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On the decision making under certainty

Stefan M. Stefanov
Department of Informatics
Neofit Rilski South-West University
2700 Blagoevgrad
Bulgaria

Abstract

In this paper, the decision making theory is considered. Some basic concepts are introduced, elements of the decision process are discussed, and decision making under certainty is studied.

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1. Introduction

In the personal, public, social, economic, and political life, certain decisions have often to be made. In order to make decisions that are of regional, national or international importance, special boards are constituted: councils, commissions, parliaments, governments, etc. The *decision making theory* is one of the important elements of management.

Most decision making theories are connected with other mathematical theories, for example, operations research, statistical theory of decision making, two-person matrix games, game theory and the theory of collective (group, collaborative) decision making involving more than two people, dynamic systems theory (in multi-step decision making), etc.

The following difficulties arise in the decision-making process.

1. There are many criteria, which often are inconsistent with each other. For example, when designing a new product for an aircraft, there

E-mail: **stefm@swu.bg**

are requirements for a minimum mass, maximum reliability, and minimum value of this product. These criteria are contradictory to each other, and therefore the question for a “compromise” between them arises. This problem is connected with the multi-objective (vector) optimization (see, e.g., [Stefanov, 29]).

2. High degree of uncertainty due to insufficient information for informed decision making.

Decision processes are also connected with matrix games (see, e.g., [Stefanov, 27, 28]).

The topics and problems, considered in this paper, and related to them, are considered in references [1]-[32], etc.

In Section 2, elements of the decision making process are considered, and classification of the decision problems is given. In Section 3, the decision making process under certainty is studied and an illustrative example is presented. In Section 4, contents of the paper is outlined.

2. Elements of the process of decision making and classification of problems

Any decision making process is characterized by the following elements:

1. A decision maker who has some responsibility for the consequences of that decision.
2. A set of variables whose values are chosen by the decision maker. These variables are called *governing factors* or *strategies*.
3. A set of variables depending on the choice of the governing factors (strategies). These variables are called *output variables* for the decision making situation or *decision outputs*.
4. A set of variables whose values are not regulated by the decision maker. These variables can remain fully determined when solving some problems, and in such a case, they are called *parameters* of the problem. In other cases, they can be changed independently of the decision maker, and in such cases, they are called *perturbations* of the environment.
5. Mathematical model of the decision problem, which is a set of relations, with which the governing factors and the parameters of the considered problem are connected with the output variables.

6. Constraints that describe the requirements, imposed on the output variables and governing factors by the situation of decision making.
7. Objective function (optimality criterion). With the use of this function, the properties of the decision made are evaluated. The objective function depends on the governing factors according to the mathematical model of the decision problem.

A classification of the theories (models) of decision making is as follows:

- according to the number of persons, involved in the decision-making process: with one, two, more than two persons;
- according to the nature and the degree of uncertainty of the input information: completely determined, probabilistic, undetermined;
- according to the number of stages, through which the decision is made: one or several stages (single-step and multi-step decision making);
- according to the number of criteria, on the basis of which the reasonableness of a decision is determined: single-criteria and multi-criteria;
- according to the degree of variability of the parameters and the external perturbations: static and dynamic. If the parameters and the external perturbations remain unchanged during the time, then the mathematical model is static. Otherwise, the model is dynamic.

  Static models can be described by graphs, tables, algorithms for calculating output variables, etc.

  Dynamic models are described by various classes of differential equations.
- according to the type of external perturbations: deterministic and random. If the perturbations are deterministic, they can be considered as parameters of the considered decision problem. If the perturbations are random, then we get a stochastic decision making model. In this case, the output parameters are also random. In general, stochastic models describe real phenomena better than deterministic models.

  etc.

An important role in the decision problems is played by constraints. Depending on the mathematical model, the constraints (the feasible region) can be as follows:

1. Static model with continuous governing factors $\mathrm{x} = (x_1, \ldots, x_n)$. The constraints can be:

   a) linear

   $$G_l = \{\mathrm{x} : A\mathrm{x} \le b, x_j \ge 0, h'_j \le x_j \le h''_j, j = 1, \ldots, n\}, \quad (1)$$

   where A is an $m \times n$ matrix, x is an $n-$ vector, and b is an $m-$ vector.

   b) nonlinear

   $$G_n = \{\mathrm{x} : g_i(\mathrm{x}) \le b_i, i = 1, \ldots, m, x_j \ge 0, j = 1, \ldots, n\}, \quad (2)$$

   where $g_i(\mathrm{x})$, $i = 1, \ldots, m$, are continuous differentiable functions of n variables $\mathrm{x} = (x_1, \ldots, x_n)$.

   c) mixed constraints of type a) and b).

2. Static model with discrete governing factors:

   $$D = \left\{ \mathrm{x} : g_i(\mathrm{x}) = \sum_{j=1}^{n} g_i(x_j) \le b_i, i = 1, \ldots, m \right\}, \quad (3)$$

   where the variables x_i, $i = 1, \ldots, n$, can only take the values from some finite set X. (For example, X can be the set of the wattage values that household light bulbs have. In this case, X consists of a finite number of values, for example $X = \{25, 40, 60, 75, 100, 150, 200\}$.)

3. Dynamic model in the time interval $\tau \in [t_0, T]$, where t_0 is the initial moment of time, and T is the final moment of time.

