual-role factor) remained constant. According to Table 4, the two categories of inputs of the third power plant remain unchanged in supply chains 10. The increase in transmitter line capacities of the first transmitters of supply chain number 7 does not create changes for other production factors. Indeed, the transmitter line of supply number 7 can utilize more capacities from transmitte line without increase power loses and specialist workforce for power loses inhibition. As indicated inTable 5, supply chain number 8's second transmitter line of confront decreases in the capacitiy and length of the transmittre line, and a slight increase in the specialist workforce as loss of transmission line and new construction of transmission lines and the number of deputy employees of transfer and exploitation remain constant.

Table 5

The optimal changes and new values of the inputs of transmitter 1, and 2

DMU	$\bar{x}^6_{1k}+\Delta\bar{x}^6_{1k}$	$\bar{x}^6_{2k}+\Delta\bar{x}^6_{2k}$	$\Delta\tilde{x}^6_{1k}$	$\tilde{x}^6_{1k}+\Delta\tilde{x}^6_{1k}$	$\Delta\bar{x}^7_{1k}$	$\Delta\bar{x}^7_{2k}$	$\bar{x}^7_{1k}+\Delta\bar{x}^7_{1k}$	$\bar{x}^7_{2k}+\Delta\bar{x}^7_{2k}$	$\Delta\tilde{x}^7_{1k}$	$\tilde{x}^7_{1k}+\Delta\tilde{x}^7_{1k}$
1	27542	8704	0	74	0	0	25086	14697	0	39
2	41011	9127.8	0	78	0	0	4938	2244.5	0	17
3	13659	8643.4	0	38	0	0	41011	9127.8	0	78
4	16545	1036.9	0	25	0	0	41011	9127.8	0	78
5	6871	2850.7	0	26	0	0	13659	8443.4	0	38
6	14068	11166.4	0	42	0	0	4938	2244.5	0	17
7	18945.187	5780.5	0	51	0	0	8762	4480.4	0	26
8	10812	8273.3	0	33	-3840.46	-3384.4	11566.54	2255.337	1799	21.201
9	25086	14697.7	0	39	0	0	7376	3776.1	0	35
10	10812	8272.3	0	33	0	0	7716.4	1453.8	0	22

Table 6

The optimal changes and new values of the inputs of distributers 1, 2

DMU	$\Delta\bar{x}^8_{1k}$	$\Delta\bar{x}^8_{2k}$	$\bar{x}^8_{1k}+\Delta\bar{x}^8_{1k}$	$\bar{x}^8_{2k}+\Delta\bar{x}^8_{2k}$	$\Delta\tilde{x}^8_{1k}$	$\tilde{x}^8_{1k}+\Delta\tilde{x}^8_{1k}$	$\Delta\bar{x}^9_{1k}$	$\Delta\bar{x}^9_{2k}$	$\bar{x}^9_{1k}+\Delta\bar{x}^9_{1k}$	$\bar{x}^9_{1k}+\Delta\bar{x}^9_{1k}$
1	1557.45	-34890	9342.452	5547	-22.40	24.6	-447.488	-4844.72	693.512	3470.278
2	0	0	11349	64702	0	292	37.305	-11340	2367.305	8399

Contd...

3	0	0	11349	64702	0	292	0	0	3068	28043
4	0	0	8612	12406	0	55	0	0	1787	8942
5	875.930	-10820	1775.93	2563	-24658	12.658	0	0	2480	26770
6	0	0	11349	64702	0	292	0	0	3175	15731
7	-314.916	-32360	3324.084	4793	-87.771	21.229	1012.329	-1431.27	2456.329	12443.734
8	-114.881	-48850	1969.119	2838	-17.424	13.424	231.301	-1440	4452.301	23249
9	1557.422	-34890	9342.423	5547	-22.400	25.400	0	0	1894	18162
10	636.207	-31080	3326.207	4526	-5.947	21.947	533.175	-38590	2617.175	13098

According to Table 7 and 8, the capacities of the first and second, and third of distribution lines increased by 40% of supply chains incontrast, the length of the distribution lines and the number of employees of the engineering assistance department and programming are decreased as power loss and dispatched electricity flow and efficiency score remaind constant. This issue shows that the three distribution lines in this supply chain need to improve their technology systems for energy losses. Thus, they lack appropriate facilities to abate power loses. It is worth mentioning that the fourth the distribution line of 70% of supply chains enables to a notable reduction of capacity and length of the distribution line and employees of the programming department as power loss and dispatchting power keeps unchanged.

