Journal of Information & Optimization Sciences
ISSN 0252-2667 (Print), ISSN 2169-0103 (Online)
Vol. 47 (2026), No. 3, pp. 1187–1211
DOI : 10.47974/JIOS-2186

An Integrated MADM framework for parking space recommendation : Combining G1, entropy weight, and grey correlation TOPSIS method

Yu Han †
Department of Artificial Intelligence
Jinhua Advanced Research Institute
Jinhua 321013
Zhejiang
China

Donghua Yu *
Department of Computer Science and Engineering
Shaoxing University
Shaoxing 312000
Zhejiang
China

Abstract

Amid China's sustained economic growth, rapid urbanization has precipitated a surge in vehicle ownership, while parking infrastructure expansion has failed to keep pace. This imbalance exacerbates urban parking scarcity, leading to inefficient unstructured parking searches by drivers. Such behavior increases energy consumption, environmental pollution, and reduces travel efficiency. The parking space recommendation problem constitutes a multi-attribute decision-making (MADM) challenge. To address this, this paper proposes an integrated optimization model combining the G1method, entropy weight method, and a grey correlation-enhanced TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) approach. The model employs a comprehensive evaluation index system incorporating both exact and interval-valued criteria. By fusing subjective weights (G1 method) and objective weights (entropy weight method), the framework overcomes the singularity and subjectivity limitations inherent in conventional weighting methods. Further, the integration of grey correlation analysis refines TOPSIS, enabling robust computation of comprehensive scores for parking schemes. A case study validates the model's applicability, generating a prioritized parking spaces ranking. Results demonstrate that the model

† *E-mail:* `hanyudeyouxiang@hrbeu.edu.cn`
* *E-mail:* `donghuayu@usx.edu.cn (Corresponding Author)`

effectively recommends contextually suitable parking spaces by harmonizing users' subjective preferences with objective scheme attributes.

Subject Classification: *90C29.*

Keywords: *MADM, G1 method, Entropy weight method, TOPSIS, Interval values, Parking spaces recommendation.*

1. Introduction

With the continuous improvement of household economic standards and the eradication of absolute poverty in China, the public has increasingly shifted focus from basic needs to the pursuit of high-quality life and happiness. This transition is vividly reflected in the dramatic growth of private vehicle ownership. From the era when motorcycles dominated personal mobility to the current prevalence of private cars, the number of motor vehicles in China has shown a sustained upward trend. By the end of 2022, the total number of private cars exceeded 300 million, accompanied by 481 million licensed drivers. In densely populated urban areas, public transportation systems (e.g., buses and subways) are often overwhelmed during peak hours, rendering private cars a practical necessity for daily commuting. However, this trend is challenged by the limited supply of parking spaces in urban centers [1]. As illustrated in Figure 1, although the total number of parking spaces has been increasing in recent years—projected to reach 195 million by 2022—the car-to-parking-space ratio still remains at approximately 1.6:1, highlighting a significant infrastructure gap.

However, urban traffic planning and parking infrastructure development have not kept pace, which significantly limits driving convenience. When searching for parking, drivers often engage in inefficient routes, wasting time and energy while worsening urban traffic congestion. This problem not only reduces travel efficiency and convenience but also raises the risk of traffic accidents. More importantly, prolonged parking searches lead to vehicles idling for longer, increasing carbon dioxide emissions and worsening the global greenhouse effect. Meanwhile, harmful gases (such as carbon monoxide and nitrogen oxides) and particulate matter from these vehicles spread widely in cities, seriously worsening environmental pollution. These impacts contradict the environmental protection principles of sustainable development, threatening long-term urban ecological balance and residents' quality of life. Therefore, the ability to quickly and accurately find parking spaces

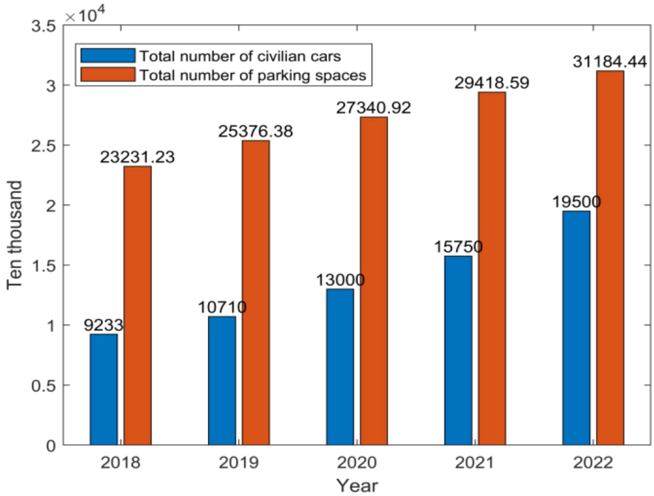

Figure 1

Total number of civilian cars and parking spaces in China.

has become a key issue in urban traffic management [2]. Intelligent parking management systems enable real-time monitoring of parking space usage, allowing prompt recommendations and reservations for users [3]. This can significantly reduce the time vehicles spend searching for parking and minimize parking spaces vacancy duration [4], thereby improving traffic efficiency [5], optimizing resource allocation [6], and boosting operational revenue [7]. Meanwhile, such systems help users plan their trips and parking more effectively, while enabling traffic management departments to implement macro-level control of overall traffic conditions [8]. Ultimately, these advancements can make a more sustainable societal environment.

The parking spaces recommendation problem is a core component of intelligent parking management systems. To recommend optimal parking spaces, it is necessary to compare and analyze factors such as parking fees and parking lot safety—this constitutes a typical multiple-attribute decision-making (MADM) problem [9]. MADM is a fundamental branch of modern decision theory, and numerous real-world challenges can be categorized as MADM problems [10].

