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A novel method for solving fuzzy multi-objective linear quadratic fractional programming problems using Yager ranking function on triangular fuzzy number

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Abstract

A fuzzy set incorporated novel approach is proposed to address fuzzy multi-objective linear quadratic fractional programming problem (MOLQFPP). The problem involves optimizing a set of conflicting quadratic fractional objective functions subject to linear constraints with triangular fuzzy number. First, we change the vague multi-objective linear quadratic fractional programming problems (FPP) into crisp multi-objective linear quadratic fractional programming problems using the Yager ranking function. The equivalent crisp single objective optimization model is obtained from crisp MOLQFPP. The outcome of the crisp problem is solved by mathematical software called LINGO. The performance of these techniques was evaluated by comparing the results to those of other existing methods. The numerical results show that the proposed technique performs better than the other techniques.

Subject Classification: *03E72, 90C20, 90C29, 90C32, 90C70.*

Keywords: *Multi-objective optimization, Fractional programming, Quadratic programming, Yager ranking function, Fuzzy number.*

1. Introduction

Multi-objective linear fractional programming (MOLFP) is a highly effective decision-making tool for modelling. Problems with multiple objectives, such as profit versus cost, actual cost versus standard cost, debt versus equity, or inventory versus sales, are analyzed under the given system constraints. [1].

The foundation of fuzzy linear programming was first introduced by Vasant, Nagarajan, and Yaacob [2]. Later, Yager [3] extended linear programming by incorporating fuzzy parameters to support decision-making in industrial production planning. Zimmermann [4] revolutionized this field in 1983 by introducing multi-objective linear programming (MOLP) and describing a method for developing a multi-objective function based on the assumption that the optimal value of each individual objective function is positive.

Sen [5] proposed an adjusted symmetric fuzzy approach to obtain an optimal feasible compromise solution for quadratic fractional programming problems (QFPP), where both the objective functions and constraints are represented as fuzzy crisp. Nawkhass and Sulaiman [6] introduced a fuzzy linear fractional programming problem in which the objective function's input parameters are triangular and trapezoidal fuzzy numbers. The constraint values on both the left and right sides have been represented as real numbers.

Abdalla et al. [7] proposed a method for solving fuzzy linear fractional programming (FLFP) problems by illustrating all variables and parameters with triangular fuzzy numbers. Furthermore, Nachammai and Thangaraj [8] modifies a ranking function for solving full FLFPP in which trapezoidal fuzzy numbers define both the objective function and the constraints. Chakraverty and Sunnela [9] initiated a new ranking function to convert fuzzy numbers into crisp values, which was used to solve completely fuzzy linear programming problems and systems of equations.

Deb and De [10] implemented the α-cut method to solve fuzzy linear fractional programming (FLFP) problems. In addition, Rasha [11] put forward a new ranking function for trapezoidal fuzzy numbers to address fully fuzzy linear fractional programming problems (FLFPP) in which both the objective function and constraints are represented by a fuzzy number, such as trapezoidal.

Deepak, Priyankm, and Gaurav [12] investigated the fuzzy quadratic multilevel fractional programming problem by employing the fuzzy goal programming procedure. Kumar and Rakshit [13] used minimax value and optimal average maximin strategies to convert multi-objective quadratic fractional programming problems (MOQFPP) to corresponding single-objective QFPPs. As an expansion of this work, we investigate and put forward a ranking of fuzzy triangular numbers used by Suleiman and Nawkhass [14]. Using the Yager ranking function, the multi-objective fuzzy linear quadratic fractional programming problem can be modified into a multi-objective linear quadratic fractional programming problem.

We subsequently employ new mean and median techniques to transform the multi-objective linear quadratic fractional programming problem into a single linear quadratic fractional programming problem. This method is simple to understand and to implement. Finally, the method generates an optimal solution, which can be expressed as a fuzzy number or in crisp form.