Table 7

The optimal changes and the new values of the inputs of distributter 3, and 4

DMU	$\Delta\tilde{x}^{9}_{1k}$	$\tilde{x}^{9}_{2k}+\Delta\tilde{x}^{9}_{2k}$	$\Delta\bar{x}^{10}_{1k}$	$\Delta\bar{x}^{10}_{2k}$	$\bar{x}^{10}_{1k}+\Delta\bar{x}^{10}_{1k}$	$\bar{x}^{10}_{2k}+\Delta\bar{x}^{10}_{2k}$	$\Delta\tilde{x}^{10}_{1k}$	$\tilde{x}^{10}_{2k}+\Delta\tilde{x}^{10}_{2k}$	$\Delta\bar{x}^{11}_{1k}$
1	-37.700	16.3	1756.879	-4657.937	5081.789	9103.063	-7.353	28.647	-2678.102
2	-21.602	39.398	378.738	-2688.322	2165.738	15433.678	8.641	-50.641	-168.407
3	0	79	0	0	3651	32533	0	115	573.586
4	0	42	0	0	1874	12075	0	115	-1457.223
5	0	122	-0.831	0	3964.169	32533	0	115	436.974
6	0	20	-137.507	-3455.908	1186.493	7645.092	5.059	24.905	-44.521
7	-78.793	36.207	612.988	-3634.156	1512.988	9748.844	-5.320	31.320	0
8	-6.755	63.755	0	0	4067	60332	0	54	-1559.637
9	0	71	942.960	-6115.908	4267.96	7645.092	-11.941	25.941	-562.026
10	11.823	41.823	0	0	4067	60332	0	54	-1559.637

Table 8

The optimal changes and the new values of the inputs of distributter 3, 4

DMU	$\Delta\bar{x}_{1k}^{11}$	$\Delta\bar{x}_{2k}^{11}$	$\bar{x}_{1k}^{11}+\Delta\bar{x}_{1k}^{11}$	$\bar{x}_{2k}^{11}+\Delta\bar{x}_{2k}^{11}$	$\Delta\tilde{x}_{1k}^{11}$	$\tilde{x}^{11}+\Delta\tilde{x}_{1k}^{11}$
1	-2678.102	289.247	1813.898	10341.247	-11.330	46.67
2	-168.407	-4512.824	1155.593	6588.176	10.732	29.641
3	573.586	-4981.910	1473.586	8401.09	1.914	37.914
4	-1457.223	-46390	1717.777	9794	-2.803	77.803
5	436.974	-8060.728	3504.974	19982.272	11.180	90.180
6	-44.521	-7617.902	1849.479	1054.098	-23.415	49.415
7	0	0	11349	64702	0	292
8	-1559.637	-30470	3835.363	21870	33.681	96.681
9	-562.026	-40350	3504.974	29862	36.180	90.180
10	-1559.637	-30470	3835.363	21870	33.681	32.681