Commonly adopted methodologies for MADM include the entropy weight method (EWM) [11], TOPSIS [12], analytic hierarchy process (AHP) [13], grey decision analysis (GDA) [14], and data envelopment analysis (DEA) [15], among others. Niu, Fan, and Li [16] used EWM to derive initial weights for safety performance indicators in the oil industry, followed by DEA to evaluate the safety performance of eight subsidiaries. Bu et al. [17] proposed an interval-valued intuitionistic fuzzy MADM approach based on an improved TOPSIS, using the TOPSIS distance metric to compute comprehensive scores for brigade rankings. Javed, Mahmoudi, and Liu [18] introduced a novel grey absolute decision analysis (GADA) method, grounded in the absolute grey relational analysis (GRA) model. In traditional MADM, exact values are typically used to represent scheme evaluations under each indicator. However, the complexity of decision environments and cognitive uncertainties render exact values insufficient to capture problem dynamics or evaluator intentions. Zadeh's [19] fuzzy set theory addresses this limitation, fostering the development of decision frameworks such as interval-valued intuitionistic fuzzy sets (IVIFSs) [20], interval-valued hesitant fuzzy sets (IVHFSs) [21], Pythagorean fuzzy sets [22], and interval-valued neutrosophic fuzzy sets [23]. Ye, Du, and Yong [24] introduced single-valued and interval-valued fuzzy multivalued sets (SIVFMSs/SIVFMEs) for decision modeling. Chen and Li [25] developed a novel weighting method for large parking lots, mitigating subjective bias and reducing discrepancies between combined weights. Demir, Basaraner, and Gumus [26] applied GIS-integrated fuzzy AHP to identify optimal parking supply locations across four Istanbul districts. Similarity measurement plays a central role in MADM. Wang et al. [27] proposed ten similarity measures for q-rung orthopair fuzzy sets, while Saqlain et al. [28] developed measures for neutrosophic hypersoft sets based on Hamming distance. Gohain et al. [29] introduced two new similarity measures of intuitionistic fuzzy set for decision makers.

Despite their effectiveness, existing methods predominantly target large-scale project evaluation [30], with limited applications to parking spaces comprehensive assessment. Their complexity and inability to handle mixed exact/interval data further hinder practical implementation. This study develops an optimized parking spaces recommendation model that integrates uncertain attribute modeling to address these gaps.

This paper has demonstrated the following contributions:

(1) The model establishes an evaluation index system for parking spaces recommendation, integrating both exact-value and interval-value data to characterize uncertainties in parking attributes. For heterogeneous data types, normalization strategies are used to

construct a standardized decision matrix, ensuring data consistency and comparability.

(2) A critical challenge in the model is indicator weight determination. The model utilizes the G1 method for subjective weight calculation and the entropy weight method for objective weight derivation, subsequently fusing these to generate comprehensive weights. This approach mitigates biases from single source weighting and enhances evaluation objectivity.

(3) Addressing the limitations of traditional TOPSIS in handling interval data and distinguishing similar schemes, the model integrates grey correlation degree to improve TOPSIS. This enhancement enables precise computation of comprehensive scores and ranking for parking space schemes, resolving the ambiguity in close evaluation results.

This research provides a more scientific, robust, and reasonable model for parking spaces recommendation problems, which contributes to advancing the further development of parking spaces recommendation technologies.

2. Main research framework

The research framework consists of four steps, as illustrated in Figure 2.

Step 1: Establishment of evaluation index system and data normalization. This step involves defining the parking space evaluation index system and normalizing the original decision matrix. Targeted normalization strategies are applied to handle mixed exact/interval data, ensuring consistency for subsequent analysis.

Step 2: Hybrid weight calculation. The G1 method is integrated with the entropy weight method to derive comprehensive indicator weights. This hybrid approach combines subjective expertise (G1) and objective data-driven analysis (entropy), mitigating single-method biases.

Step 3: Improved TOPSIS with grey correlation. An enhanced TOPSIS model is developed by incorporating grey correlation degree, enabling precise computation of comprehensive scores for parking space schemes.

This improvement addresses the limitations of traditional TOPSIS in handling interval data and distinguishing similar evaluations.

Step 4: Model application and decision support. The final step involves applying the model to real-world parking spaces recommendation scenarios, followed by generating data-driven recommendations for users. This ensures the model's practical relevance and decision-making utility.

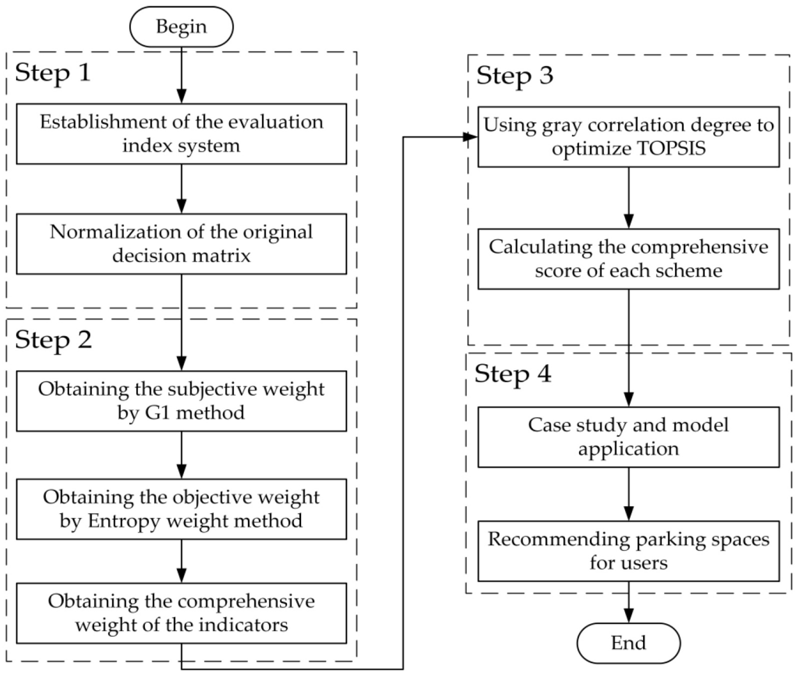

Figure 2

Main research framework.

3. Research model

3.1 *Establishment of evaluation index system and data Normalization*

3.1.1 Establishment of the evaluation index system

The driving travel scenario comprises a sequential activity chain: road navigation, parking space identification and selection, parking maneuver execution, and subsequent pedestrian movement to the destination. Our analysis identifies parking lot entrances as frequent congestion bottlenecks. When multiple parking options exist near the destination, drivers anticipate arrival times at candidate parking spaces with personal preferences to make informed decisions.

