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Process mean and economic manufacturing quantity settings by considering time value of money

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Abstract

We present the extended production problem with defective product and time value of money. The mean and production quantity are got based on the minimization of the expected cost.

Subject Classification: *60E05 Probability distributions: general theory.*

Keywords: *Production quantity, Time value of money.*

1. Introduction

The classical production problem discusses the excellent quality. Sana et al. [1, 2], Tahera et al. [3], and Sana [4, 5] have proposed the work about imperfect manufacturing process.

Recently, Chen [6, 7] applied the asymmetric loss concept for getting mean and production quantity. Chen and Chou [8, 9] applied Taguchi's [10] criteria for getting the mean.

Teran et al. [11] transferred the quality performance measurement into the present worth per unit. Hence, Teran et al.'s study also considered the time value of money per unit when customer holds it. Chou & Liang [12] and Chou & Chen [13] have presented the bivariate/multivariable quality loss with the time value of money. Their work addressed the sensitivity analyses of parameters for the effects of biases at production time and associated quality loss.

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Tahera et al. [14] considered mean and production time in the defective production. Chen and Chou [13] further presented the extended Tahera et al.'s one about obtaining mean, specification limits, and EMQ under the specified process capability.

In this work, we try to address a modified one with time value of money for obtaining the mean and EMQ.

2. Modified Tahera et al.'s Model

For two nominal and independent characteristics, the loss function is

$$L(y)=k_{11}(y_1-T_1)^2+k_{22}(y_2-T_2)^2+k_{12}(y_1-T_1)(y_2-T_2) \tag{1}$$

where k_{11}, k_{22}, and k_{12} is quality loss coefficients; T_1 and T_2 are target values of characteristics.

The expected quality cost, $E[L(y)]$ is

$$k_{11}E[(y_1-T_1)^2]+k_{22}E[(y_2-T_2)^2]+k_{12}E[(y_1-T_1)(y_2-T_2)] \tag{2}$$

Similar to Tahera et al., the expected quality cost, $E[L(t)]$ is

$$\begin{aligned}
&\{k_{11}[\sigma_1^2+(\mu_1-T_1)^2]+k_{22}[\sigma_2^2+(\mu_2-T_2)^2]+k_{12}[(\mu_1-T_1)(\mu_2-T_2)]\}\exp(-\lambda_1 t)\\
&\quad+k_{11}[(\mu_1+\delta_1-T_1)^2+\sigma_1^2](1-\exp(-\lambda_1 t))+k_{11}[2(\mu_1+\delta_1-T_1)\theta_1\\
&\quad+\sigma_1^2]\left[t-\frac{1}{\lambda_1}(1-\exp(-\lambda_1 t))\right]+k_{11}\theta_1^2\left[t^2-2\frac{t}{\lambda_1}+\frac{2}{\lambda_1^2}(1-\exp(-\lambda_1 t))\right]\\
&\quad+k_{22}[(\mu_2+\delta_2-T_2)^2+\sigma_2^2](1-\exp(-\lambda_1 t))\\
&\quad+k_{11}[2(\mu_2+\delta_2-T_2)\theta_2+\sigma_2^2]\left[t-\frac{1}{\lambda_1}(1-\exp(-\lambda_1 t))\right]\\
&\quad+k_{22}\theta_2^2\left[t^2-2\frac{t}{\lambda_1}+\frac{2}{\lambda_1^2}(1-\exp(-\lambda_1 t))\right]\\
&\quad+k_{12}[(\mu_1+\delta_1-T_1)(\mu_2+\delta_2-T_2)](1-\exp(-\lambda_1 t))\\
&\quad+k_{12}(\mu_1+\delta_1-T_1)\theta_2\left[t-\frac{1}{\lambda_1}(1-\exp(-\lambda_1 t))\right]\\
&\quad+k_{12}(\mu_2+\delta_2-T_2)\theta_1\left[t-\frac{1}{\lambda_1}(1-\exp(-\lambda_1 t))\right]\\
&\quad+k_{12}\theta_1\theta_2\left\{t^2(1-\exp(-\lambda_1 t))-2t\left[\frac{1}{\lambda_1^2}(1-\exp(-\lambda_1 t))-\frac{t}{\lambda_1}\exp(-\lambda_1 t)\right]\right.\\
&\quad\left.+\frac{2}{\lambda_1}\left[\frac{1}{\lambda_1^2}(1-\exp(-\lambda_1 t))-\frac{t}{\lambda_1}\exp(-\lambda_1 t)\right]-\frac{t^2}{\lambda_1}\exp(-\lambda_1 t)\right\}
\end{aligned} \tag{3}$$