The structure of this study is as follows:

In Section 2, a literature review is related to the fuzzy multi-objective linear quadratic fractional programming problem presented. Section 3 describes the problem's methodology. Section 4 presents some fundamental definitions related to MOFLQFPP. Section 5 develops the mathematical model for these problems and outlines the proposed solution approach. In Section 6, we discussed the algorithm of the proposed method. Section 7 provides numerical examples for illustrating the proposed approaches and compare them with other existing methods. In Section 8, there is a comparative study of different methods. Finally, in Section 9, the conclusion is presented.

2. Literature Review

This section reviews various studies concerning mean and median scalarization approaches applied to fuzzy multi-objective linear quadratic fractional programming problems.

The process of reducing multi-objective optimization issues to single-objective optimization problems and employing various approaches to solve them has been extensively researched and examined in numerous academics. Listed among them are a few who are to solve multi-optimization linear programming problems [15], improved scalarization algorithms utilizing geometric, harmonic, and mean have been employed. Sulaiman, and Salih [16] Implemented medians and means and a solution for the multi-objective linear fractional programming problem was determined.

Nahar and Alim[17] proposed innovative average methods which were both geometric and mathematical. Akhtar and Modi [18] describe methods to solve multi-objective linear fractional programming problems with suggested and advanced harmonic average approaches. The multi-objective linear programming problem was solved through new statistical averaging methods and advanced transformation techniques [19–20].

Furthermore, Mustafa and Sulaiman [21] developed new and improved mean deviations to handle MOFPP. According to Abdalla and Abdullah [22], a MOLFPP can be reduced to a single-objective one using Pearson's second skewness coefficient, which is based on the mean and median. Lachhwani [25] has recently proposed a fuzzy goal programming (FGP) approach for solving multi-objective quadratic programming problems. Subsequently, Lachhwani [24] employed this strategy to solve problems in multi-objective quadratic fractional programming.

To address more complicated multi-level linear fractional programming problems (ML-LFPPs), Lachhwani and Nehra [28] recommended a modified fuzzy goal programming (FGP) technique in conjunction with MATLAB implementation. Additionally, Li and Hu [23] employed the same approach to solve fuzzy MOO problems. Using the same approach, they established a promising technique for handling fuzzy problems involving multiple objectives.

Lachhwani [27] implemented FGP to solve MOO problems that incorporated fuzzy relational equations (FREs) as constraints. Similarly, in 2014 [26], Lachhwani presented a novel solution technique for multi-level multi-objective linear programming problems that uses fuzzy goal programming (FGP). Kumar and Rakshit [29] used a fuzzy goal programming procedure to investigate fuzzy multilevel quadratic fractional programming problems. Using recently established mean and median base methodologies, the main goal of this study is to solve a fuzzy multi-objective linear quadratic fractional programming problem with triangular fuzzy integers.

The suggested methodology also addresses the drawbacks of the existing methods.

3. Solution Procedure

In the present approach, we have solved a fuzzy MOLQFPP involving triangular fuzzy numbers using the new mean and median techniques First, the suggested method is used to convert the multi-objective fuzzy linear quadratic fractional programming problem into an equivalent crisp MOLQFPP. Second, the new mean and median techniques 1 are used to transform the linear MOQFPP into a linear quadratic programming problem, which is then solved with LINDO mathematical software. The outcomes are contrasted with those derived from other currently used techniques. Lastly, numerical examples are presented to demonstrate the efficacy of the suggested approach.

4. Preliminaries

The definitions, theorems, and basic ideas necessary for the study of MOFLQFPPs are presented in this section.

Definition [30]: A triangular fuzzy number, (Figure 1) $\widetilde{A} = (a_l, a_m, a_u), a_l \leq a_m \leq a_u$ is defined with its membership function as:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_l}{a_m-a_l}, a_l \leq x \leq a_m \\ \frac{a_u-x}{a_u-a_m}, a_m \leq x \leq a_u \\ 0, otherwise \end{cases} \tag{1}$$