Generally, the oil fields of 40% of supply chains have the necessary abilities for waste energy inhibition by applying less resources and labor without any change in the amount sold oil other companies and flare gas emission. Indeed, the oil fields belonging to 40% of supply chains earned appropriate capacities for flare emission reduction. Also, the gas field should be equipped with the necessary technologies for flare abatement and wasted energy inhibition because the decrease in resource allocation of gas fields of 50% of supply chains requires to an increment of specialist workforce for control flare gas. In particular, the first power plant of supply chain number 2 has appropriate abilities to confront greenhouses by applying less nominal capacity and maintaining the level of harmful emissions and the amount of electricity sent to transmitter lines. Similarly, the second transmitter of supply chain number 8 is equipped with newer technologies and construction and commissions of improved engineering systems for power loses inhibition. Therefore, the second transmitter of this supply chain is able to fewer capacities utilization to keep constant power losses and power production. Finally, the fourth distributor line has acceptable abilities to decreasing capacity and length of distributer line in 70% of supply chains while wasted energy and dispatched electricity to power costumers remained constant. According to the obtained result of input-oriented inverse model performance, supply chains are divided into two categories as follows: (1) There exist divisions whose resources decrease with maintaining the level of desirable outputs, and other factors are possible in the case that increases investment resources to undesireble outputs decrement. The gas field of 50% supply chains and the fourth distributer line of 60% of supply chains were required to specialist and programming workforce for flare gas return to gas natural cycle and power loss abatement. (2) The divisions of the supply chain in which resource allocation decrease provide investment capacities reduction while desirable output and all production factor remain unchanged. The resource utilization decrease of oil fields belonging to supply chains number 3, 6, 7, and 10, implying especially abatement of the specialist workforce while the sold gas to other companies and the cost of cleaning flare keep constant.

5. Conclusions

In recent years, fossil fuel consumption, such as oil, natural gas, and coal, has raised atmospheric concentration of CO2 and contributed to climate change. Hence, the management and utilization of available resources is critical in the power sector. The inverse DEA model was suggested in this study for the power industry's sustainable supply chain improvement. The suggested model's ability to determine the ideal allocation of resources in supply chain divisions through the best possible adjustments to two categories of inputs while maintaining the other production indexes and the divisions' efficiency is kept unchanged. According to the results of the inverse DEA input-oriented model, the oil field of supply chains 60% applied the optimal value of resources. It used appropriate investment capacities to mitigate flare gas and waste energy, while the gas field of supply chains 60% utilized the large number of gas ressource in power production. Hence, they should reduce fuel consumptions and apply more investment approtunities to flare gas abatement. Also, the supply chains of 90% of power plant sectors and transmitter lines indicate acceptable results in optimal resource utilization and investment capacities allocation for environmental preservation and energy inhibition.

References

[1] R. D. Banker, A. Charnes, and W. W. Cooper, "Some models for estimating technical and scale inefficiencies in data envelopment analysis," *Management Science*, vol. 30, pp. 1078–1092 (1984).

[2] A. Charnes, W. W. Cooper, and E. Rhodes, "Measuring the efficiency of decision-making units," *European Journal of Operational Research*, vol. 2, pp. 429–444 (1978).

[3] F. Colver and T. Sueyoshi, "Contribution of Professor William W. Cooper in operation research and management science," *European Journal of Operational Research*, vol. 196, pp. 1–16 (2009).

[4] R. Farzipoor Saen, "Developing a new data envelopment analysis methodology for supplier selection in the presence of both undesirable outputs and imprecise data," *Journal of Intelligent Manufacturing*, vol. 51, pp. 1243–1250 (2010).

[5] S. R. Ghyeghrn, A. Amirteimoori, A. Malek, and S. Kordrostami, "Cost efficiency analysis in data envelopment analysis framework: An application in sugar industries," *Journal of Information and Optimization Science*, vol. 42, no. 5, pp. 1137–1161 (2021), doi: 10.1080/02522667.2021.1885749.

[6] A. Hadi-Vancheh and A. A. Foroughi, "A generalized model for inputs/outputs estimation," *Mathematics and Computer Modelling*, vol. 43, no. 5, pp. 447–457 (2006).

[7] S. Han, "An inverse DEA model with stochastic factors," *Mathematics Practice and Theory*, vol. 33, no. 3, pp. 23–29 (2013).

[8] A. Hassenzadeh, S. Yousefi, R. Farzipoor Saen, and S. S. Seyede Hosseininia, "How to assess sustainability of countries via inverse data envelopment analysis?" *Clean Technologies and Environmental Policy* (2017), doi: 10.1007/s10098-017-1450-x.

[9] Iran Power Generation and Transmission Company, "Detailed statistics of Iran's power industry electricity transmission" (in Persian), Iran Power Generation and Transmission Company (2015). [Online]. Available: https://web.archive.org/web/20210514014836/http://amar.tavanir.org.ir/

[10] G. R. Jahanshahloo, F. H. Lotfi, N. Shoja, J. Tohid, and S. Razavyn, "Input estimation and identification of extra inputs in inverse DEA models," *Applied Mathematics and Computation*, vol. 156, no. 2, pp. 427–437 (2014).