In large-scale parking facilities, extra walking distance from the parking space to the destination is inevitable. Notably, driver parking choices often prioritize spatial proximity to the target location. Additionally, significant safety disparities exist across parking facilities: those staffed with security personnel are generally considered safer than unmanned outdoor lots.

Even within a single parking lot, safety conditions vary among parking spaces. Spaces near lot entrances or on different floors are more susceptible to vehicle scratches. Concurrently, parking spaces exhibit distinct levels of convenience, influenced by factors such as accessibility, lighting, and pedestrian flow patterns.

The parking space recommendation problem involves multiple attributes, some of which are inherently non-quantifiable. Through a comprehensive literature review [31, 32], expert consultations, and field investigations, this study identifies five core attributes to construct the evaluation index system: the period of arriving at the parking spaces (U_1), parking fee per hour (U_2), convenience of parking (U_3), the walking distance from the parking space to the destination (U_4), safety of parking lots (U_5).

Through extensive consultations with experts and drivers, we found that the period of arriving at the parking spaces and convenience of parking involve uncertainties due to their stochastic nature. To accurately model these characteristics, this study uses interval values for representation. By contrast, indicators such as parking fee per hour, walking distance from parking space to destination, and safety of parking lots are quantified using exact values, as summarized in Table 1.

The convenience of parking and safety of parking lots were evaluated by experts using a 1–9 rating scale, where a higher score indicates better performance. Specifically, a rating of 1 corresponds to the lowest level of parking convenience, while a rating of 9 denotes the highest convenience.

Table 1

Evaluation Index System of Parking Spaces Recommendation scheme.

	U_1	U_2	U_3	U_4	U_5
Indicator	the period of arriving at the parking spaces	parking fee per hour	convenience of parking	walking distance from the parking pace to the destination	safety of parking lots
Type of indicators	cost indicator	cost indicator	benefit indicator	cost indicator	benefit indicator
Type of value	interval value	exact value	interval value	exact value	exact value

3.1.2 Normalization of the original decision matrix

The set of schemes (parking spaces) exist near the destination is $S=\{S_1,S_2,\dots,S_n\}$, $n\geq 2$. The set of indicators for each scheme is $P=\{P_1,P_2,\dots,P_m\}$, $m\geq 2$. a_{ij} represents the value of scheme j on indicator i, $1\leq i\leq m, 1\leq j\leq n$. The original decision matrix $A=(a_{ij})_{m\times n}$ is

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix} \tag{1}$$

$A=(a_{ij})_{m\times n}$ has exact and interval values. The superscript L is the lower limit and U is the upper limit of the interval values.

$$\begin{cases} a_{ij}=a_{ij}, \text{if } a_{ij} \text{ is exact value} \\ a_{ij}=[a_{ij}^L, a_{ij}^U], \text{if } a_{ij} \text{ is interval value} \end{cases} \tag{2}$$

Owing to the heterogeneity in dimensions and units of the indicators, direct comparison is infeasible. To eliminate the influence of dimensional discrepancies on calculation results, Eqs (3)-(8) were utilized to normalize a_{ij}, and the normalization matrix, yielding the normalized decision matrix $Z=(z_{ij})_{m\times n}$. When the data are exact values, for the benefit indicators,

$$z_{ij} = \frac{a_{ij} - \min_i a_{ij}}{\max_i a_{ij} - \min_i a_{ij}} \tag{3}$$

and for the cost indicators,

$$z_{ij} = \frac{\max_i a_{ij} - a_{ij}}{\max_i a_{ij} - \min_i a_{ij}} \tag{4}$$

When the data are interval values, for the benefit indicators,

$$z_{ij}^L = \frac{a_{ij}^L}{\sqrt{\sum_{j=1}^{n} (a_{ij}^U)^2}} \tag{5}$$

$$z_{ij}^U = \begin{cases} \dfrac{a_{ij}^U}{\sqrt{\sum_{j=1}^{n} (a_{ij}^L)^2}}, & \dfrac{a_{ij}^U}{\sqrt{\sum_{j=1}^{n} (a_{ij}^L)^2}} \le 1 \\ 1, & \dfrac{a_{ij}^U}{\sqrt{\sum_{j=1}^{n} (a_{ij}^L)^2}} > 1 \end{cases} \tag{6}$$

and for the cost indicators,

$$z_{ij}^L = \begin{cases} \dfrac{1/a_{ij}^U}{\sqrt{\sum_{j=1}^{n} (1/a_{ij}^L)^2}}, & \dfrac{1/a_{ij}^U}{\sqrt{\sum_{j=1}^{n} (1/a_{ij}^L)^2}} \le 1 \\ 1, & \dfrac{1/a_{ij}^U}{\sqrt{\sum_{j=1}^{n} (1/a_{ij}^L)^2}} > 1 \end{cases} \tag{7}$$

$$z_{ij}^U = \begin{cases} \dfrac{1/a_{ij}^L}{\sqrt{\sum_{j=1}^{n} (1/a_{ij}^U)^2}}, & \dfrac{1/a_{ij}^L}{\sqrt{\sum_{j=1}^{n} (1/a_{ij}^U)^2}} \le 1 \\ 1, & \dfrac{1/a_{ij}^L}{\sqrt{\sum_{j=1}^{n} (1/a_{ij}^U)^2}} > 1 \end{cases} \tag{8}$$

Following normalization, all elements in $A = (a_{ij})_{m \times n}$ are scaled to the interval [0,1], though the matrix still contains a mix of exact and interval values. To facilitate data processing,, the concepts of the upper bound matrix, Z^U, and the lower bound matrix, Z^L, are introduced. The elements z_{ij}^U and z_{ij}^L of Z^U and Z^L are derived from normalized decision matrix $Z=(z_{ij})_{m\times n}$ as follows. If z_{ij} is an exact value, then

$$z_{ij}^U = z_{ij}^L = z_{ij} \tag{9}$$

If z_{ij}=$[u,v]$ is an interval value, then

$$\begin{cases} z_{ij}^L = u \\ z_{ij}^U = v \end{cases} \tag{10}$$

3.2 *Comprehensive weight of the indicators*

In parking spaces recommendation decision-making, evaluation indicators exhibit significant variations in importance. Indicator weight calculation methods are fundamentally divided into two types: subjective weighting methods and objective weighting methods.

