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Joint settings of economic manufacturing quantity (EMQ), process variance, quality investment, and parameters of acceptance sampling

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Abstract

This work discusses inventory model based on EMQ, process variance setting, quality investment and parameters of sampling inspection. It also considers a long-term quality improvement tool for decreasing bias of product.

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Keywords: *EMQ, Acceptance sampling, Process variance, Investment of quality.*

1. Introduction

Traditional EMQ model is based on the perfect production without bas unit. Porteus [1] & Rosenblatt & Lee [2, 3] have discussed the imperfect EMQ one.

Taguchi's [4] loss function can evaluate new quality. Shin et al.'s [5] work applied Taguchi's function with production expense. Chen & Chou [6] proposed the mean setting with quality and expense. Chen & Chou [7] discussed production run and credit period for inventory's model.

Improvement activity is a long-term tool for increasing product quality. Tsou [8], Chen & Tsou [9], Ganeshan et al. [10], and Hong et al. [11] have applied this method.

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We address investment of quality & variance in this work. The optimal EMQ, process variance, investment of quality, and parameters of acceptance sampling need to be obtained.

2. Model

Consider to maximize

$$ETP(Q_1,\sigma_0, Inv, n_1, d_0) = M_1 \cdot W_{22} - S_t \cdot \frac{M_1}{Q_1} - 0.5 \cdot Q_1 \cdot \left(1 - \frac{O_1}{I_1}\right) h_1 \tag{1}$$

subject to

$$\max_{0 \le p \le 1} AOQ = p_L \tag{2}$$

where

$$W_{22} = \left\{(A_{22} - A_{11}) + RI \cdot p_1 + \left(1 - \frac{n_1}{Q_1}\right) IC + \left(1 - \frac{n_1}{Q_1}\right)(LF1 - LF0) - \frac{n_1 p_1 RI}{Q_1}\right\} P(X_1 \le d_0) + (A_{11} - RI \cdot p_1 - IC - C_1 m_I - LF1) \tag{3}$$

$$LF0 = k_1 \left\{[(m_I - T_1)^2 + \sigma_0^2]\Phi\left(\frac{T_1 - m_I}{\sigma_0}\right) - \sigma_0 (m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right\} + k_2 \left\{[(m_I - T_1)^2 + \sigma_0^2]\left[1 - \Phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right] + \sigma_0 (m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right\} \tag{4}$$

$$\begin{aligned} LF1 = k_1 \Bigg\{ & [(m_I - T_1)^2 + \sigma_0^2]\left[\Phi\left(\frac{T_1 - m_I}{\sigma_0}\right) - \Phi\left(\frac{LSL - m_I}{\sigma_0}\right)\right] - \\ & \sigma_0 \left[(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right) - (m_I - 2T_1 + LSL)\phi\left(\frac{LSL - m_I}{\sigma_0}\right)\right]\Bigg\} + \\ k_2 \Bigg\{ & [(m_I - T_1)^2 + \sigma_0^2]\left[\Phi\left(\frac{USL - m_I}{\sigma_0}\right) - \Phi\left(\frac{T_1 - m_I}{\sigma_0}\right)\right] - \\ & \sigma_0 \left[-(m_I - T_1)\phi\left(\frac{T_1 - m_I}{\sigma_0}\right) + (m_I - 2T_1 + USL)\phi\left(\frac{USL - m_I}{\sigma_0}\right)\right]\Bigg\} \end{aligned} \tag{5}$$

$$m_I^2 = T_1^{\,2} + (m_0^2 - T_1^{\,2})\exp(-\beta Inv) \tag{6}$$

$$p_1 = 1 - \Phi\left(\frac{USL - m_0}{\sigma_0}\right) + \Phi\left(\frac{LSL - m_0}{\sigma_0}\right) \tag{7}$$

$$P(X_1 \le d_0) = \sum_{x=0}^{d_0} \frac{e^{-n_1 p_1}(n_1 p_1)^{x_1}}{x_1!} \tag{8}$$

$ETP(Q_1, \sigma_0, Inv, n_1, d_0)$] is expected total profit; W_{22} is ETP; A_{11} is sold price with rejected product; A_{22} is sold price for accepted product; Q_1 is production size; X is no. of bad unit; n is sampling no.; d_0 is available no. of bad unit; IC is inspection expense; $RI$is replacement expense; p_1 is bad prob.; C_1 is expense of material; $LF0$ is expected loss for no inspection; $LF1$ is expected loss for inspection; $\phi(\cdot)$ is p.d.f. of SND; $\Phi(\cdot)$ is c.d.f. of SND; LSL is min level; USL is max level; m_0 is process expectation; σ_0 is SD; M_1 is sold product (time); O_1 is sold product (day); I_1 is production product per day; S_t is preparing expense; h_1 is store expense; k_1 & k_2 are coefficients of loss; Inv is quality investment; m_I is improved mean; β is declined coefficient.