The expected quality loss with time value of money, ETC_Q is

$$\{k_{11}[\sigma_1^2+(\mu_1-T_1)^2]+k_{22}[\sigma_2^2+(\mu_2-T_2)^2]$$
$$+k_{12}(\mu_1-T_1)(\mu_2-t_2)]\}\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))$$
$$+k_{11}[(\mu_1+\delta_1-T_1)^2+\sigma_1^2][\tfrac{1}{r_1}(1-\exp(-r_1Q/p))$$
$$-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]$$
$$+k_{11}[2(\mu_1+\delta_1-T_1)\theta_1+\sigma_1^2]\{\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{Q\exp(-r_1Q/p)}{pr_1}$$
$$-\tfrac{1}{\lambda_1}[\tfrac{1}{r_1}(1-\exp(-r_1Q/p))-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]\}$$
$$+k_{11}\theta_1^2\{\tfrac{2}{r_1}[\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{Q\exp(-r_1Q/p)}{pr_1}]-\tfrac{(Q/p)^2\exp(-r_1Q/p)}{r_1}$$
$$-\tfrac{2}{\lambda_1}[\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{Q\exp(-r_1Q/p)}{pr_1}]$$
$$+\tfrac{2}{\lambda_1^2}[\tfrac{1}{r_1}(1-\exp(-r_1Q/p))-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]\}$$

$$+k_{22}[(\mu_2+\delta_2-T_2)^2+\sigma_2^2][\tfrac{1}{r_1}(1-\exp(-r_1Q/p))$$
$$-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]+k_{22}[2(\mu_2+\delta_2-T_2)\theta_2$$
$$+\sigma_2^2]\{\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{(Q/p)^2\exp(-r_1Q/p)}{r_1}$$
$$-\tfrac{1}{\lambda_1}[\tfrac{1}{r_1}(1-\exp(-r_1Q/p))-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1\ )Q/p))]\}$$
$$+k_{22}\theta_2^2\{\tfrac{2}{r_1}[\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{Q\exp(-r_1Q/p)}{pr_1}]-\tfrac{(Q/p)^2\exp(-r_1Q/p)}{r_1}$$
$$-\tfrac{2}{\lambda_1}[\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{Q\exp(-r\ Q/p)}{pr_1}]$$
$$+\tfrac{2}{\lambda_1^2}[\tfrac{1}{r_1}(1-\exp(-r_1Q/p))-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]\}$$
$$+k_{12}[(\mu_1+\delta_1-T_1)(\mu_2+\delta_2-T_2)][\tfrac{1}{r_1}(1-\exp(-r_1Q/p))$$
$$-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]$$
$$+k_{12}(\mu_1+\delta_1-T_1)\theta_2\{\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{Q\exp(-r_1Q/p)}{pr_1}$$
$$-\tfrac{1}{\lambda_1}[\tfrac{1}{r_1}(1-\exp(-r_1Q/p))-\tfrac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]\}$$
$$+k_{12}(\mu_2+\delta_2-T_2)\theta_1\{\tfrac{1}{r_1{}^2}(1-\exp(-r_1Q/p))-\tfrac{Q\exp(-r_1Q/p)}{pr_1}]$$