Subjective weighting methods depend on experts or user evaluations, primarily reflecting subjective biases and judgments in weight determination. In contrast, objective weighting methods derive weights from the inherent attribute data of indicators, ensuring results are free from human prejudice. It is critical to note that weights obtained solely from either approach may not fully reflect the actual significance of indicators in practical applications.

Parking space schemes involve a mix of quantitative and qualitative indicators, introducing complexity to indicator weight calculation. The accuracy of weights is pivotal for ensuring the fairness and scientific validity of scheme evaluations—deviations in weight assignment can mislead final decisions, underscoring the need for a comprehensive methodology. This study proposes a hybrid weighting strategy that integrates subjective and objective methods, deriving comprehensive indicator weights through systematic combination. Our approach balances the precision of objective data with expert empirical knowledge embedded in subjective judgments, ensuring evaluation results are not only more accurate but also better aligned with real-world parking space recommendation scenarios.

3.2.1 G1 method

For subjective weighting, the G1 method proposed by Guo [33] is adopted. Unlike AHP, G1 does not require a consistency test, resolving the challenge of achieving consensus on attribute relative importance when numerous indicators are involved [34]. The G1 method allows decision-makers to assign weights based on domain expertise, aligning with real-world preference elicitation. Therefore, this study uses the G1 method, whose calculation steps are as follows:

Firstly, decision-makers sort the attributes U_i, $i = 1,2,\cdots m-1,m$, according to their ideas, and the sorted attribute relationship formula is $I_1 \geq I_2 \geq, \ldots \geq I_{m-1} \geq I_m$.

$I_p \geq I_q$ means that the importance of attribute p is greater than or equal to that of q. Then, the relative importance ri between I_{i-1} and Ii is also determined by the decision maker, as shown in Table 2. For example, if the decision maker thinks $I2$ is slightly more important than $I3$, then $r2$ =1.2.

Table 2

Assignment table of *ri*.

Value	Assignment Description
1	Ii-1 is as important as Ii
1.2	Ii-1 is slightly more important Than Ii
1.4	Ii-1 is more important than Ii
1.6	Ii-1 is obviously more important than Ii
1.8	Ii-1 is too more important than Ii
1.1, 1.3, 1.5, 1.7	The importance is between the above levels

Meanwhile, ri is obtained by comparative judgment of $\omega s,i$-1 and $\omega s,i$

$$r_i = \frac{\omega_{s,\,i-1}}{\omega_{s,\,i}}, i = 2, 3, \cdots m-1, m \tag{11}$$

From Eq (11), we can know

$$\prod_{i=k}^{m} r_i = \frac{\omega_{s,k-1}}{\omega_{s,k}} \frac{\omega_{s,k}}{\omega_{s,k+1}} \cdots \frac{\omega_{s,m-2}}{\omega_{s,m-1}} \frac{\omega_{s,m-1}}{\omega_{s,m}} = \frac{\omega_{s,k-1}}{\omega_{s,m}}, k \geq 2 \tag{12}$$

In summing k from 2 to m, the result is

$$\sum_{k=2}^{m}\left(\prod_{i=k}^{m} r_i\right) = \sum_{k=2}^{m}\frac{\omega_{s,k-1}}{\omega_{s,m}} \tag{13}$$

Because $\sum_{k=1}^{m}\omega_{s,k} = 1$

$$1+\sum_{k=2}^{m}\left(\prod_{i=k}^{m} r_i\right) = \frac{\omega_{s,m}}{\omega_{s,m}} + \sum_{k=2}^{m}\frac{\omega_{s,k-1}}{\omega_{s,m}} = \omega_{s,m}^{-1} \tag{14}$$

The weight of $\omega_{s,m}$ can be obtained via Eq (15).

$$\omega_{s,m} = \left(1+\sum_{k=2}^{m}\left(\prod_{i=k}^{m} r_i\right)\right)^{-1} \tag{15}$$

The weights of other indicators can be obtained via Eq (16).

$$\omega_{s,i-1} = r_i\omega_{s,i}, i = 2,3,\cdots m-1, m \tag{16}$$

Finally, we reorder ωs,i, and get the weights of ωs,Ui.

3.2.2 Entropy method

The dispersion of indicator values across decision schemes for *Ui* reflects its impact on decision-making and ranking. Specifically, greater dispersion indicates higher significance, justifying a larger objective weight. Conversely, smaller dispersion implies lower significance, warranting a smaller objective weight. The degree of indicator dispersion can be quantified by information entropy: higher entropy of an indicator signifies smaller value dispersion, implying weaker decision influence and thus meriting a lower objective weight. For the calculation of objective weights, the following steps are adopted: Firstly,

$$b_{ij}^{L} = \frac{z_{ij}^{L}}{\sum_{j=1}^{n} z_{ij}^{L}} \tag{17}$$

$$b_{ij}^{U} = \frac{z_{ij}^{U}}{\sum_{j=1}^{n} z_{ij}^{U}} \tag{18}$$

then calculate the entropy values of b_{ij}^{L} and b_{ij}^{U}

$$E_i^{L} = -\frac{1}{\ln n}\sum_{j=1}^{n} b_{ij}^{L}\ln b_{ij}^{L} \tag{19}$$

$$E_i^U = -\frac{1}{\ln n}\sum_{j=1}^{n} b_{ij}^U \ln b_{ij}^U \tag{20}$$

From E_i^L and E_i^L, we can get lower and upper bounds of the indicator objective weight

$$\omega_{o,i}^L = \frac{1-E_i^L}{m-\sum_{i=1}^{m} E_i^L} \tag{21}$$

$$\omega_{o,i}^U = \frac{1-E_i^U}{m-\sum_{i=1}^{m} E_i^U} \tag{22}$$

Finally, these weights are integrated to obtain the final weight for the indicators via Eq (23)

$$\omega_{o,i} = \frac{1}{2}(\omega_{o,i}^U + \omega_{o,i}^L), i = 1,2,3,\cdots m-1, m \tag{23}$$

3.2.3 Comprehensive weight

This study integrates multiple weighting methods, using the G1 method for subjective weight calculation and the entropy weight method for objective weight calculation. The G1 method relies on decision-makers' subjective judgments to establish indicator importance orders, a process that involves significant subjectivity. In contrast, the entropy weight method focuses on data variability and intervariable correlations, demonstrating high sensitivity to data fluctuations. Leveraging the complementary characteristics of these approaches, comprehensive weights are calculated using Eq (24), which skillfully integrates subjective expertise and objective data to ensure balanced and holistic evaluations.