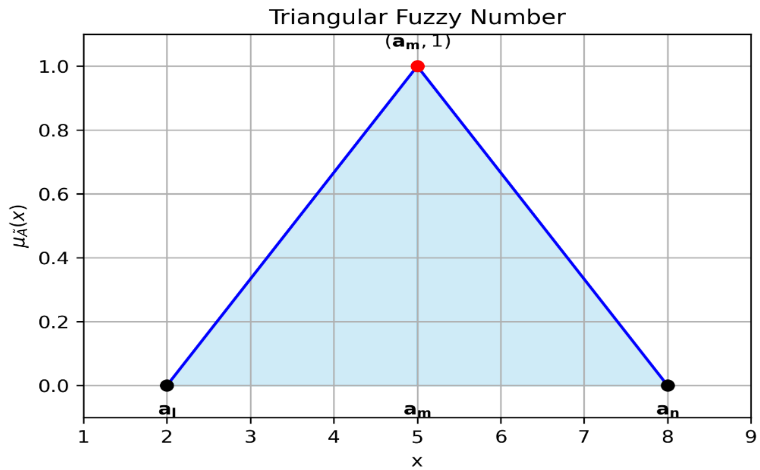

Figure 1
Triangular fuzzy number $\widetilde{A}$

Definition (Yager's ranking function [14]): A ranking approach that fulfills the properties of compensation, linearity, and additivity, and yields outcomes consistent with human intuition. If $\tilde{A} = (a_l, a_m, a_u)$ is a triangular fuzzy number then the Yager's ranking is defined by:

$$R(\tilde{A}) = \frac{1}{2}\int_0^1 (a^L{}_\alpha + a^U{}_\alpha) d\alpha, \tag{2}$$

where $(a^L{}_\alpha, a^U{}_\alpha) = \{(a_m - a_l) + a_l, a_u - (a_u - a_m)\alpha\}$

5. Mathematical Formulation

5.1. *Quadratic Programming*

Quadratic programming (QP) belongs to the family of nonlinear programming problems and involves optimizing a quadratic objective function under a system of linear constraints, which may include both equalities and inequalities.

A quadratic programming problem general form, where the objective functional is quadratic and the constraints are linear in $x \in R^n$ is defined as:

$$Max/Min\ Z. = \frac{1}{2}x^T Gx + c^T x + a$$

Subject to (3)

$$Ax \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b$$
$$x \geq 0$$

where $(a_{ij})_{m\times n}$ =A, matrix of coefficients, $\forall_i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$, $(b_1, b_2, \ldots, b_M)^T = b, x = (x_1, x_2, \ldots, x_n)^T, (c_1, c_2, \ldots, c_n)^T = C^T, and\ G = (g_{ij})_{n\times n}$ is symmetric square matrix with positive definite or positive semi-definite and the constraints are linear and the objective function is quadratic. Such optimization problem is said to be a programming QP.

5.2. *Fractional Quadratic Programming Problem*

Below is the formulation of the quadratic fractional programming model:

$$Max\ z. = \frac{(c^T x + \delta + Gx\frac{1}{2}x^T)}{(\gamma + C^T x)}$$

Subject to (4)

$$Ax \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b$$
$$x \geq 0$$

Here, G is an n × n matrix of coefficients and is assumed to be symmetric. Unless explicitly transposed (T), all vectors are considered column vectors. Where x is an n-dimensional vector of decision variables, c, C is the n – dimensional vector of constants, b is n -dimensional vector of constants.γ, δ are scalars.

5.3. *Multi-Objective Quadratic Fractional Programming Problem (MOQFPP)*

The objective functions in a MOQFPP are defined as the ratio of two functions, where the numerator is quadratic and the denominator is linear. The goal of these problems is to optimize several fractional objective functions at once. The following is a formal definition of the MOQFPP:

$$\begin{aligned} Max. Z_1 &= \frac{({c_{21}}^T x + \beta_1)}{{C_1}^T x + \gamma_1}({c_{11}}^T x + \alpha_1) \\ &\vdots \\ Max. Z_r &= \frac{({c_{1r}}^T x + \alpha_r)({c_{2r}}^T x + \beta_r)}{\gamma_r + {C_r}^T x} \\ Min\ Z_{r+1} &= \frac{({c_{1r+1}}^T x + \alpha_{r+1})({c_{2r+1}}^T x + \beta_{r+1})}{{C_{r+1}}^T x + \gamma_{r+1}} \\ &\vdots \\ Min\ Z_s &= \frac{({c_{1s}}^T x + \alpha_s)({c_{2s}}^T x + \beta_s)}{\gamma_s + {C_s}^T x} \end{aligned} \tag{5}$$