$$-\frac{1}{\lambda_1}[\frac{1}{r_1}(1-\exp(-r_1Q/p))-\frac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]\}$$
$$+k_{12}\theta_1\theta_2\{\frac{2}{r_1}[\frac{1}{r_1^2}(1-\exp(-r_1Q/p))-\frac{(Q/p)^2\exp(-r_1Q/p)}{r_1}$$
$$-\frac{2}{\lambda_1+r_1}[\frac{1}{(\lambda_1+r_1)^2}(1-\exp(-(\lambda_1+r_1)Q/p))]-\frac{(Q/p)^2\exp(-(\lambda_1+r_1)Q/p)}{\lambda_1+r_1}$$
$$-\frac{2}{\lambda_1^2}[\frac{1}{r_1^2}(1-\exp(-r_1Q/p))-\frac{Q}{pr_1}\exp(-r_1Q/p)]$$
$$+2[\frac{1}{(\lambda_1+r_1)^2}(1-\exp(-(\lambda_1+r_1)Q/p))]-\frac{(Q/p)\exp(-(\lambda_1+r_1)Q/p)}{\lambda_1+r_1}]$$
$$+\frac{2}{\lambda_1}\{\frac{2}{\lambda_1+r_1}[\frac{1}{(\lambda_1+r_1)^2}(1-\exp(-(\lambda_1+r_1)Q/p))]-\frac{(Q/p)^2\exp(-(\lambda_1+r_1)Q/p)}{\lambda_1+r_1}\}$$
$$+\frac{2}{\lambda_1}\{\frac{1}{\lambda_1^2}[\frac{1}{r_1}(1-\exp(-r_1Q/p))-\frac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]$$
$$-\frac{1}{\lambda_1}[\frac{1}{(\lambda_1+r_1)^2}(1-\exp(-(\lambda_1+r_1)Q/p))-\frac{(Q/p)\exp(-(\lambda_1+r_1)Q/p)}{\lambda_1+r_1}]\}$$
$$-\frac{1}{\lambda_1}\{\frac{2}{\lambda_1+r_1}[\frac{1}{(\lambda_1+r_1)^2}(1-\exp(-(\lambda_1+r_1)Q/p))]-\frac{(Q/p)^2\exp(-(\lambda_1+r_1)Q/p)}{\lambda_1+r_1}\}\} \quad (4)$$

where Q is production quantity; p is production rate; r is interest rate.

The expected material cost with time value of money, ETC_R is

$$C_R\{(\mu_1+\mu_2)\frac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r\ )Q/p))+(\mu_1+\mu_2+\delta_1+\delta_2)$$
$$[\frac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))-\frac{1}{r_1}(1-\exp(-r_1Q/p))]\ \ +(\theta_1+\theta_2)$$
$$\{\frac{1}{(\lambda_1+r_1)^2}(1-\exp(-(\lambda_1+r_1)Q/p))-\frac{(Q/p)\exp(-(\lambda_1+r_1)Q/p)}{\lambda_1+r_1}$$
$$-[\frac{1}{r_1^2}(1-\exp(-r_1Q/p))-\frac{(Q/p)\exp(-r_1Q/p)}{r_1}]\}+(\theta_1+\theta_2)$$
$$\{\frac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))-\frac{(Q/p)\exp(-(\lambda_1+r_1)Q/p)}{\lambda_1+r_1}-\frac{1}{\lambda_1}$$
$$[\frac{1}{r_1}(1-\exp(-r_1Q/p))-\frac{1}{\lambda_1+r_1}(1-\exp(-(\lambda_1+r_1)Q/p))]\} \quad (5)$$

where C_R is unit cost of material.

The handing cost with time value of money, ETC_H is

$$H(p-D)[\frac{1}{r_1^2}(1-\exp(-r_1Q/p))-\frac{Q}{pr_1}\exp(-r_1Q/p)] \quad (6)$$

where H is holding cost; D is demand rate.

The total average maintenance cost with time value of money, ETC_M is

$$\frac{\alpha}{r_1}(1-\exp(-r_1Q/p))+\beta[\frac{1}{r_1^{\,2}}(1-\exp(-r_1Q/p))-\frac{Q}{pr_1}\exp(-r_1Q/p)] \tag{7}$$

where $M(t)$ is maintenance cost function, $M(t)=\alpha+\beta t$, $\alpha>0$, $\beta>0$.

Hence, the expected total cost, $ETC(\mu_1, \mu_2, Q)$ is

$$\frac{ETC_Q+ETC_R+S+ETC_H+C_A+ETC_M}{Q/p} \tag{8}$$

where S is set-up cost; C_A is adjustment cost.

Eq. (8) needs to find the combination of $(\mu_1^*,\ \mu_2^*,\ Q^*)$ which minimizes the expected total cost. The procedure of solution is similar to Chen et al. [15].