$$\omega_i = \frac{\omega_{s,U_i}\omega_{o,i}}{\sum_{i=1}^{m}\omega_{s,U_i}\omega_{o,i}}, i = 1,2,3,\cdots m-1, m \tag{24}$$

3.3 *Improved TOPSIS method*

TOPSIS is a classic multiple-criteria decision analysis method based on ideal solution approximation. Ideal solutions are categorized into positive ideal solution and negative ideal solution. By comparing each evaluation unit's parameters against these ideal solutions, a comprehensive score is derived: closer proximity to positive ideal solution indicates a higher score, while nearness to negative ideal solution corresponds to a lower score. The traditional TOPSIS method uses Euclidean distance to calculate the distance to an ideal solution, which has certain defects. Failing to consider linear relationships between criteria renders Euclidean distance calculations invalid, severely compromising evaluation accuracy. When scores of different units are nearly identical, the method cannot distinguish their spatial distributions, thus failing to reflect their true status objectively.Traditional TOPSIS is ill-equipped to address the vagueness inherent in uncertain decision environments, limiting its applicability to complex scenarios.In response to these limitations, this study proposes an enhanced TOPSIS method.

3.3.1 grey correlation degree

In the upper bound matrix, Z^U, let

$$\overline{Z}^U = \left\{\overline{z}_1, \overline{z}_2, \ldots \overline{z}_m\right\}^T \tag{25}$$

where $\overline{z}_i$ is the best value in indicator i, be the positive ideal solution.

In the lower bound matrix, Z^L, let

$$\underline{Z}^L = \{\underline{z}_1, \underline{z}_2, \ldots \underline{z}_m\}^T \tag{26}$$

where $\underline{z}_i$ is the worst value in indicator i, be the negative ideal solution. To address the aforementioned limitations, this study uses an enhanced TOPSIS method for comprehensive evaluation of candidate parking spaces. By replacing Euclidean distance with grey correlation degree, the method evaluates the global trend similarity between each unit's indicator parameters and the positive/negative ideal solutions. This approach effectively captures the internal variation patterns within evaluation units, overcoming the linear dependency ignorance of traditional TOPSIS.

Matrix ξ^U represents the grey correlation degree with Z^U and $\overline{Z}^U$, of which the elements are

$$\xi_{ij}^U = \frac{\min_i \min_j \left|z_{ij}^U - \overline{z}_i\right| + \rho \max_i \max_j \left|z_{ij}^U - \overline{z}_i\right|}{\left|z_{ij}^U - \overline{z}_i\right| + \rho \max_i \max_j \left|z_{ij}^U - \overline{z}_i\right|} \tag{27}$$

Matrix ξ^L represents the grey correlation degree with Z^L and $\underline{Z}^L$, of which the elements are

$$\xi_{ij}^{L} = \frac{\min_i \min_j \left| z_{ij}^{L} - \underline{z}_i \right| + \rho \max_i \max_j \left| z_{ij}^{L} - \underline{z}_i \right|}{\left| z_{ij}^{L} - \underline{z}_i \right| + \rho \max_i \max_j \left| z_{ij}^{L} - \underline{z}_i \right|} \tag{28}$$

In Eqs (27) and (28), ρ is the grey discrimination coefficient, which is generally $\rho = 0.5$.

3.3.2 Ranking of schemes

According to the grey correlation coefficient matrix, the weighted grey correlation degree of each scheme calculated are L_j^{+} and L_j^{-}, which also make up the improved distance.

$$\begin{cases} L_j^{+} == \sum_{i=1}^{m} \omega_i \xi_{ij}^{U} \\ L_j^{-} == \sum_{i=1}^{m} \omega_i \xi_{ij}^{L} \end{cases} \tag{29}$$

In Eq(29) , ωi is the comprehensive weight obtained in Eq (24). The overall similar trend between the schemes and the positive and negative ideal solutions is taken as the comprehensive score, Tj, to conduct a comparative analysis of the attributes, and the calculation formula for Tj is as follows:

$$T_j = \frac{L_j^{+}}{L_j^{+} + L_j^{-}} \tag{30}$$

As Tj approaches 0, scheme j will become closer to the negative ideal solution and the scores will become lower. As Tj approaches 1, scheme j will become closer to the positive ideal solution and the scores will become higher. Then, we can get the ranking of schemes.

3.3.3 Analysis of improved TOPSIS method

In order to verify whether the improved TOPSIS method is reasonable, the proposed method was applied to the case in Reference [35], and those results have been compared to ours.

$$A = \begin{pmatrix} 39.5 & 40.97 & 38.88 \\ 115.4 & 122.1 & 137.5 \\ 172.29 & 162.91 & 156.71 \\ 2000 & 2200 & 2500 \\ 28.59 & 10.42 & 20.33 \\ 31.952 & 28.558 & 27.452 \\ 26.7 & 16.1 & 18.3 \\ 1202 & 1399 & 1513 \\ [7.0, 8.5] & [5.5, 7.0] & [3.0, 4.5] \\ [3.0, 4.5] & [4.5, 5.5] & [7.0, 8.5] \\ [1.5, 3.0] & [5.5, 7.0] & [7.0, 8.5] \\ [4.5, 5.5] & [5.5, 7.0] & [5.5, 7.0] \\ [4.5, 5.5] & [7.0, 8.5] & [5.5, 7.0] \\ [1.5, 3.0] & [5.5, 7.0] & [7.0, 8.5] \\ [3.0, 4.5] & [4.5, 5.5] & [3.0, 4.5] \\ [3.0, 4.5] & [5.5, 7.0] & [4.5, 5.5] \\ [4.5, 5.5] & [3.0, 4.5] & [3.0, 4.5] \\ [4.5, 5.5] & [5.5, 7.0] & [5.5, 7.0] \end{pmatrix} \tag{31}$$

Railway routing scheme selection is a complex MADM problem involving qualitative and quantitative indicators. The Hotan–Ruoqiang railway, located in China's Xinjiang province, connects Hotan to Ruoqiang, with a design speed of 120 km/h and a total length of 825.5 km. TheMinfeng–Yuhu area is an important part of the Hotan–Ruoqiang railway, which has a dry climate, sparse vegetation and a fragile ecology. It is difficult for this railway to avoid all environmentally sensitive areas (such as Niya National Wetland Park) and windy sand areas (such as moving sand dunes) along the route.