Subject to

$$Ax = b$$
$$x \geq 0$$

Here, x is an n-dimensional vector of decision variables, b is an m-dimensional vector of constants, and r represents the number of objective functions to be maximized. Additionally, s denotes the total number of objective functions, with (s - r) being the number of objective functions to be minimized.

A is nxm matrix, all vectors are assumed to be column vectors unless transposed (T). c_{1i}, c_{2i} , C_i where $i = 1, \dots, s$ are the n-dimensional vector of constants, $\alpha_i, \beta_i, \gamma_i$

5.4. *Fuzzy Multi-Objective Linear Quadratic Fractional Programming Problem (FMOLQFPP)*

The programming problem involving fuzzy numbers with multi-objective linear quadratic fractional functions can be formulated as follows:

$$\begin{aligned} Max. Z_1 &= \frac{\tilde{C}_{11}x + \frac{1}{2}x^T \tilde{D}_{11}x}{\tilde{C}_{12}x + \frac{1}{2}x^T \tilde{D}_{12}x} \\ &\;\;\vdots \\ Max. Z_r &= \frac{\tilde{C}_{r1}x + \frac{1}{2}x^T \tilde{D}_{r1}x}{\tilde{C}_{r2}x + \frac{1}{2}x^T \tilde{D}_{r2}x} \\ Min\ Z_{r+1} &= \frac{\tilde{C}_{(r+1)1}x + \frac{1}{2}x^T \tilde{D}_{(r+1)1}x}{\tilde{C}_{(r+1)2}x + \frac{1}{2}x^T \tilde{D}_{(r+1)2}x} \\ &\;\;\vdots \\ Min\ Z_s &= \frac{\tilde{C}_{s1}x + \frac{1}{2}x^T \tilde{D}_{s1}x}{\tilde{C}_{s2}x + \frac{1}{2}x^T \tilde{D}_{s2}x} \end{aligned} \tag{6}$$

Subject to

$$\tilde{A}x \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} \tilde{b}$$
$$x \geq 0$$

Here, x is an n-dimensional vector of decision variables, b is an m-dimensional vector of constants, r denotes the number to be maximized, (s - r) represents the number to be minimized, and s is the total number of objectives.A is $m \times n$ matrix, all vectors are assumed to be column vectors unless transposed (T). $\tilde{c}_{1i}$, $\tilde{c}_{2i}$, $\tilde{C}_{3i}$ with $i = 1, \dots, s$ are the n-dimensional fuzzy vectors and constants, $\tilde{\alpha}_i, \tilde{\beta}_i, \tilde{\gamma}_i$ with $i = 1, \dots, s$ are fuzzy scalars.

5.5. *Proposed Technique*

To scalarize fuzzy linear multi-objective quadratic fractional programming problem, we use statistical measures like the mean, median, and standard deviation. Below, we briefly outline the recommended method that relies on the idea of mean and median. The steps for changing a MOLQFPP into a single-objective linear QFPP are as follows:

$$MaxZ=\frac{\sum_{i=1}^{r} Z_i-\sum_{i=r+1}^{n} Z_i}{\text{NMMT1}} \tag{7}$$

Here, NMMT denotes new mean and median technique 1, and given by:

$$\text{NMMT1} = \left(\frac{\min\{M,M_d,S\}}{M+M_d+S}\right)\left(\frac{1}{N}\right) \tag{8}$$

where, M=Mean of $|\gamma_i|$, $\forall i$=1,2, … , n.
M_d =Median of $|\gamma_i|$, $\forall i$=1,2, … , n.
S= Standard deviation of $|\gamma_i|$, $\forall i$=1,2, … , n.
N =Total number of objective functions.