3. Numerical Example

Consider some parameters: $T_1=8$, $T_2=12$, $\delta_1=1$, $\delta_2=0.5$, $\theta_1=1$, $\theta_2=0.4$, $\sigma_1=1.2$, $\sigma_2=1.2$, $\lambda_1=0.05$, $\alpha=10$, $\beta=3$, p = 8000, D = 2000, C_R = 40, S = 100, $r_1=0.03$, H = 5, and $C_A=50$. By solving Eq. (8), we have $Q^*=41$, $\mu_1^*=7.7$, and $\mu_2^*=10.9$ with $ETC(\mu_1, \mu_2, Q)=63.14$.

Table 1 presents sensitivity results. We have some conclusions:

1. The P, S, C_A, k_{12}, θ_1, θ_2, and λ have an important effect on EMQ.
2. The S, H, k_{12}, θ_2, λ, m_1, and m_2 have an important effect on mean of characteristic 1.
3. The S, C_R, k_{22}, θ_2, λ, σ_2, and m_2 have an important contribution for mean value of characteristic 2.
4. The P, S, C_A, k_{11}, k_{12}, k_{22}, θ_1, θ_2, λ, H, C_R, D, r, m_1, m_2, σ_1, and σ_2 have an important effect on the expected total cost.

4. Conclusions

We have addressed the extended production problem with defective product and time value of money. We should extend the modified one with quality, reliability, and maintenance for further study.

Table 1

Sensitivity results

k_{11}	Q	μ_1	μ_2	ETC
24	41	7.6	10.9	51.47
36	41	7.8	10.8	74.53
k_{12}	Q	μ_1	μ_2	ETC
16	51	7.6	10.9	260.16
24	34	5.6	11.2	0.10
k_{22}	Q	μ_1	μ_2	ETC
12	41	7.8	10.5	53.35
18	41	7.6	11.1	70.78
δ_1	Q	μ_1	μ_2	ETC
0.8	41	7.7	10.9	63.06
1.2	41	7.7	10.9	63.22
δ_2	Q	μ_1	μ_2	ETC
0.4	41	7.7	10.9	63.12
0.6	41	7.7	10.9	63.15
θ_1	Q	μ_1	μ_2	ETC
0.8	51	7.7	10.9	349.71
1.2	33	7.7	10.9	1296.64
θ_2	Q	μ_1	μ_2	ETC
0.32	51	7.7	10.9	298.15
0.48	34	5.6	10.2	2.47
λ	Q	μ_1	μ_2	ETC
0.04	20	7.7	10.9	102.79
0.06	95	6.3	11.4	0.87
α	Q	μ_1	μ_2	ETC
8	41	7.7	10.9	61.13
12	41	7.7	10.9	65.14
β	Q	μ_1	μ_2	ETC
2.4	41	7.7	10.9	63.13
3.6	41	7.7	10.9	63.14

Contd...

r	Q	μ_1	μ_2	ETC
0.024	43	7.7	10.9	320.35
0.036	42	7.7	10.9	519.62
p	Q	μ_1	μ_2	ETC
6400	32	7.7	10.9	404.22
9600	49	7.7	10.9	27.14
D	Q	μ_1	μ_2	ETC
1600	41	7.7	10.9	86.39
2400	41	7.7	10.9	39.88
S	Q	μ_1	μ_2	ETC
80	36	6.4	11.3	0.23
120	46	7.7	10.9	524.19
C_A	Q	μ_1	μ_2	ETC
40	38	7.7	10.9	202.76
60	43	7.7	10.9	639.86
σ_1	Q	μ_1	μ_2	ETC
0.96	41	7.7	10.9	47.50
1.44	41	7.72	10.8	82.27
σ_2	Q	μ_1	μ_2	ETC
0.96	41	7.7	10.88	55.30
1.44	41	7.7	10.82	72.70
C_R	Q	μ_1	μ_2	ETC
32	41	7.7	11.1	42.47
48	41	7.7	10.6	81.58
H	Q	μ_1	μ_2	ETC
4	41	7.7	10.9	0.68
6	41	7.7	10.9	132.90
m_1	Q	μ_1	μ_2	ETC
6.4	41	5.9	11.0	0.15
9.6	41	9.3	10.9	127.11
m_2	Q	μ_1	μ_2	ETC
9.6	41	6.6	8.6	1.11
14.4	41	7.7	13.3	159.10

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