Niya National Wetland Park, which stretches from the Kunlun Mountains to the hinterland of the Taklimakan Desert, is an important environmental barrier of the region. In order to reduce the damage from railway construction to the sand area and the environment, and considering the scope of the park, the distribution of sand hazards and the terrain conditions, railway engineers have proposed the 1-3 scheme. The multiple-attribute evaluation matrix formed by the three schemes is as matrix A.

Using Eqs (3) to (10), the matrix A is normalized and the upper bound matrix is obtained as Z^U and Z^L .

$$Z^u = \begin{pmatrix} 0.7033 & 0.0000 & 1.0000 \\ 1.0000 & 0.6968 & 0.0000 \\ 0.0000 & 0.6021 & 1.0000 \\ 0.0000 & 0.4000 & 1.0000 \\ 1.0000 & 0.0000 & 0.5500 \\ 0.0000 & 0.7542 & 1.0000 \\ 0.0000 & 1.0000 & 0.7925 \\ 1.0000 & 0.3666 & 0.0000 \\ 0.4940 & 0.6287 & 1.0000 \\ 0.5087 & 0.6218 & 0.9609 \\ 1.0000 & 0.4769 & 0.3747 \\ 0.8176 & 0.6689 & 0.6689 \\ 0.8565 & 0.5507 & 0.7008 \\ 1.0000 & 0.4769 & 0.3747 \\ 0.9181 & 0.6121 & 0.9181 \\ 1.0000 & 0.5669 & 0.6929 \\ 0.6121 & 0.9181 & 0.9181 \\ 0.6121 & 0.7790 & 0.7790 \end{pmatrix} \tag{32}$$

For a better comparison, the original weights were used for the indicators, and the calculated results obtained are shown in Table 3.

Table 3

Calculated results.

	Scheme A	Scheme B	Scheme C
L_j^+	0.7422	0.5533	0.6843
L_j^-	0.5749	0.6301	0.5577
T_j	0.5635	0.4676	0.5509

Using the enhanced TOPSIS method proposed in this study, the schemes were ranked as A > B > C, designating Scheme A as the optimal railway alignment. As shown in Table 3, the ranking results demonstrate high consistency with the method reported in Reference [35], validating the proposed approach's potential to address similar multiple-attribute decision-making (MADM) problems.

4. Case study

There are eight eligible parking spaces near the destination. Due to the uncertainty of the real environment, the period of arriving at the parking spaces and the convenience of parking are represented by interval values. Detailed information is shown in Table 4.

Table 4

Information of parking spaces.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
U_1 (min)	[30,35]	[25,38]	[24,40]	[28,34]	[30,39]	[36,38]	[26,37]	[34,42]
U_2 (Yuan)	1	1.5	1.5	2	2	1.5	1.5	2
U_3	[5,7]	[8,9]	[3,5]	[6,9]	[4,5]	[6,7]	[4,6]	[6,8]
U_4 (m)	430	520	330	270	540	770	380	570
U_5	7	9	5	9	5	7	5	7

Using Eqs (3) to (10), the original decision matrix A is normalized. The upper bound matrix and the lower bound matrix are obtained. There are two kinds of users, they sort and assign values to the abovementioned indicators according to their preferences, shown in Table 5.

Table 5

Evaluation Indicators.

	The order of indicators	**The relative importance**
User 1	$U1 > U2 > U3 > U4 = U5$	$r2 = 1.8$; $r3 = 1.2$; $r4 = 1.4$; $r5 = 1.0$
User 2	$U2 > U3 > U1 = U4 > U5$	$r2 = 1.6$; $r3 = 1.4$; $r4 = 1$; $r5 = 1.2$

The information above was substituted into Eqs (11) and (16) to obtain the subjective weights for each indicator, as shown in Table 6.

Table 6

The subjective weight for Users.

	ω_{s,u_1}	ω_{s,u_2}	ω_{s,u_3}	ω_{s,u_4}	ω_{s,u_5}
User 1	0.3731	0.2073	0.1728	0.1234	0.1234
User 2	0.1545	0.3640	0.2163	0.1545	0.1287

The objective weights can be calculated via Eps(17) to (23), as shown in Table 7.

Table 7

The objective weight.

$\omega_{o,1}$	$\omega_{o,2}$	$\omega_{o,3}$	$\omega_{o,4}$	$\omega_{o,5}$
0.1490	0.2422	0.1193	0.2473	0.2422

Then, the comprehensive weights of users can be calculated via Eq (24) , as shown in Table 8.

Table 8

Comprehensive weight for Users

	ω_1	ω_2	ω_3	ω_4	ω_5
User 1	0.2975	0.2687	0.1104	0.1633	0.1601
User 2	0.1115	0.4272	0.1251	0.1851	0.1511

The positive ideal solution is obtained as

$$\overline{Z}^{U} = \{0.5834, 1.0000, 0.5545, 1.0000, 1.0000\}^{T} \tag{34}$$

The negative ideal solution is obtained as

$$\underline{Z}^{L} = \{0.1482, 0.0000, 0.2387, 0.0000, 0.0000\}^{T} \tag{35}$$

Using Eq (27) to (28), matrix ξ^{U} represents the grey correlation degrees with Z^{U} and $\overline{Z}^{U}$. Matrix ξ^{L} represents the grey correlation degrees with Z^{L} and $\underline{Z}^{L}$.

Using Eq (29) to (30), the weighted grey correlation degree of each parking space L_j^+, L_j^- and the comprehensive scores T_j are calculated; the data thereof are shown in Table 9 and Table 10.

Table 9

Improve TOPSIS evaluation results of user 1.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
L_j^+	0.7304	0.7543	0.5485	0.7258	0.5103	0.6129	0.5848	0.6779
L_j^-	0.6805	0.6531	0.8631	0.7320	0.8894	0.6307	0.7523	0.7437
T_j	0.5177	0.5360	0.3886	0.4976	0.3646	0.4928	0.4374	0.4769
Ranking	2	1	7	3	8	4	6	5

Table 10

Improve TOPSIS evaluation results of user 2.