6. Algorithm of the proposed method

We present an algorithm for solving the fuzzy multi-objective linear quadratic fractional programming problem (6) as follows:

Step 1: Convert FMOLQFPP into MOLQFPP by using Yager's ranking technique as given equation (2).

Step 2: Determine the best value for each fractional objective function based on the specified constraints by using a variable transformation.

Step 3: After determining the specific optimal values for each fractional obective function through variable transformation, compute NMMT1 using formula (8).

Step 4: Using formula (7) and the individual optimal values found in the previous step, .to form a single, combined fractional objective function.

Step 5: Then, with the same set of constraints, optimize this combined fractional objective function.

$$Max\ Z=\frac{\sum_{i=1}^{r} Z_i-\sum_{i=r+1}^{n} Z_i}{\text{NMMT1}}$$

7. Numerical Examples

To illustrate the use of the suggested methods and algorithm, this section provides illustrative examples. A few examples drawn from earlier research are used to illustrate how our method differs from others.

Example 1: Consider the following fuzzy multi-objective linear quadratic fractional programming problem.

$$Maximize\ Z_1 = \frac{(6,7,8)x_1{}^2+(4,5,7)x_1x_3+(6,7,8)}{(3,4,5)x_1{}^2+(3,4,5)x_2{}^2+(7,8,9)}$$

$$Maximize\ Z_2 = \frac{(1,2,3)x_3{}^2+(4,6,7)x_2+(-5-4,-2)}{(1,2,3)x_1{}^2+(1,2,3)x_2{}^2+(3,4,5)}$$

$$Maximize\ Z_3 = \frac{(4,5,6)x_3x_2+(6,8,9)x_3x_1+(1,2,3){x_2}^2+(3,5,7)}{(1,2,3){x_1}^2+(1,2,3){x_2}^2+(3,4,5)}$$

Subject to (9)

$$(2,3,4)x_1 + (4,5,6)x_2 - (1,2,3)x_3 \leq (6,7,8)$$
$$(7,8,9)x_1 - (11,12,13)x_2 + (5,6,7)x_3 \leq (11,12,13)$$
$$x_1, x_2, x_3 \geq 0$$

Solution: By using equation (2) the given problem is given as:

$$Maximize\ Z_1 = \frac{7{x_1}^2+6.25x_1x_3+7}{4{x_1}^2+4{x_2}^2+8}$$
$$Maximize\ Z_2 = \frac{2{x_3}^2+5.75x_2-3.75}{2{x_1}^2+2{x_2}^2+4}$$
$$Maximize\ Z_3 = \frac{5x_3x_2+7.75x_3x_1+2{x_2}^2+5}{2{x_1}^2+2{x_2}^2+4}$$

Subject to (10)

$$3x_1 + 5x_2 - 2x_3 \leq 7$$
$$8\,x_1 - 12x_2 + 6x_3 \leq 12$$
$$x_1, x_2, x_3 \geq 0$$

Table 1

Results of Example 1 after individually optimizing all objective functions.

No.	**Optimal solution (x_1, x_2, x_3, x_4)**	**Optimal value $Z_i^* = \|\psi_i\|$**	**Mean (Z_i^*)**	**Median (Z_i^*)**	**Standard deviation(S)**
1.	(1.43, 1.12, 2.34)	1.99			
2.	(0, 2.05, 6.11)	6.65	5.32	6.65	2.37
3.	(0.92,3.43, 7.64)	7.32			

Equation (7) is then used to formulate the combined quadratic fractional programming problem using the data from Table 1 and the New Mean and Median Techniques 1 as follows:

$$Max\ Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{\text{NMMT1}},$$

where, $\text{NMMT1} = \left(\frac{\min\{5.32, 6.65, 2.37\}}{5.32+6.65+2.37}\right)\left(\frac{1}{3}\right) = \left(\frac{2.37}{43.02}\right) = 0.055$