If τ is continuous, we get a *dynamic model with continuous time*, which is described by differential equations.

If the time interval τ, in which a decision is made, is defined by a certain step (for example, when bus travels along a certain route at 15-minute intervals), we get a *dynamic model with discrete time*.

Depending on the conditions of the environment and how informed the decision maker is, the following classification of the decision problems can be proposed:

- decision making under certainty,

- decision making in risk,
- decision making under uncertainty,
- decision making in conflict situations.

The first class of the decision problems is considered in the next section, and other classes will be considered in a separate paper.

3. Decision making under certainty

The decision-making process under certainty is characterized by a single-valued or deterministic relationship between the decision made and the output (the result) of this decision. The main difficulty is that the results have to be compared with respect to *several* criteria (objectives). In such cases, the decision problem arises under the so-called *vector optimality criterion*. The corresponding mathematical models usually are optimization problems with several objective functions, and they are called *multi-criteria decision problems* (*multi-criteria optimization problem*).

Consider the problem of choosing the most appropriate decisions. This problem is important, especially when there is an uncountable set of feasible governing factors (strategies), satisfying constraints that are described by the mathematical model of the decision problem. The totality of governing factors, satisfying constraints that are described by the mathematical model of the decision problem, is said to be the *set of alternatives* A, among which the choice is made. The alternatives are denoted by x and if necessary, it is indicated what constraints the alternatives satisfy. For example, $x \in G_l$ means that a decision problem subject to linear constraints is considered.

In order to compare different alternatives and to choose the best one, a property (or a set of properties) of the evaluated alternatives must be chosen, a quantitative measure of the properties must be defined, and the choice must be done by using the quantitative measure. This quantitative measure is called the *utility function* $u(\mathrm{x})$, and the decision rules use the so-called *utility theory*, developed by John von Neumann and Oscar Morgenstern ([7]). The mathematical basis of this theory is a system of axioms, stating that there exists a measure of the value, which allows the alternatives (results of the decision) be ranked. This measure is called the *utility function* or *utility of the results*.

Axioms.

1. The result (the alternative) x_i is preferable to the alternative x_j, which is written as follows $x_i > x_j$, if and only if $u(x_i) > u(x_j)$, where $u(x_i)$ and $u(x_j)$ are the utilities of the alternatives x_i and x_j, respectively.
2. Transitivity: if $x_i > x_j$ and $x_j > x_k$, then $x_i > x_k$ and $u(x_i) > u(x_k)$.
3. Linearity: if a result can be represented in the form $x = (1-l)x_1 + lx_2$, where $0 \le l \le 1$, then $u(x) = (1-l)\, u(x_1) + lu(x_2)$.
4. Additivity: if $u(x_1, x_2)$ is the utility of achieving simultaneously the results x_1 and x_2, then $u(x_1, x_2) = u(x_1) + u(x_2)$.

Similarly, if we have n results $x_1, \ldots, x_n$, which are achieved simultaneously, then

$$u(x_1, \ldots, x_n) = \sum_{i=1}^{n} u_i(x_i).$$

Example 1: Let an expert has ranked five results $x_1, \ldots, x_5$ by assigning them the following ratings: $u_0(x_1) = 7$, $u_0(x_2) = 4$, $u_0(x_3) = 2$, $u_0(x_4) = 1.5$, $u_0(x_5) = 1$.

Considering the possible choices, he proposed the following evaluations of some combinations:

1) $x_1 < x_2 + x_3 + x_4 + x_5$,
2) $x_1 < x_2 + x_3 + x_4$,
3) $x_1 < x_2 + x_3 + x_5$,
4) $x_1 > x_2 + x_3$,
5) $x_2 < x_3 + x_4 + x_5$,
6) $x_2 > x_3 + x_4$,
7) $x_3 > x_4 + x_5$.

Utility of the results must be evaluated so that all the above inequalities be satisfied.

Substituting the initial estimates in inequality 7), we get

$$u_0(x_3) = 2 < u_0(x_4) + u_0(x_5) = 1.5 + 1 = 2.5.$$

Therefore 7) is not satisfied.

Change the utility of the result x_3 as follows $u_1(x_3) = 3$ and check inequality 6):

$$u_0(x_2) = 4 < u_1(x_3) + u_0(x_4) = 4.5.$$

Therefore 6) is also not satisfied.

Take $u_1(x_2) = 5.$ For this choice, 5) is satisfied. Check the inequality 4):

$$u_0(x_1) = 7 < u_1(x_2) + u_1(x_3) = 5 + 3 = 8.$$

Therefore 4) is not satisfied. Hence we take $u_1(x_1) = 8.5.$ Now the inequalities 1), 2), 3), 4), and 5) are satisfied. Check the inequalities 6) and 7) for the new values:

$$5 > 3 + 1.5 \quad \text{and} \quad 3 > 1.5 + 1,$$

that is, both inequalities 6) and 7) are now satisfied.

Therefore, the final ratings of the utility of the results are

$$u_1(x_1) = 8.5, \quad u_1(x_2) = 5, \quad u_1(x_3) = 3, \quad u_1(x_4) = 1.5, \quad u_1(x_5) = 1.$$

4. Conclusions

The decision problems are considered in this paper. Some basic concepts are introduced, elements of the decision process are discussed, and classification of the decision problems is given. Decision making under certainty is studied, and an illustrative example is presented.

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