	S_1	S_2	S_3	S_4	S_5	S_6	S_7	S_8
L_j^+	0.7611	0.6804	0.5242	0.6226	0.4555	0.5616	0.5464	0.5791
L_j^-	0.5934	0.6201	0.7835	0.7629	0.9100	0.5869	0.6933	0.7813
T_j	0.5619	0.5232	0.4009	0.4494	0.3336	0.4890	0.4408	0.4257
Ranking	1	2	7	4	8	3	5	6

5. Discussion

The ranking of the eight parking spaces derived from Table 9 is $S_2 > S_1 > S_4 > S_6 > S_8 > S_7 > S_3 > S_5$. S_2 achieve the highest comprehensive score (0.5360), and S_5 has the lowest comprehensive score (0.3646). Thus, S_2 is recommended for User 1.

For User 2, the ranking from Table 10 is $S_1 > S_2 > S_6 > S_4 > S_7 > S_8$ $S_1 > S_2 > S_6 > S_4 > S_7 > S_8 >$ S_1 has the highest comprehensive score (0.5619), and S_5 has the lowest comprehensive score (0.3336). Thus, S_1 is recommended for User 2.

User 1 represents a time-sensitive individual who prioritizes the arrival period. Despite similar arrival time windows for S_2 ([25,38]) and S_3 ([24,40]) ,the moderate weight of arrival time(U1 ,0.2975) does not dominate decision-making. Notably, S_2 significantly outperforms S_3 in parking convenience (U_3, comprehensive weight 0.0114),driving the recommendation.

User 2 is price-sensitive, with the weight of parking fee per hour (U_2, 0.4272), which is significantly higher than that of other indicators. Upon examination, it is evident that the hourly parking fee for S_1 is lower than that of other parking spaces. Therefore, S_1 is recommended for User 2.

The results demonstrate that the proposed model can comprehensively assess parking space schemes, incorporating both users' subjective preferences and objective information derived from various parking space attributes. Notably, the comprehensive evaluation reveals significant differences among different parking spaces, enabling meaningful comparisons. Consequently, this methodology is capable of effectively recommending suitable parking spaces tailored to different user types.

6. Conclusion

The intelligent reservation and management of parking spaces constitute a pivotal component of smart city development, with potential for broader application in smart business districts, communities, and campuses, thereby advancing informatization in related fields and fostering industrial transformation and upgrading. The problem of parking spaces recommendation is the core difficulty in this domain.

This study addresses the core challenge of parking space recommendation by proposing an optimized decision model for uncertain attribute scenarios. The model integrates exact and interval-valued data within a comprehensive evaluation framework, established through systematic literature synthesis, expert elicitation, and field validation. By fusing G1-based subjective weights with entropy weight-derived objective weights, and enhancing TOPSIS via weighted grey correlation analysis, the model enables precise comprehensive scoring of parking schemes. A case study involving 8 parking spaces and 2 user profiles validates its capability to deliver personalized recommendations, outperforming traditional methods in handling interval uncertainties.

Commercial implementation of the proposed approach is expected to yield multi-faceted benefits: reducing redundant vehicle trips , decreasing energy consumption, alleviating traffic congestion, and mitigating vehicular emissions. These outcomes align with sustainable urban development goals, demonstrating the model's potential to catalyze industrial transformation in intelligent transportation systems.

However, it should be noted that this study assumes positive and uniformly distributed interval values, which may not reflect the complexity of real-world scenarios. Future research could explore negative interval values and various probability distributions, as well as further refine indicators to better describe parking spaces and optimize the approach.

7. Acknowledgements

This research was funded by the Scientific Research Projects of Jinhua Advanced Research Institute, grant number G202402; the Research Project of the Cyber Range Training Platform, grant number BC202405; the Research Project of Jinhua Science and Technology Bureau, grant number 2026-3-017.

References

[1] A. Dehghani and A. Soltani, "Site selection of car parking with the gis-based fuzzy multi-criteria decision making," *International Journal of Information Technology & Decision Making*, pp. 1–26 (2023).

[2] H. Chai, R. Ma, and H. M. Zhang, "Search for parking: A dynamic parking and route guidance system for efficient parking and traffic management," *J. Intell. Transp. Syst.*, vol. 23, pp. 541–556 (2019).

[3] H. Errousso, E. A. A. Alaoui, S. Benhadou, and A. Nayyar, "Intelligent parking space management: a binary classification approach for detecting vacant spots," *Multimedia Tools* & *Applications*, vol. 84, no. 8 (2025).

[4] M. Xie, H. Wang, Y. Gao, and Y. Wang, "A rolling-horizon framework for managing shared parking and electric vehicle charging," *Sust. Cities Soc.*, vol. 98, p. 104810 (2023).

[5] A. Kalašová, K. čulík, M. Poliak, and Z. Otahálová, "Smart parking applications and its efficiency," *Sustainability*, vol. 13, no. 11, p. 6031 (2021).

[6] J. Parmar, P. Das, and S. M. Dave, "A machine learning approach for modelling parking duration in urban land-use," *Physica A*, vol. 572, p. 125873 (2021).

[7] O. B. Okeke, "The impacts of shared autonomous vehicles on car parking space," *Case Studies on Transport Policy*, vol. 8, no. 4, pp. 1307–1318 (2020).

[8] Janak, Parmar, Pritikana, Das, Sanjaykumar, and M. Dave, "Study on demand and characteristics of parking system in urban areas:a review," *Journal of Traffic and Transportation Engineering(English Edition)*, vol. 7, no. 1, pp. 116–129, (2020).

[9] S. Zeng, Y. Hu, and X. Xie, "Q-rung orthopair fuzzy weighted induced logarithmic distance measures and their application in multiple attribute decision making," *Eng. Appl. Artif. Intell.*, vol. 100, p. 104167 (2021).

[10] J. Zhan, H. Jiang, and Y. Yao, "Three-way multiattribute decision-making based on outranking relations," *IEEE Trans. Fuzzy Syst.*, vol. 29, pp. 2844–2858 (2021).