So,

$$Maximize\ Z = \frac{1}{0.055}\left(\frac{7{x_1}^2+4{x_2}^2+4{x_3}^2+21.75x_1x_3+10x_3x_2+11.5x_2+9.5}{4{x_1}^2+4{x_2}^2+8}\right)$$

Subject to

$$3x_1 + 5x_2 - 2x_3 \leq 7 \tag{11}$$
$$8\,x_1 - 12x_2 + 6x_3 \leq 12$$

$$x_1, x_2, x_3 \geq 0$$

Solving problem (11), $Max\ Z = 243.21$ at (0.32,2.46,6.48) is resulted

Now, Example 1 is solved by using alternative techniques, such as:

1. By mean [10], Max Z=2.51 at (0.32,2.46,6.48) is resulted
2. From Median [10], the result obtained is Max Z = 2.01 at the point (0.32, 2.46, 6.48).
3. By Standard Mean Deviation [14], Max Z = 5.64 at (0.32,2.46,6.48) is the result.
4. Using the new Arithmetic Average [10], Max Z = 3.36 at the point (0.32, 2.46, 6.48) is the result.
5. Using the Pearson 2 Skewness Coefficient [15], the result is Max Z = 7.96 at the point (0.32, 2.46, 6.48).

Example 2: Consider the following problem.

$$Maximize\ Z_1 = \frac{(5,6,7)x_1{}^2+(4,4,5)x_2x_3+(7,8,9)}{(1,2,3)x_1{}^2+(3,5,7)x_2{}^2+(1,3.5)}$$

$$Maximize\ Z_2 = \frac{(1,2,3)x_2{}^2+(4,4,5)x_4+(3,5,7)}{(1,2,3)x_1{}^2+(3,5,7)x_2{}^2+(1,3.5)}$$

$$Minimize\ Z_3 = \frac{(4,4,5)x_3x_2+(5,6,7)x_3x_1+(1,2,3)x_1+(3,5,7)}{(1,2,3)x_4{}^2+(3,4,5)x_4x_1+(3,4,5)x_3+(7,8,9)}$$

$$Minimize\ Z_3 = \frac{(8,10,12)x_3+(4,4,5)x_4x_1+(1,2,3)}{(1,2,3)x_4{}^2+(3,4,5)x_4x_1+(3,4,5)x_3+(7,8,9)}$$

Subject to (12)

$$(1,3,5)x_1 + (3,5,7)x_2 - (1,2,3)x_3 + (4,4,5)x_4 \leq (15,20,25)$$

$$(7,8,9)x_1 - (11,12,13)x_2 + (5,6,7)x_3 - (11,12,13)x_4 \leq (8,10,12)$$

$$x_i \geq 0 \text{ , I =1,2,3,4}$$

Solution: By using equation (2) the given problem is converted from the fuzzy form to the crisp form then the problem is formulated as follow:

$$Maximize\ Z_1 = \frac{6x_1{}^2+4.25x_2x_3+8}{2x_1{}^2+5x_2{}^2+3}$$

$$Maximize\ Z_2 = \frac{2x_2{}^2+4.25x_4+5}{2x_1{}^2+5x_2{}^2+3}$$

$$Minimize\ Z_3 = \frac{4.25x_3x_2+6x_3x_1+2x_1+5}{2x_4{}^2+4x_4x_1+4x_3+8}$$

$$Minimize\ Z_4 = \frac{10x_3+4.25x_4x_1+2}{2x_4{}^2+4x_4x_1+4x_3+8}$$

Subject to (13)

$$3x_1 + 5x_2 - 2x_3 + 4.25x_4 \leq 20$$

$$8x_1 - 12x_2 + 6x_3 - 12x_4 \leq 10$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Table 2
Combined quadratic fractional problem using Table 1 and NMMT1 via equation (7).