[11] M. Khodakaremi, A. Shabani, R. Farzipoor Saen, M. Zolfaghari, and M. Tavana, "Developing distinctive two-stage data envelopment analysis models: An application evaluating the sustainability of supply chain management," *Measurement* vol. 70, pp. 62–74 (2015).

[12] S. Lertworasirikul, P. Chamsthikul, and S. C. Fang, "Inverse data envelopment analysis model to preserve relative efficiency values: The

case of variable returns to scale," *Computers & Industrial Engineering*, vol. 61, no. 4, pp. 1017–1021 (2011).

[13] M. Pouralizadeh, "A DEA model to sustainability improvement of the electricity supply chain in presence dual-role factors and undesirable outputs: A case on power industry," *AIMS Energy*, vol. 8, no. 4, pp. 580–614 (2020).

[14] M. Pouralizadeh, A. Amirteimoori, R. Riccardi, F. G. Z. de Moura, M. G. M. Udupa, and S. P. S. P. G. S. Swamy, "Supply chain performance evaluation in the presence of undesirable outputs: A case on power industry," *AIMS Energy*, vol. 8, pp. 48–80 (2020).

[15] M. Pouralizadeh, "Sustainability assessment of electricity supply chain via resource waste reduction and pollution emissions management: A case study of the power industry," *Smart and Sustainable Manufacturing Systems*, vol. 5, no. 1, pp. 290–309 (2021).

[16] M. Pouralizadeh, "Marginal profit maximization estimation of supply chains by waste energy: A case study of the power industry," *RIARO Operation Research*, vol. 58, pp. 3143-1369 (2024). doi: 10.1051/ro/2024103.

[17] A. M. Rad, R. Dehghan, and F. H. Lotfi, "Inverse DEA with fuzzy data for output estimation," *Iranian Journal of Optimization*, vol. 2, no. 2, pp. 388–411 (2011).

[18] T. Sueyoshi and M. Goto, "Measurement of a linkage among environmental, operational and financial performance in Japanese manufacturing firms: A use of data envelopment analysis with strong complements slackness condition," *European Journal of Operational Research*, vol. 207, pp. 1742–1753 (2010).

[19] T. Sueyoshi and M. Goto, "Should the US clean air act include CO2 emission control? Examination by data envelopment analysis," *Journal of Energy Policy*, vol. 38, pp. 5902–5911 (2010).

[20] T. Sueyoshi and M. Goto, "Environmental assessment for corporate sustainability by resource utilization and technology innovation: DEA radial measurement on Japanese industrial sectors," *Energy Economics*, vol. 46, pp. 295–307 (2014).

[21] M. Tavana, H. Mirzagoltabar, S. M. Mirhedayatian, M. Zolfaghari, and R. Saen, "A new network Epsilon-based DEA model for supply chain performance evaluation," *Journal of Computational and Industrial Engineering*, vol. 66, pp. 501–513 (2013).

[22] K. Tone and M. Tsutsui, "An epsilon-based measure of efficiency in DEA-A third pole of technical efficiency," *European Journal of Operational Research*, vol. 207, pp. 1554–1566 (2010).

[23] Q. Wei, J. Zhang, and X. Zhang, "An inverse DEA for inputs/outputs estimate," *European Journal of Operational Research*, vol. 121, no. 1, pp. 151–163 (2000).

[24] G. Yu, P. Wei, and A. Brocket, "A generalized data envelopment analysis model: A unification and extension of existing methods units," *Annals of Operations Research*, vol. 66, pp. 47–89 (1996).

[25] M. Zhang and J. S. Cui, "The extension and integration of the inverse DEA method," *Journal of Operational Research Society*, pp. 1–9 (2016).

[26] X. S. Zhang and J. C. Cui, "A project evaluation system in the state economic information system of China: An operation research practice in public sectors," *International Transactions in Operational Research*, vol. 6, no. 5, pp. 441–452 (1999).

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