[11] R. Zhang, D. Tian, H. Wang, X. Kang, G. Wang, and L. Xu, "Risk assessment of compound dynamic disaster based on ahp-ewm," *Appl. Sci.*, vol. 13, no. 18 (2023). [Online]. Available: https://www.mdpi.com/2076-3417/13/18/10137

[12] T. Midrar, S. Khan, S. Abdullah, and T. Botmart, "Entropy based extended toposis method for mcdm problem with fuzzy credibility numbers," *AIMS Math.*, vol. 7, pp. 17, pp. 286–17 312 (2022).

[13] G. Büyüközkan, E. Mukul, and E. Kongar, "Health tourism strategy selection via swot analysis and integrated hesitant fuzzy linguistic ahp-mabac approach," *Socio-Econ. Plan. Sci.*, vol. 74, p. 100929 (2021).

[14] A. Torkayesh, S. Hashemkhani Zolfani, M. Kahvand, and P. Khazaelpour, "Landfill location selection for healthcare waste of urban areas using hybrid bwm-grey marcos model based on gis," *Sust. Cities Soc.*, vol. 67, p. 102712 (2021).

[15] C. Wang, X. Nguyen, and Y. Wang, "Automobile industry strategic alliance partner selection: The application of a hy- brid dea and grey theory model," *Sustainability*, vol. 8, p. 173 (2016).

[16] Y. Niu, Y. Fan, and Y. Li, "Safety performance measurement in collectivized oil companies in china: Contribution of leading indicators to lagging indicators," *J. Loss Prev. Process Ind.*, vol. 83, p. 105090 (2023).

[17] F. Bu, J. He, H. Li, and Q. Fu, "Interval-valued intuitionistic fuzzy madm method based on topsis and grey correlation analysis," *Math. Biosci. Eng.*, vol. 17, pp. 5584–5603 (2020).

[18] S. Javed, A. Mahmoudi, and S. Liu, "Grey absolute decision analysis (gada) method for multiple criteria group decision- making under uncertainty," *Int. J. Fuzzy Syst.*, vol. 22, pp. 1073–1090 (2020).

[19] L. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, pp. 338–353 (1965).

[20] X. Shu, T. Bao, Y. Li, K. Zhang, and B. Wu, "Dam safety evaluation based on interval-valued intuitionistic fuzzy sets and evidence theory," *Sensors*, vol. 20, p. 2648 (2020).

[21] R. Pérez-Fernández, P. Alonso, H. Bustince, I. Díaz, and S. Montes, "Applications of finite interval-valued hesitant fuzzy preference relations in group decision making," *Inf. Sci.*, vol. 326, pp. 89–101 (2016).

[22] X. Zhang, "Multi-criteria pythagorean fuzzy decision analy- sis: A hierarchical qualiflex approach with the closeness index- based ranking methods," *Inf. Sci.*, vol. 330, pp. 104–124 (2016).

[23] C. Da Costa Pereira and A. Tettamanzi, "Reasoning about actions with imprecise and incomplete state descriptions," *Fuzzy Sets Syst.*, vol. 160, pp. 1383–1401 (2009).

[24] J. Ye, S. Du, and R. Yong, "Multi-criteria decision-making model using trigonometric aggregation operators of single- valued neutrosophic credibility numbers," *Inf. Sci.*, vol. 644, p. 118968 (2023).

[25] F. Chen and X. Li, "Preferred parking space recommendation method based on multi-attribute decision-making algorithm," *J. Phys.: Conf. Ser.*, vol. 1972, p. 12067 (2021).

[26] S. Demir, M. Basaraner, and A. Taskin Gumus, "Selection of suitable parking lot sites in megacities: A case study for four districts of Istanbul," *Land Use Policy*, vol. 111, p. 105731 (2021).

[27] P. Wang, J. Wang, G. Wei, and C. Wei, "Similarity measures of q-rung orthopair fuzzy sets based on cosine function and their applications," *Mathematics*, vol. 7, p. 340 (2019).

[28] M. Saqlain, M. Riaz, M. Saleem, and M. Yang, "Distance and similarity measures for neutrosophic hypersoft set (nhss) with construction of nhss-topsis and applications," *IEEE Access*, vol. 9, pp. 30 803–30 816 (2021).

[29] B. Gohain, R. Chutia, P. Dutta, and S. Gogoi, "Two new simi- larity measures for intuitionistic fuzzy sets and its various applications," *Int. J. Intell. Syst.*, vol. 37, pp. 5557–5596 (2022).

[30] A. M. Al-Abadi, A. M. Handhal, M. A. Abdulhasan, W. L. Ali, J. Hassan, and A. H. Al Aboodi, "Optimal siting of large photovoltaic solar

farms at basrah governorate, southern iraq using hybrid gis-based entropy-topsis and ahp-topsis models," *Renewable Energy*, vol. 241, p. 122308 (2025). [Online]. Available: https://www.sciencedirect.com/science/article/pii/ S0960148124023760

[31] L. Pan, N. Yang, L. Zhang, R. Zhang, B. Xie, and H. Yan, "Assessment of the impact of multi-agent model-based traffic optimization interventions on urban travel behavior," *Electronics*, vol. 14, no. 1 (2025). [Online]. Available: https://www.mdpi.com/2079-9292/14/1/13

[32] M. Li, J. Cheng, and J. Fu, "Matching methods of shared parking slots considering overdue parking behavior," *Sustainability*, vol. 16, no. 24 (2024). [Online]. Available: https://www.mdpi.com/2071-1050/16/24/11253

[33] F. Ye, J. Sun, Y. Wang, N. Nedjah, and W. Bu, "A novel method for the performance evaluation of institutionalized collaborative innovation using an improved g1critic comprehensive evaluation model," *J. Innov. Knowl.*, vol. 8, p. 100289 (2023).

[34] X. Zhong, D. Wang, L. Mo, W. Zhang, L. Xiao, X. Wu, Y. Chen, and L. Yang, "Evaluation of the quality of covid-19 prevention and control by a novel comprehensive evaluation model in a tertiary general hospital: A prospective observational study," *BMC Public Health*, vol. 21, pp. 1–17 (2021).

[35] Y. Gao, X. Dong, F. Han, and Z. Li, "An optimization model for a desert railway route scheme based on interval number and topsis," *Appl. Sci.*, vol. 12, p. 10728 (2022).

Received July, 2025