No.	Optimal solution (x_1, x_2, x_3, x_4)	Optimal value $Z_i^* = \lvert\psi_i\rvert$	Mean (Z_i^*)	Median (Z_i^*)	Standard deviation(S)
1.	(0,0.73,119.94,58.40)	67.11			
2.	(0,0,124.33,61.33)	88.56	38.94		
3.	(0,0.73,119.94,58.40)	0.00062		33.60	39.63
4	(0, 0,0, 2.82)	0.084			

Then, we find combined quadratic fractional programming problem using the information of Table 2 and New Mean and Median Techniques 1 by equation (7), which is

$$Max\ Z = \frac{\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{n} Z_i}{\text{NMMT1}},$$

Where, $\text{NMMT1} = \left(\frac{\min\{38.94, 33.60, 39.63\}}{38.94+33.64+39.63}\right)\left(\frac{1}{4}\right) = \left(\frac{33.60}{448.68}\right) = 0.075$

So,

$$Max\ Z = \frac{1}{0.075}\left(\frac{6{x_1}^2+4.25x_2x_3+2{x_2}^2+4.25x_4+13}{2{x_1}^2+5{x_2}^2+3}\right) - \left(\frac{4.25x_3x_2+6x_3x_1+2x_1+10x_3+4.25x_4x_1+7}{2{x_4}^2+4x_4x_1+4x_3+8}\right)$$

Subject to (14)

$$3x_1 + 5x_2 - 2x_3 + 4.25x_4 \le 20$$
$$8x_1 - 12x_2 + 6x_3 - 12x_4 \le 10$$
$$x_1, x_2, x_3, x_4 \ge 0$$

After solving problem (14), we get $Max\ Z = 1662.384$ at $(0, 0.39, 121.97, 59.76)$.

Now, we solve Example 2 using alternative techniques, such as:

1. Using the Mean [10], the maximum value obtained is Z = 3.20 at (0, 0.39, 121.97, 59.76).
2. From Median [10], the maximum value obtained is Z = 3.71 at (0, 0.39, 121.97, 59.76).
3. Using the Standard Mean Deviation [14], the maximum value obtained is Z = 3.15 at (0, 0.39, 121.97, 59.76).
4. (0, 0.39, 121.97, 59.76) Using the new Arithmetic Average [10], the maximum value obtained is Z = 3.72.
5. (0, 0.39, 121.97, 59.76) Using the New Geometric Average [10], the maximum value obtained is Z = 611.17.
6. (0, 0.39, 121.97, 59.76) Using the Pearson 2 Skewness Coefficient [15], the maximum value obtained is Z = 311.7.

8. Comparative Study

The table below presents a summary of the FMOLQFPP results using various techniques, comparing the proposed method with existing ones. As shown, the proposed technique yields improved outcomes over previously studied approaches.

Moreover, the proposed technique is capable of solving problems that some existing methods fail to address.

Table 3

Comparison of five numerical results obtained from the previous examples.

Techniques	Example 1	Example 2
Mean	2.45	3.20
Median	2.39	3.71
Standard Mean Deviation	5.89	3.15
New Arithmetic average	9.5	3.72
New Geometric Average	---	611.17
Pearson2 Skewness coefficient	25.69	311.7
Proposed Technique	83.38	1662.384

In Table 3, '---' shows that the techniques unable to solve the given problems. The results for examples 1 and 2 clearly demonstrate that the proposed method, New Mean and Median Technique 1, outperforms the other techniques.

9. Conclusion

This article presents a novel approach to solving the fuzzy MOLQFPP susing the fuzzy relation equation. All coefficients and parameters are represented as triangular fuzzy numbers. The method converts the fuzzy multi-objective problem to a crisp linear quadratic fractional programming problem. We use the New Mean and Median Technique 1 to simplify the problem to a single objective. The problem obtained is then solved by LINDO software in a straight and very efficient way. The proposed approach is more generalized and can handle the conflicting fuzzy multi-objective problems better. Numerical results are given in a few examples to illustrate the solution procedure and its efficiency in comparison with methods considered earlier. Numerical results validate that our proposed method can obtain superior optimization solutions over other methods.

Author’s Contribution:

Individual authors contributed towards, literature survey of existing methodologies, the gaps in the existing work, interfacing of various components and generating result of models, testing the and comparing different methods, technical content writing, table formation and overall formatting, citing references etc. All authors declare that they have no conflicts of interest.

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