Consider the following steps:

Step 1. Let maximum Q_1 be $Q_{\max}$, maximum Inv be $Inv_{\max}$, and initial $Q_1 = n_1$.

Step 2. Let initial $Inv = 0$.

Step 3. From Dodge-Romig, one has (n_1, d_0) satisfying Eq. (2).

Step 4. Search interval $0.01 \le \sigma_0 \le (USL - LSL)/8$, we have $ETP(Q_1, \sigma_0, Inv, n_1, d_0)$ from Eq. (1).

Step 5. For $Inv = Inv+1$, continue Steps 3-4 with Inv be $Inv = Inv_{\max}$.

Step 6. Set $Q_1 = Q_1 + 1$ and continue Steps 2-5 with $Q_1 = Q_{\max}$. We have $(Q_1, \sigma_0, Inv, n_1, d_0)$ with maximum $ETP(Q_1, \sigma_0, Inv, n_1, d_0)$

3. Numerical Example

Table 1 presents the change of $\pm 20\% \sim \pm 30\%$ for the basic numerical data. From Eqs. (1)-(8), we have $Q_1 = 241$, $\sigma_0 = 0.01$, $Inv = 44$, $n_1 = 18$, $d_0 = 0$ and $ETP(Q_1, \sigma_0, Inv, n_1, d_0)$ = 23486.35.

Table 1 presents some results:

1. The variance approaches to zero, i.e., $\sigma_0^2 \to 0$.
2. The M_1, O_1, I_1, S_t, h_1, T_1, and β have a large contribution on EMQ.
3. The T_1 and β have a large contribution on Inv.
4. Parameters C_1, A_2, M_1, and T_1 show a large contribution on ETP.

4. Conclusions

This work considers investment of quality and variance. Further study should consider the integrated design with process capability, unknown parameters of production process, and tolerance.

Table 1

Some results

C_1	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
0.8	0.01	45	242	(18, 0)	31484.55
1.2	0.01	44	241	(18, 0)	15488.30
A_{22}	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
64	0.01	44	241	(18, 0)	10686.35
96	0.01	44	241	(18, 0)	36286.35
A_{11}	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
54	0.01	44	241	(18, 0)	23486.35
81	0.01	44	241	(18, 0)	23486.35
M_1	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
640	0.01	44	215	(18, 0)	18743.44
960	0.01	45	267	(18, 0)	28234.46
O_1	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
64	0.01	43	179	(18, 0)	23321.60
96	0.01	48	545	(18, 0)	23756.97
I_1	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
90	0.01	46	329	(18, 0)	23610.36
120	0.01	43	186	(18, 0)	23345.91
S_t	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
16	0.01	44	234	(18, 0)	23499.78
24	0.01	45	250	(18, 0)	23473.34
h_1	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
8	0.01	45	270	(18, 0)	23537.68
12	0.01	44	220	(18, 0)	23440.22
RI	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
24.4	0.01	44	241	(18, 0)	23486.35
36.6	0.01	44	241	(18, 0)	23486.35
k_1	σ_0	Inv	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
160	0.01	43	239	(18, 0)	23494.45

Contd...

240	0.01	46	244	(18, 0)	23479.48
k_2	σ_0	*Inv*	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
240	0.01	45	242	(18, 0)	23486.60
360	0.01	44	241	(18, 0)	23486.13
T_1	σ_0	*Inv*	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
49.5	0.01	37	229	(18, 0)	23909.68
50.5	0.01	49	250	(18, 0)	23073.34
β	σ_0	*Inv*	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
0.08	0.01	50	251	(18, 0)	23442.96
0.12	0.01	37	230	(18, 0)	23511.52
IC	σ_0	*Inv*	Q_1	(n_1, d_0)	$ETP\ (Q_1, \sigma_0, Inv, n_1, d_0)$
0.4	0.01	44	238	(18, 0)	23492.35
0.6	0.01	44	244	(18, 0)	23480.43

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